produces an increased electron injection from the source and a pronounced increase in the current drive as the drain voltage increases. On the other hand, the electric field intensity exhibits a sharp increase at the drain end of the gate for the HEMT with modified p-channel doping. Both electric field intensity and electron concentration at the source end for the HEMT with modified p-channel doping are much lower and less sensitive to the applied drain voltage compared with corresponding values for the HEMT's with both a modified n channel and an undoped channel. The low electron density and less spread in the electric field profile do not significantly increase the electron injection into the channel. As a result, drain current is less sensitive to applied drain voltage and output conductance is considerably reduced for the device with modified p-channel doping.

In summary, we have studied the effects of modified n- and p-channel doping on the device characteristics of 0.25 μm gate length $In_{0.52}Al_{0.48}As/In_{0.53}Ga_{0.47}As/InP$ HEMT's. As gate voltage increases, the transconductance for the HEMT with modified p channel doping is comparable to that of the undoped channel HEMT and is much higher than that for the HEMT with modified n-channel. Simultaneously, devices with modified p channel doping exhibit reduced output conductance compared with those for the modified n channel and undoped channel counterparts. Ensemble Monte Carlo simulations reveal that enhanced carrier confinement and reduced spread in the channel field profile are the primary physical mechanisms for the experimentally-observed improvement of the HEMT with modified p-channel doping. Results from this study indicate that employing proper modified p-channel doping is a promising approach for improved $In_{0.52}Al_{0.48}As/In_{0.53}Ga_{0.47}As/InP$ HEMT operations.

References

[1] U. K. Mishra, J. F. Jensen, A. S. Brown, M. A. Thompson, L. M. Jelloian, and R. S. Beaubien, "Ultra-high-speed digital circuit performance in 0.2 μm gate-length AlInAs/GaInAs HEMT technology," *IEEE Electron Device Lett.*, vol. EDL-9, pp. 482–484, 1988.

[2] G.-G. Zhou, A. Fischer-Colbrie, J. Miller, Y.-C. Pao, B. Hughes, L. Studebake, and J. S. Harris, Jr., "High output conductance of InAlAs/InGaAs/InP MODFET due to weak impact ionization in the InGaAs channel," *IEDM Tech. Dig.*, p. 247, 1991.

[3] Y.-C. Pao, C. Nishimoto, M. Riaziat, R. Majidi-Ahy, N. G. Bechtel, and J. S. Harris, Jr., "Impact of surface layer on $In_{0.52}Al_{0.48}As/In_{0.53}Ga_{0.47}As/InP$ high electron mobility transistors," *IEEE Electron Device Lett.*, vol. 11, pp. 312–314, 1990.

[4] K. H. G. Duh, P.-C. Chao, S. M. J. Liu, P. Ho, M. Y. Kao, and J. M. Ballingall, "A super low noise 0.1 μm T-gate InAlAs-InGaAs-InP HEMT," *IEEE Microwave Guided Wave Lett.*, vol. 1, pp. 114–116, 1991.

[5] J. B. Kuang, P. J. Tasker, S. Ratanaphanyarat, W. J. Schaff, L. F. Eastman, G. W. Wang, Y. K. Chen, O. A. Aina, H. Hier, and A. Fathimulla, "Low frequency and microwave characterization of submicron-gate $In_{0.52}Al_{0.48}As/In_{0.53}Ga_{0.47}As/In_{0.52}Al_{0.48}As$ heterojunction metal-semiconductor field-effect transistor grown by molecular-beam epitaxy," *J. Appl. Phys.*, vol. 66, pp. 6168–6174, 1989.

[6] Y.-C. Pao and J. S. Harris, Jr., "Low-conductance drain (LCD) design of InAlAs/InGaAs/InP HEMT's," *IEEE Electron Device Lett.*, vol. 13, pp. 535–537, 1992.

[7] A. S. Brown, U. K. Mishra, C. S. Chou, C. E. Hooper, M. A. Melendes, M. Thompson, L. E. Larson, S. E. Rosenbaum, and M. J. Delaney, "AlInAs-GaInAs HEMT's utilizing low temperature AlInAs buffers grown by MBE," *IEEE Electron Device Lett.*, vol. 10, pp. 565–567, 1989.

