

# Two-population SIR model and strategies to reduce mortality in pandemics

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**Abstract.** Despite many studies on the transmission mechanism of the Severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2), it remains still challenging to efficiently reduce mortality. In this work, we apply a two-population Susceptible-Infected-Removed (SIR) model to investigate the COVID-19 spreading when contacts between elderly and non-elderly individuals are reduced due to the high mortality risk of elderly people. We discover that the reduction of connections between two populations can delay the death curve but cannot reduce the final mortality. We propose a merged SIR model, which advises elderly individuals to interact less with their non-elderly connections at the initial stage but interact more with their non-elderly relationships later, to reduce mortality. Finally, immunizing elderly hub individuals can also significantly decrease mortality.

**Keywords:** COVID-19 pandemic, Mortality reduction, Two-population SIR model, Complex networks

## 1 Introduction

In many countries, the first wave of the Coronavirus disease 2019 (COVID-19) appeared in early 2020. In the summer of 2020, the spread of COVID-19 was significantly reduced due to strict restrictions [1] and weather effects [2]. At the beginning of autumn, students went back to school, marked the beginning of the second wave. However, rising infections were not taken seriously because infections were mainly among the young population and no significant hospitalization and deaths were observed [3]. Simultaneously, the high decrease rate and self-preservation have caused that many elderly individuals reduced their contact with young people [4]. In October 2020, the hospitalization rates in many countries started increasing and the second COVID-19 wave was born. At the beginning of 2021, more contagious mutations of the coronavirus marked the third wave [5]. The third wave might not be the last COVID-19 pandemic and efficient strategies to reduce mortality will remain on the agenda.

The Susceptible-Infected-Removed (SIR) model [6, 7] and its variations are commonly applied to describe the COVID-19 pandemic [8–10] and to forecast the number of infected and deceased cases in a population [11–13]. The ratio

of new deceased elderly cases to new deceased non-elderly cases each day is expected to be constant over time in classic epidemic models but is time-varying in reality. Recent works start to consider the age-structured SIR model to describe the COVID-19 pandemic more realistic [14–18]. The age-structured SIR model divides the whole population into several age groups and the infection rates are age-dependent. Real data reveal that the elderly infected had a 30- to 100- fold higher risk of dying than younger individuals in many European countries [19]. Here, elderly and non-elderly individuals are respectively defined as individuals who are  $\geq 65$  years old and  $< 65$  years old [19]. The elderly population accounts for a proportion of around 20% in many European countries [20]. Since the main difference in the COVID-19 pandemic is between elderly and non-elderly individuals, we construct a two-population SIR model [21] as follows:

1. There are two sub-populations: non-elderly and elderly individuals uniformly distributed over the social contact network. The virus spreading in a region is likely to start from non-elderly individuals because the virus can be carried into a community from other areas by commuters [22] and most commuters are non-elderly individuals.
2. There are four infection rates between and within non-elderly and elderly individuals. We believe that the highest infection rate is among elderly people. Elderly individuals are advised to a kind of self-isolation to protect themselves [23]. Staying in relative isolation from non-elderly people could be feasible, but some strong connections among elderly individuals, e.g., couples and people in the same nursing home, cannot be cut off. Conversely, the ties among elderly individuals will be stronger when their connections with non-elderly individuals are significantly reduced. The second highest infection rate is among the non-elderly population. The inter-group infection rates are the smallest since elderly individuals are afraid of being infected by non-elderly individuals. The infection rates between non-elderly and elderly individuals are low, but not zero, as elderly people still depend on younger people one way or another.

This article first investigates the features of fatality-related curves in the two-population SIR model when the connection between two populations is reduced. It shows that non-elderly deceased cases are prone to occur at the initial stage and most elderly deceased patients appear more often at a later stage. The final mortality, however, cannot be reduced by only limiting the connection between two populations. Moreover, reducing the infection rate among non-elderly individuals, e.g., closing schools, also cannot efficiently reduce mortality. In this work, we propose a merged SIR model to reduce the final mortality significantly. There are two stages in the merged SIR model: in the first stage, the model is the same as the two-population SIR model of Magal *et al.* [21] and in the second stage, the merged SIR model reduces to the standard SIR model. The physical meaning of the merged SIR model is that elder people are advised to reduce their connections with non-elderly individuals at the beginning of the pandemic and interact more with non-elderly individuals later. The merged SIR model benefits

the mortality reduction since many recovered non-elderly people can protect the susceptible elderly individuals.

