The Arithmetic-Geometric Mean: a pearl of Gauss

Similar to my 2016-talk on the history of the Primes in Delft and Toscany, I will talk about a fascinating path that Gauss, Prince of Mathematics, took, but never published during his life. Only sketches of his amazing creation have been found after his death and were published in Gauss Werke, the collection of all his achievements.

Our story starts with the mean, moves to Gauss's arithmetic-geometric mean (AGM), that he already discovered at the age of 14, but which kept him fascinating throughout his entirely life. In his mysterious diary, Gauss wrote after his first discovery on AGM, that "an entirely new field of mathematics will arise".
At the end, I will link Gauss's AGM to the computation of π, one of oldest numbers that pervades mathematics nearly everywhere. The other three mathematical genii in the top 4, Archimedes, Newton and Euler will also enter our story on π.

Only high school mathematics is assumed.