[8] U. K. Mishra, L. M. Jelloian, M. Liu, M. Thompson, S. E. Rosenbaum, and K. W. Kim, "Effect of n and p channel doping on the I-V characteristics of AlInAs-GaInAs HEMT's," *Inst. Phys. Conf. Ser.*, no. 120, pp. 287–292, 1991.

[9] K. W. Kim, H. Tian, and M. A. Littlejohn, "Analysis of delta-doped and uniformly doped AlGaAs/GaAs HEMT's by ensemble Monte Carlo simulation," *IEEE Trans. Electron Devices*, vol. 38, pp. 1737–1742, 1991.

[10] A. P. Long, P. H. Beton, and M. J. Kelly, "Hot-electron transport in $In_{0.53}Ga_{0.47}As$," *J. Appl. Phys.*, vol. 62, pp. 1842–1849, 1987; K. F. Brennan and D. H. Park, "Theoretical comparison of electron real-space transfer in classical and quantum two-dimensional heterojunction structure systems," *J. Appl. Phys.*, vol. 65, pp. 1156–1163, 1989.

[11] H. S. Kim, H. Tian, K. W. Kim, and M. A. Littlejohn, "Electron velocity-field characteristics of $In_{0.52}Al_{0.48}As$," *Appl. Phys. Lett.*, vol. 61, pp. 1202–1204, 1992.

[12] J. R. Waldrop, E. A. Kraut, C. W. Farley, and R. W. Grant, "Measurement of $InP/In_{0.53}Ga_{0.47}As$ and $In_{0.53}Ga_{0.47}As/In_{0.52}Ga_{0.48}As$ heterojunction band offsets by x-ray photoemission spectroscopy," *J. Appl. Phys.*, vol. 69, pp. 372–378, 1991.

Origin of the Difference Between the Capacitance Intercept Voltage and the Built-In Potential

P. Van Mieghem

***Abstract*—For a diode with an abrupt, constant doping profile, the difference between the built-in potential V_{bi} and the capacitance intercept voltage V_{int} is shown to be at least $2(k_BT/q)$. The difference lies in the fact that $q \cdot V_{bi}$ reflects the potential energy of the equilibrium space charge in the diode whereas $q \cdot V_{int}$ is related to the work needed to create this space charge.**

The built-in potential V_{bi} of a pn-junction is a basic parameter appearing in many fundamental formulas that describe electrical phenomena in semiconductor devices [1]. Despite its important role it is not straightforward how to determine V_{bi} experimentally. A common technique is to extract the capacitance intercept voltage V_{int} from a junction capacitance measurement $C(V)$. This is broadly considered a good measure for V_{bi}. Although the theory of the capacitance intercept voltage has a long history (see [2] for the references) a coherent physical understanding of both quantities has been lacking.

In this brief, we investigate the relation between V_{bi} and V_{int}. The aim of the paper is basically of device physical interest although the determination of the doping concentration N and the bandgap E_g from capacitance measurements may benefit from the presented physical interpretation. For mathematical simplicity, we restrict ourselves to abrupt junctions with constant doping profiles in one dimension. For these diodes V_{int} is defined as the asymptotic voltage for which $C^{-2}(V_{int}) = 0$. As there exists no voltage for which $C^{-2}(V)$ vanishes, V_{int} is extrapolated from capacitance values from the reverse and low forward biased region.

Manuscript received February 19, 1993; revised August 16, 1993. This work was supported by NATO and the Belgian National Fund for Scientific Research (NFWO). The review of this brief was arranged by Associate Editor J. R. Hauser.

The author is with the Department of Electrical Engineering and Computer Science, Massachusetts Institute of Technology, Cambridge, MA 02139, on leave from IMEC, B-3001 Heverlee, Belgium.

IEEE Log Number 9212789.

Let us start with the classical formula's in the literature [1] that assume Boltzmann statistics,

$$V_{bi} = E_g + \frac{k_B T}{q}\left\{\ln\left(\frac{N_D}{N_c}\right) + \ln\left(\frac{N_A}{N_v}\right)\right\} = \frac{k_B T}{q}\ln\left|\frac{N_D N_A}{n_i^2}\right| \quad (1)$$

and

$$V_{int} = E_g + \frac{k_B T}{q}\left\{\ln\left(\frac{N_D}{N_c}\right) + \ln\left(\frac{N_A}{N_v}\right) - 2\right\} \quad (2)$$

where the bandgap E_g is expressed in meV.