Compartmental epidemic models assume that social contact networks are homogeneous with an infinite network size, but the actual network size is finite and the degree distributions of many real social networks follow a power law [24] with an exponent  $\gamma \in [2, 3]$ . We thus simulate the two-population SIR model on a scale-free network with a realistic network size to investigate the effect of network topology on the reduction of mortality. By comparing the simulation results of the two-population SIR model for the scale-free network and the Erdős–Rényi (ER) random network [25], the epidemic spreading in the heterogeneous network is much faster due to the star (or super spreader) effect. The reduction of connections between elderly and non-elderly individuals cannot decrease mortality in the compartmental epidemic model, but can reduce the mortality in the two-population SIR epidemic on the scale-free network. The merged SIR model is the best strategy to mitigate mortality. Finally, we illustrate that mortality can also be efficiently reduced by only immunizing rare elderly hub individuals.

## 2 Two-population SIR model

The two-population SIR model was first proposed by Magal *et al.* [21]. Similar models, that incorporate the underlying contact graph, are the networked SIR model proposed by Youssef and Scoglio [26], that was later entirely generalized to GEMF in [27]. Our work here applies the two-population SIR model to a realistic scenario related to the COVID-19 pandemic, systematically analyzes the death-related curve features, explores the effect of restrictions on mortality reduction and proposes an improved model to reduce the final mortality.

In the two-population SIR model, the non-elderly and elderly susceptible people are infected by non-elderly and elderly infectious individuals with different infection rates. Suppose that the elderly and non-elderly populations are well-mixed and large enough, the fractions of susceptible individuals  $S(t)$ , infectious individuals  $I(t)$  and removed (recovered or deceased) individuals  $R(t)$  at time  $t$  are reasonably well modeled by the following well-known differential equations:

$$\begin{cases} \frac{dS(t)}{dt} = -\text{diag}(S(t))BI(t), \\ \frac{dI(t)}{dt} = \text{diag}(S(t))BI(t) - EI(t), \\ \frac{dR(t)}{dt} = EI(t), \end{cases} \quad (1)$$

where  $\text{diag}(\ast)$  denotes a  $2 \times 2$  diagonal matrix whose diagonal entries are represented as a  $2 \times 1$  vector. The vectors of fractions of susceptible individuals  $S(t)$ , infectious individuals  $I(t)$  and removed individuals  $R(t)$  for non-elderly and elderly individuals are respectively,

$$S(t) = \begin{pmatrix} S_n(t) \\ S_e(t) \end{pmatrix}, I(t) = \begin{pmatrix} I_n(t) \\ I_e(t) \end{pmatrix} \quad \text{and} \quad R(t) = \begin{pmatrix} R_n(t) \\ R_e(t) \end{pmatrix}. \quad (2)$$

The subscripts  $n$  and  $e$  denote non-elderly and elderly individuals, respectively. The matrices  $E$  (removed rates) and  $B$  (infection rates) are respectively

$$E = \begin{pmatrix} \delta_n & 0 \\ 0 & \delta_e \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} \beta_{nn} & \beta_{ne} \\ \beta_{en} & \beta_{ee} \end{pmatrix}, \quad (3)$$

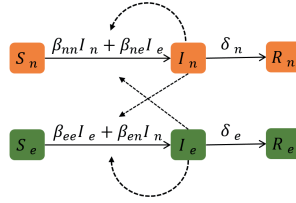
where  $\beta_{ne}$  denotes the infection rate from elderly infectious individuals to non-elderly susceptible individuals,  $\beta_{en}$  denotes the infection rate from non-elderly infectious individuals to elderly susceptible individuals,  $\beta_{nn}$  denotes the infection rate among non-elderly individuals,  $\beta_{ee}$  denotes the infection rate among elderly individuals,  $\delta_n$  denotes the removed rate for non-elderly infectious individuals and  $\delta_e$  denotes the removed rate for elderly infectious individuals. To simplify, we let the infection rates between two populations be equal,  $\beta_{ne} = \beta_{en} = \epsilon\beta_{nn}$ , and thus the matrix  $B$  can be rewritten as

$$B = \beta_{nn} \begin{pmatrix} 1 & \epsilon \\ \epsilon & \kappa \end{pmatrix}. \quad (4)$$