These expressions differ by an amount $\Delta = V_{bi} - V_{int} = 2(k_B T/q)$. The difference Δ has been attributed to the influence of free carriers in the Debye regions that compensate a fraction of the depleted ion charge. This results in a lowering of V_{int} [1]. But, the space charge in the reverse biased voltage regime from which V_{int} is extracted, is more depleted than the equilibrium space charge. Hence, we expect even more pronounced free carrier effects for V_{bi}. Why then do we not see their influence on V_{bi}? This observation poses the question as what Δ means or what phenomenon allows for this difference. Intuitively, one is eager to conclude that both quantities should be the same and that V_{int} is the only accurate (and simple) way to measure V_{bi} (as is usually done as already mentioned above). Equation (2) for V_{int} is exact under the assumptions of constant quasi-fermi levels over the diode and Boltzmann statistics. The former assumption has been justified in [2] for the applied voltage interval from which V_{int} is obtained. This suggests that we focus on V_{bi} to derive a more accurate expression than (1).

In order to gain a deeper understanding, it is instructive to briefly review the physical meaning of V_{bi}. The built-in potential V_{bi} arises from an electrostatic reaction to charge diffusion. Specifically, V_{bi} is the potential internally generated over the whole diode structure by an electric field $E(x)$ that precisely balances the diffusion force in the case that no external fields are applied. This definition implies that V_{bi} is the only electrostatic potential difference between the left and right diode contacts obeying the condition that the current through the diode is exactly zero. A straightforward derivation is then as follows. From the exact balances between electrical field and diffusion force,

$$E(x) = -\frac{k_B T}{q}\frac{1}{n}\frac{dn}{dx} \quad (3)$$

we find by integration over the n-p diode structure (say from the left contact $x = -L$ in the n-type region over the metallurgical junction at $x = 0$ to the right contact $x = K$ in the p-type region)

$$V_{bi} = -\int_{-L}^{K} E(x)\,dx = \frac{k_B T}{q}\ln\left|\frac{n(-L)}{n(K)}\right|. \quad (4)$$

Since at equilibrium (and only for Boltzmann statistics), we have

$$n(K)\cdot p(K) = n_i^2 = N_c N_v \exp\left(-E_g/k_B T\right) \quad (5)$$

yielding

$$V_{bi} = E_g + \frac{k_B T}{q}\left\{\ln\left(\frac{n(-L)}{N_c}\right) + \ln\left(\frac{p(K)}{N_v}\right)\right\}. \quad (6)$$

Invoking the neutrality condition, equation 30 on p. 27 in [1] gives

$$V_{bi} = E_g + \frac{k_B T}{q}\left\{\ln\left|\frac{N_D}{2N_c}\left(1 + \sqrt{1 + \frac{4n_i^2}{N_D^2}}\right)\right| + \ln\left|\frac{N_A}{2N_v}\left(1 + \sqrt{1 + \frac{4n_i^2}{N_A^2}}\right)\right|\right\}. \quad (7)$$

Applying $\sqrt{1+x} \approx 1 + x/2 - x^2/8 + o(x^2)$ and further $\ln(1+x) \approx x + x^2/2 + o(x^2)$ for $|x| < 1$, we arrive at

$$V_{bi} = E_g + \frac{k_B T}{q}\left\{\ln\left(\frac{N_D}{N_c}\right) + \ln\left(\frac{N_A}{N_v}\right)\right\} + \frac{k_B T}{q}\left\{\frac{n_i^2}{N_D^2} + \frac{n_i^2}{N_A^2} - \frac{n_i^4}{2N_D^4} - \frac{n_i^4}{2N_D^4} + o\left(\frac{n_i^4}{N_D^4} + \frac{n_i^4}{N_A^4}\right)\right\}. \quad (8)$$

The last terms are at most of order $n_i^2/N_D^2 + n_i^2/N_A^2 << 1$ and more than sufficient to justify (1). In addition their sign is such that Δ becomes even larger than $2(k_B T/q)$. In conclusion so far, within the assumption of Boltzmann statistics, we find that the classical expression of V_{bi} is accurate within a fraction much smaller than $2(k_B T/q)$. However, this derivation does not give a hint to enravel a deeper physical reason for the difference Δ.

In order to investigate the origin of the discrepancy, we now turn our attention to V_{int}. Only for abrupt junctions with a constant doping profile, a general description assuming the Fermi–Dirac statistics and an arbitrary density of state $g(E)$ can be presented. From previous work [2], [3] on the capacitance of an abrupt diode (written for a symmetrical junction $N_D = N_A = N$ for reasons explained in [2]), we find

$$V_{int} = E_g + \frac{k_B T}{q}\left(R_c^{-1}\left(\frac{N}{N_c}\right) + R_v^{-1}\left(\frac{N}{N_v}\right) - Z(N)\right) \quad (9)$$

while

$$Z(N) = \frac{N_c}{N}\cdot \mathfrak{I}_c\left(R_c^{-1}\left(\frac{N}{N_c}\right)\right) + \frac{N_v}{N}\cdot \mathfrak{I}_v\left(R_v^{-1}\left(\frac{N}{N_v}\right)\right) \quad (10)$$

where the notation is clarified in the appendix. The built-in potential, written in the same general formalism, is

$$V_{bi} = \frac{1}{N_c}\frac{k_B T}{q}\int_{n(-L)}^{n(K)} \frac{dn}{\mathcal{P}\left(\frac{dg(\xi)}{d\xi}, R_c^{-1}\left(\frac{n}{N_c}\right)\right)} \quad (11)$$

or within the same approximation as (8) (neglecting the very small correction terms),

$$V_{bi} = E_g + \frac{k_B T}{q}\left(R_c^{-1}\left(\frac{N}{N_c}\right) + R_v^{-1}\left(\frac{N}{N_v}\right)\right) \quad (12)$$

As a result, we observe that the difference $\Delta = (k_B T/q)\,Z(N)$ is now doping concentration dependent and is plotted for GaAs in Fig. 1. For low doping concentrations, $Z(N)$ tends to 2 (the Boltzmann non-degenerate regime), while it increases significantly (roughtly $Z(N) \sim N^{2/3}$) for heavy doping. Now, expression (9) can be transformed into an integral as demonstrated in appendix B, yielding

$$V_{int} = E_g + \frac{k_B T}{q}\left(\frac{1}{N}\int_0^N \left[R_c^{-1}\left(\frac{n}{N_c}\right) + R_v^{-1}\left(\frac{n}{N_v}\right)\right] dn\right) \quad (13)$$

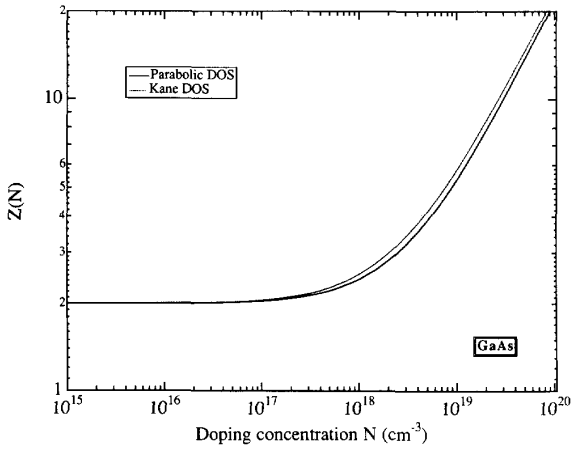

Fig. 1. The function $Z(N)$ calculated for a parabolic DOS and a tailed Kane DOS [5] in GaAs.

or in the assumption that the bandgap E_g does not depend on doping concentration (and thus disregarding heavy doping effects),

$$V_{\text{int}} = \frac{1}{N}\int_0^N \left[E_g + \frac{k_BT}{q}\left(R_c^{-1}\left(\frac{n}{N_c}\right) + R_v^{-1}\left(\frac{n}{N_v}\right)\right)\right] dn. \quad (14)$$

A quite interesting observation is that we can rewrite (14) as

$$V_{\text{int}}(N) = \frac{1}{N}\int_0^N V_{\text{bi}}(n)\, dn = \langle V_{\text{bi}}(n)\rangle_N \quad (15)$$

or, alternatively,

$$qN \cdot V_{\text{int}}(N) = \int_0^N V_{\text{bi}}(n)\, q \cdot dn. \quad (16)$$