We have that the non-elderly fractions  $S_n(t) + I_n(t) + R_n(t) = N_n$  and the elderly fractions  $S_e(t) + I_e(t) + R_e(t) = N_e$ , where  $N_n$  denotes the fraction of non-elderly population and  $N_e$  denotes the fraction of elderly population. The two-population SIR model assumes that the total population is unchanged and thus  $N_n + N_e = 1$ . We denote the initial state by  $v[0] = (S_n[0], I_n[0], R_n[0], S_e[0], I_e[0], R_e[0])$ . A schematic depiction of the two-population SIR model is shown in Fig. 1. The infectious individuals will turn to be immune with a recovery rate ( $\xi_n$  for non-elderly individuals and  $\xi_e$  for elderly individuals) or deceased with a fatality rate ( $\eta_n$  for non-elderly individuals and  $\eta_e$  for elderly individuals). It holds that the removed rates  $\delta_n = \eta_n + \xi_n$  and  $\delta_e = \eta_e + \xi_e$ . This work focuses on the fractions of *new* deceased non-elderly and elderly cases that are  $\eta_n I_n(t)$  and  $\eta_e I_e(t)$ , respectively. We are also interested in the fractions of deceased non-elderly and elderly cases that are  $D_n(t) = \eta_n R_n(t)/\delta_n$  and  $D_e(t) = \eta_e R_e(t)/\delta_e$ , respectively. By numerical solving Equations (1), we analyze the effect of infection rates on the following four death-related curve features,

1. maximum of  $\eta_n I_n(t)$  and  $\eta_e I_e(t)$ :  $\max_{t \geq 0} \eta_n I_n(t)$  and  $\max_{t \geq 0} \eta_e I_e(t)$ ,
2. time points at which the maximum of  $\eta_n I_n(t)$  and  $\eta_e I_e(t)$  occur:  $\operatorname{argmax}_{t \geq 0} \eta_n I_n(t)$  and  $\operatorname{argmax}_{t \geq 0} \eta_e I_e(t)$ ,
3. time difference between two arguments of the maxima:  $\operatorname{argmax}_{t \geq 0} \eta_e I_e(t) - \operatorname{argmax}_{t \geq 0} \eta_n I_n(t)$ ,
4. fractions of final deceased non-elderly cases  $D_n(\infty)$  and elderly cases  $D_e(\infty)$ .

In this work, we set the fraction of non-elderly individuals as  $N_n = 0.8$ , the fraction of elderly individuals as  $N_e = 0.2$  and the removed rates as  $\delta_n = \delta_e = 0.1$ . The fatality rates for non-elderly and elderly infections are set to be  $\eta_n = 0.0001$  and  $\eta_e = 0.01$ , respectively. The initial state is set as  $v[0] = (0.7999, 0.0001, 0, 0.2, 0, 0)$ . These parameters are set based on real data. The elderly population makes up around 20% of the whole population in many European countries [20]. Elderly people who were infected had 30- to 100- fold higher



**Fig. 1.** Schematic depiction of the two-population SIR model. There are two populations: non-elderly people (highlighted in orange) and elderly people (highlighted in green). There are four different infection rates  $\beta_{nn}$ ,  $\beta_{ne}$ ,  $\beta_{en}$  and  $\beta_{ee}$  between and among the populations and two different removed rates  $\delta_n$  and  $\delta_e$  for each population.

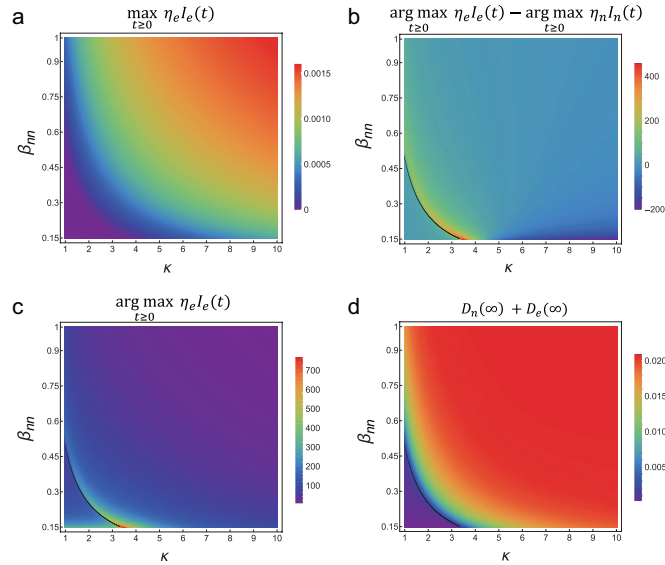
risk of dying than younger people in several European countries [19]. The time to recovery or death is on average around 10 days [28]. We also investigate various parameter settings and find that the changing of these parameters has no much effect on the main conclusions drawn in this paper.