The interpretation of (16) is as follows. For a certain doping concentration n, a well-defined space charge is built up in the diode that generates a potential $V_{\text{bi}}(n)$ over the diode. This state corresponds to a potential energy $q \cdot V_{\text{bi}}(n)$. The energy needed to increase the space charge at $q \cdot n$ by an amount $q \cdot dn$, equals $q \cdot dn$. $V_{\text{bi}}(n)$. Thus, from (16), we see that V_{int} is related to the total energy $qNV_{\text{int}}(N)$ needed to build up a space charge from 0 to the final concentration N. Equation (16) shows the essential difference between the built-in potential V_{bi} and the capacitance intercept voltage V_{int} for a doping concentration N: qV_{bi} represents the potential energy needed to add (or remove) an arbitrary small charge to (or from) the established space charge at doping concentration N while V_{int} is related to the work involved to create this space charge. Clearly, these two energies are in general not the same [4], hence V_{int} does not equal V_{bi}. In fact from (15), V_{int} equals the built-in potential averaged over all carrier concentration up to N, $\langle V_{\text{bi}}(n)\rangle_N$, which is always smaller than $V_{\text{bi}}(N) = \max_n V_{\text{bi}}(n)$.

The way in which V_{int} is obtained, is consistent with the interpretation above. The capacitance intercept voltage is found from extrapolation of $C^{-2}(V)$ in reverse (or low forward) biased region. For increasing negative applied voltages, an increasing space charge is built up. This information of the space charge built-up energy must be enclosed in V_{int}. More specific as suggested by its definition, V_{int} reflects the creation energy of the equilibrium space charge.

Finally, notice that (15) is general and also applies to Boltzmann statistics as readily may be verified. Equations (15) and (16) are proposed based on a particular doping profile. A proof of (15) and (16) for an arbitrary profile seems hardly possible due to the lack of a general solution—even assuming constant quasi-Fermi levels—of the nonlinear Poisson equation (of the form $d^2\phi/dx^2 = g(\phi(x)) + f(x)$, where $g(\phi(x))$ and $f(x)$ are related to the free carriers and doping profile, respectively) preventing accurate analytical calculations. Relying on the space charge layer approximation for this problem may introduce errors as large as the discrepancy between V_{int} and V_{bi} and is thus here inadequate.

Appendix A
The $\mathcal{P}$ Transform

In order to account for an arbitrary density of state (DOS) function, the $\mathcal{P}$ transform is introduced which is defined as

$$\mathcal{P}(g(\xi), y) = \int_{-\infty}^{\infty} \frac{g(\xi)}{1 + \exp(\xi - y)}\, d\xi = R(y) \quad (A1)$$

The number of electrons (holes) $n(p)$ in a semiconductor with a DOS $g_{c(v)}(E)$ and quasi-Fermi level $\zeta_{n(p)}$, then reads

$$\begin{cases} n = N_C \cdot \mathcal{P}\left(g_c(\xi), \dfrac{\zeta_n - E_c}{k_BT}\right) \\ p = N_V \cdot \mathcal{P}\left(g_v(\xi), \dfrac{-\zeta_p + E_v}{k_BT}\right). \end{cases} \quad (A2)$$

Particularly, if the density of states is parabolic, i.e., $g(E) = \sqrt{E}$ for $E > 0$ and $g(E) = 0$ for $E < 0$, the number of electrons written in the formalism of (A1) with Heavyside's step function $\theta(x)$, yields

$$n = N_C \cdot \mathcal{P}\left(\frac{2}{\sqrt{\pi}}\sqrt{\xi}\,\theta(\xi), \frac{\zeta_n - E_c}{k_BT}\right) = N_C F_{1/2}\left(\frac{\zeta_n - E_c}{k_BT}\right) \quad (A3)$$

where $F_p(x)$ is the Fermi–Direct integral of order p.

Finally, $y = R_\lambda^{-1}(z)$ denotes the inverse function or solution of $z = R_\lambda(y)$ with respect to y and the subscript λ refers either to the conduction band (c) or valence band (v), while the integrated $\mathcal{P}$ transform corresponds to

$$\mathfrak{I}_c(y) = \mathcal{P}\left(\int_{-\infty}^{\xi} g_\lambda(\tau)\, d\tau, y\right) \quad (A4)$$

with the following, easily verified property,

$$\int_a^y \mathcal{P}(g(\xi), u)\, du = \mathfrak{I}_c(y) - \mathfrak{I}_c(a) \quad (A5)$$

Appeendix B
The Equivalence Between (9) and (13)

The demonstration that (13) is equivalent to (9) uses the following identity of inverse $f^{-1}(y)$.