To understand the effect of parameters  $\beta_{nn}$  and  $\kappa$  in (4) on death-related curves, we plot the heatmaps as shown in Fig. 2. It indicates that there are large time difference  $\operatorname{argmax}_{t \geq 0} \eta_e I_e(t) - \operatorname{argmax}_{t \geq 0} \eta_n I_n(t)$  when the infection rate  $\beta_{ee}$  is around the epidemic threshold  $\delta_e/N_e$  (shown as the black curves in Fig. 2). This epidemic threshold is obtained assuming that the infection rate between two populations  $\beta_{en} \rightarrow 0$ . The mortality cannot be significantly reduced by only reducing the infection rate among non-elderly individuals  $\beta_{nn}$ , e.g., closing schools, given that the infection rate  $\beta_{ee}$  is above the epidemic threshold. The only efficient way to well reduce mortality is to keep the infection rate  $\beta_{ee}$  among elderly individuals below the epidemic threshold.

Although mortality can be well reduced by decreasing the infection rates among elderly individuals  $\beta_{ee}$ , this strategy is not realistic since elderly people need a sufficient amount of social interaction. This work discusses strategies to reduce mortality considering the social needs of all the people. Elderly people reduce their social connections with non-elderly individuals and increase their interactions with elderly relationships. Thus, their interaction frequency [29], which is the total number of social interactions per unit time, is unchanged. We study the effect of reducing connections between elderly and non-elderly individuals on mortality reduction by comparing the mortality in the standard SIR model and the two-population SIR model. To keep the interaction frequency in the standard SIR model and the two-population SIR model to be at the same level, the equivalent infection rate in the standard SIR model is

$$\beta = \beta_{nn} N_n^2 + \beta_{ee} N_e^2 + (\beta_{ne} + \beta_{en}) N_n N_e. \quad (5)$$

Figure 4a indicates that the fractions of the final deceased individuals for the standard SIR model and the two-population SIR model are the same. The effect



**Fig. 2.** Curve features for the two-population SIR model with different parameters  $\beta_{nn}$  and  $\kappa$ . The parameter  $\epsilon$  is set to be  $\epsilon = 0.001$ . The black curves show the parameters when the infection rate  $\beta_{ee}$  is at the epidemic threshold  $\delta_e/N_e$ . The time difference  $\arg\max_{t \geq 0} \eta_e I_e(t) - \arg\max_{t \geq 0} \eta_n I_n(t)$  will be large when the infection rate  $\beta_{ee}$  is slightly above the epidemic threshold. The fraction of total deceased individuals will be small when the the infection rate  $\beta_{ee} < \delta_e/N_e$ .

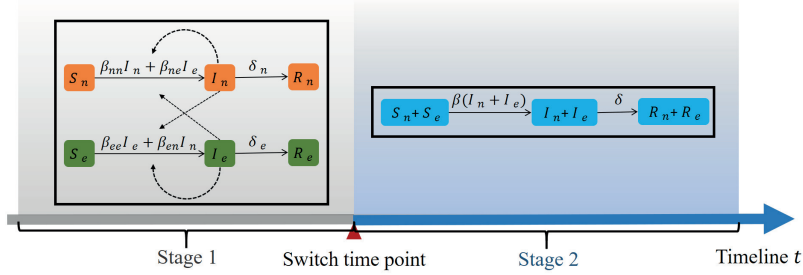
of reducing the connection between elderly and non-elderly groups is only to delay the deceased curve, but not to effectively reduce mortality.

### 3 Merged SIR model to reduce mortality

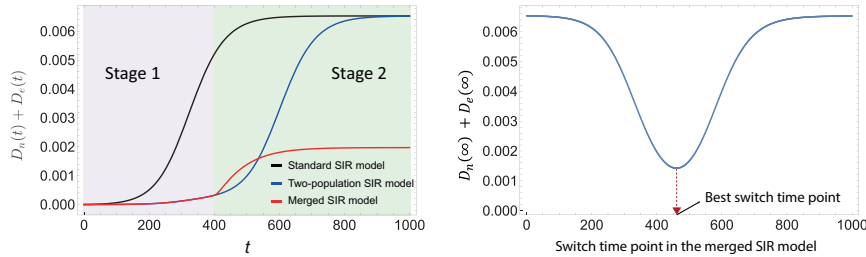
To effectively reduce mortality, we propose a merged SIR model in which the epidemic spreading follows the two-population SIR model in the first stage and follows the standard SIR model in the second stage. The illustration of the merged SIR model is shown in Fig. 3. The reduction of the connection between two populations can delay the pandemic among elderly people. The reconnection of these two populations further protect elderly people due to the herd immunity effect of recovered non-elderly individuals. Figure 4 shows that the merged SIR model can significantly reduce the final deceased fractions and there is the best switch time point to minimize the final mortality. Heatmaps in Fig. 5 show the effect of parameters  $\beta_{nn}$  and  $\epsilon$  on the best switch time point and reduced rate of the final mortality. The reduced rate of the final mortality is defined as

$$\frac{D_e(\infty) + D_n(\infty) - \tilde{D}_e(\infty) - \tilde{D}_n(\infty)}{D_e(\infty) + D_n(\infty)}, \quad (6)$$

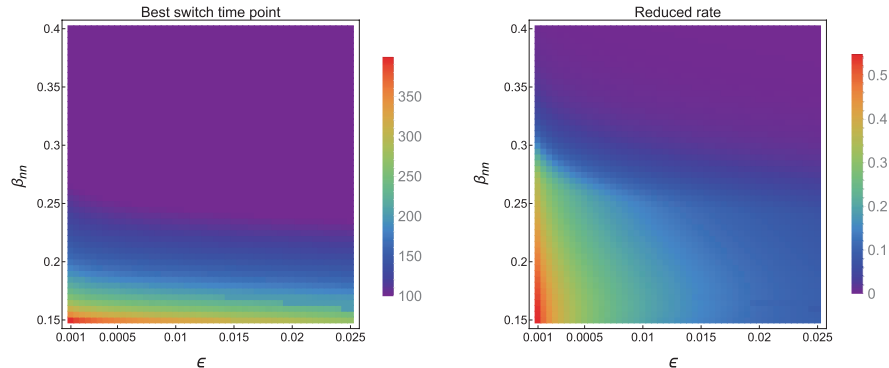
where  $D_e(\infty)$  and  $D_n(\infty)$  are respectively the elderly and non-elderly mortality for the two-population SIR model and  $\tilde{D}_e(\infty)$  and  $\tilde{D}_n(\infty)$  are respectively the elderly and non-elderly mortality for the merged SIR model. Figure 5 reveals that the first stage (reducing the connection between two populations) should take a longer time if parameters  $\beta_{nn}$  and  $\epsilon$  are smaller. Besides, the final mortality can be reduced more significantly for smaller parameters  $\beta_{nn}$  and  $\epsilon$ .



**Fig. 3.** Schematic depiction of the merged SIR model. This model has two stages: the first stage follows the two-population SIR model and the second stage follows the standard SIR model. The physical meaning of this model is to reduce the connection between the elderly and non-elderly populations initially and reconnect these two populations after many non-elderly infected individuals have been recovered. There is a switch time point between these two stages.



**Fig. 4.** The fractions of deceased individuals in the standard SIR model, the two-population SIR model and the merged SIR model. The parameters for the two-population SIR model are  $\beta_{nn} = 0.15$ ,  $\epsilon = 0.0001$  and  $\kappa = 4$ . We set the infection rate  $\beta = 0.12$  for the standard SIR model to keep the interaction frequency the same among models. The left figure reveals that the two-population SIR model cannot, but the merged SIR model can efficiently reduce mortality. The right figure shows the final deceased fractions with different switch time points, indicating that there is the best switch time point to minimize the final mortality.