$$\int_a^b f^{-1}(y)\, dy = bf^{-1}(b) - af^{-1}(a) - \int_{f^{-1}(a)}^{f^{-1}(b)} f(y)\, dy \quad (B.1)$$

The proof of (B.1):

- Partial integration of the left hand side gives

$$\int_a^b f^{-1}(y)\, dy = y \cdot f^{-1}(y)\Big|_a^b - \int_a^b y \frac{d}{dy}[f^{-1}(y)]\, dy$$

- From the definition of an inverse function, we have $f(f^{-1}(y)) = y$ which yields after differentiation,

$$\frac{d}{dy}[f^{-1}(y)] = \frac{1}{f'(f^{-1}(y))}.$$

- Substitution into the integral and making a change in the independent variable $y = f(u)$ or $u = f^{-1}(y)$ finally leads to (B.1)

Using (B.1) with $f^{-1}(y) = R_c^{-1}(y/N_c)$ gives

$$\int_a^b R_c^{-1}(y/N_c)\,dy = b\left(R_c^{-1}\left(\frac{b}{N_c}\right)\right) - a\left(R_c^{-1}\left(\frac{a}{N_c}\right)\right) - N_c \int_{R_c^{-1}(a/N_c)}^{R_c^{-1}(b/N_c)} R_c(y)\,dy.$$

Since $R^{-1}(y) = O(\ln(y))$, by choosing $a = 0$, $a\,R^{-1}(a)$ vanishes and $R^{-1}(a)$ tends to minus infinity. Applying (A5) with $\mathfrak{I}_c(R^{-1}(a)) = 0$ as $a = 0$, we obtain with $b = N$,

$$\int_0^N R_c^{-1}\left(\frac{y}{N_c}\right) dy = N\left(R_c^{-1}\left(\frac{N}{N_c}\right)\right) - N_c\mathfrak{I}_c\left(R_c^{-1}\left(\frac{N}{N_c}\right)\right).$$

A similar relation can written for the valence band. By adding these together we establish the equivalence between (9) and (13).

As an example, we consider a parabolic DOS and Fermi-Dirac statistics (A3) to obtain

$$V_{\text{int}} = E_g + \frac{k_B T}{q}\left(F_{1/2}^{-1}\left(\frac{N}{N_c}\right) + F_{1/2}^{-1}\left(\frac{N}{N_v}\right) - Z_{\text{parab}}(N)\right)$$

with

$$Z_{\text{parab}}(N) = \frac{N_c}{N}\cdot F_{3/2}\left[F_{1/2}^{-1}\left(\frac{N}{N_c}\right)\right] + \frac{N_v}{N}\cdot F_{3/2}\left[F_{1/2}^{-1}\left(\frac{N}{N_v}\right)\right]$$

that is equivalent to

$$V_{\text{int}} = E_g + \frac{k_B T}{q}\left(\frac{1}{N}\int_0^N \left[F_{1/2}^{-1}\left(\frac{n}{N_c}\right) + F_{1/2}^{-1}\left(\frac{n}{N_v}\right)\right] dn\right).$$

The Boltzmann counterpart follows from

$$\frac{1}{N}\int_0^N \left[\ln\left(\frac{n}{N_c}\right) + \ln\left(\frac{n}{N_v}\right)\right] dn = \ln\left(\frac{N}{N_c}\right) + \ln\left(\frac{N}{N_v}\right) - 2$$

ACKNOWLEDGMENT

We thank J. del Alamo, J. H. Smet and C. C. Eugster for useful discussions.

REFERENCES

[1] S. M. Sze, *Physics of Semiconductor Devices*. New York: J. Wiley, 1981, p. 74.
[2] P. Van Mieghem, R. P. Mertens and R. J. Van Overstraeten, "Theory of the junction capacitance of an abrupt diode," *J. Appl. Phys.*, vol. 67, no. 9, pp. 4203–4211, 1990.
[3] P. Van Mieghem, R. P. Mertens, G. Borghs, and R. J. Van Overstraeten, "Band-gap narrowing in GaAs using a capacitance method," *Phys. Rev. B.*, vol. 41, no. 9, pp. 5952–5959, 1990.
[4] See, e.g., M. Zahn, *Electromagnetic Field Theory*. Robert E. Krieger, 1979, pp. 204–213.
[5] P. Van Mieghem, "Theory of bandtails in heavily doped semiconductors," *Rev. Mod. Phys.*, vol. 64, no. 3, pp. 755–794, 1992.