**Fig. 5.** The best switch time point and reduced rate of the final mortality. We choose different parameters  $\beta_{nn}$  and  $\epsilon$  and a fixed parameter  $\kappa = 4$  for the two-population SIR model. The best switch time point will be larger and the final mortality will be reduced more significantly if parameters  $\beta_{nn}$  and  $\epsilon$  are smaller.

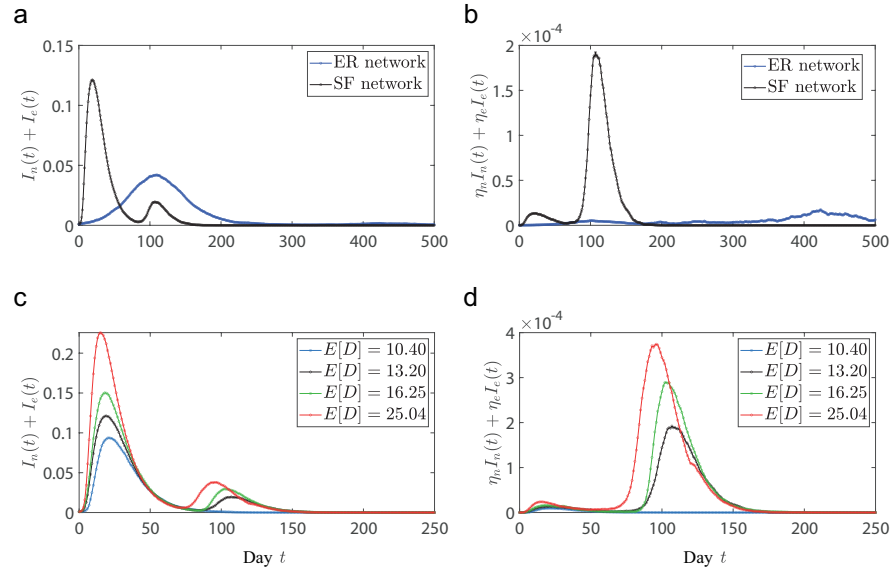
#### 4 Two-population SIR epidemic on large complex networks

We apply the Monte Carlo method [30] to simulate the two-group SIR epidemic on complex networks. In this work, we consider large networks with network size  $N = 10^5$  generated by the configuration model [31] and the simulation starts from 100 non-elderly infected individuals. We first compare the simulation results on the scale-free network and the Erdős–Rényi random network to analyze the effect of network heterogeneity on epidemic curves. The network size  $N$  and mean degree  $E[D]$  of the Erdős–Rényi random network are the same as the scale-free network. Figure 6a and Fig. 6b indicate that the epidemic spreading in the scale-free network is much faster than the spreading in the Erdős–Rényi random network due to the super spreaders. Figure 6c and Fig. 6d illustrate that the epidemic spreads quicker when the mean degree  $E[D]$  is higher.

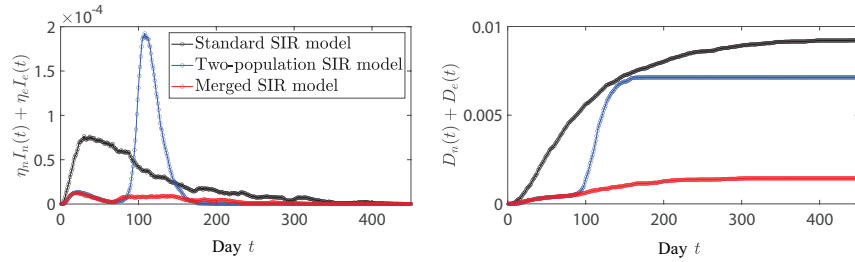
We simulate the standard SIR model, the two-population SIR model and the merged SIR model on the scale-free network as shown in Fig. 7. Different from the results as demonstrated in Fig. 4, for the epidemic spreading on complex networks, the final mortality for the two-population SIR model is lower than the standard SIR model since a part of susceptible elderly people can be protected by their recovered non-elderly relationships. This type of local immunity, which differs from the herd immunity, can only be observed in the epidemic spreading on networks. The merged SIR model is the best strategy to reduce mortality.

Given that there have been a limited number of COVID-19 vaccines, it is valuable to study the strategy to reduce mortality by immunizing specific population. There are rare elderly hub individuals in social networks, e.g., the priests,



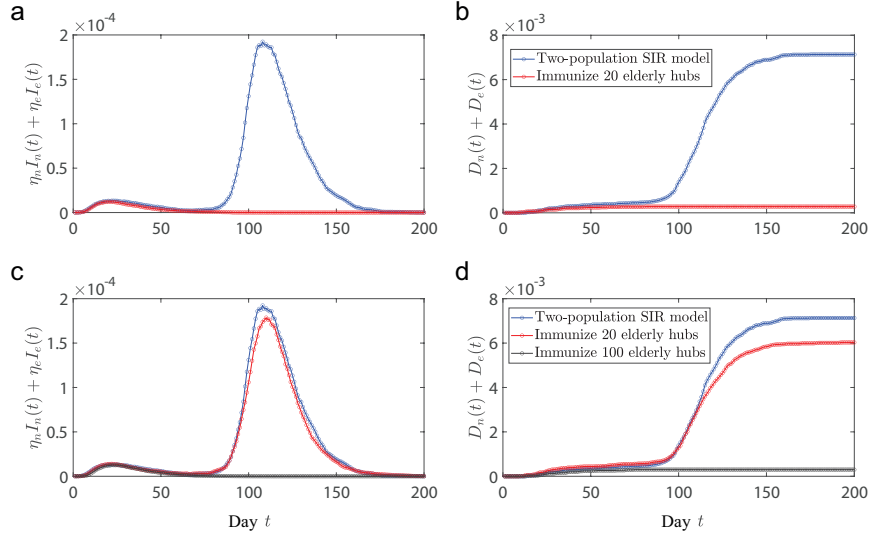


**Fig. 6.** Fractions of infectious cases  $I_n(t) + I_e(t)$  and daily deceased cases  $\eta_n I_n(t) + \eta_e I_e(t)$  for the two-population SIR epidemic on the scale-free network and the Erdős–Rényi network with the network size  $N = 10^5$ . The infection parameters are set to be  $\beta_{nn} = 0.015$ ,  $\kappa = 4$  and  $\epsilon = 0.001$ . The spreading in the scale-free network is much faster than the spreading in the Erdős–Rényi network. Figures c and d show the effect of mean degree  $E[D]$  of the scale-free network on the two-population SIR epidemic. With the increase of the scale-free networks' link density, there are more individuals infected and deceased. The exponent in the scale free networks is set as  $\gamma = 2.5$ . The minimum degree of the scale-free network is set to be 5.



**Fig. 7.** Fractions of daily deceased cases  $\eta_n I_n(t) + \eta_e I_e(t)$  and deceased cases  $D_n(t) + D_e(t)$  for the standard SIR epidemic, the two-population SIR epidemic and the merged SIR epidemic on the scale-free network with the network size  $N = 10^5$ . The infection rate in the standard SIR model is  $\beta = 0.012$ . The infection rates in the two-population SIR model are the same as Fig. 6. Different from the result in Fig. 4, the final deceased fraction for the two-population SIR model is lower than the standard SIR model. The final deceased fraction for the merged SIR model is the smallest, indicating that the merged SIR model is the best strategy to reduce the mortality.

which are the virus’s primary route of transmission from non-elderly to elderly people. Figure 8a and Fig. 8b reveal that the final mortality can be significantly reduced by only immunize 20 elderly hub individuals in  $10^5$  population assuming that the vaccines are 100% effective. We further analyze the situation when the vaccines are 80% effective. Figure 8c and Fig. 8d illustrate that more elderly hub individuals require to be immunized to reduce mortality efficiently.



**Fig. 8.** Effect of immunizing rare elderly hub individuals on reducing the final mortality. Figures a and b respectively show the fractions of daily deceased cases  $\eta_n I_n(t) + \eta_e I_e(t)$  and deceased cases  $D_n(t) + D_e(t)$  for the two-population SIR epidemic with and without immunizing elderly hub individuals. We immunize 20 elderly individuals with the largest degree in the simulation of the two-population SIR model on the scale-free network with  $10^5$  population assuming that the vaccines are 100% effective. Figures c and d show the fractions when the vaccines are 80% effective. It requires more vaccine doses to effectively reduce the mortality if the vaccines are less effective.

## 5 Conclusions

Since early 2020, scientists have found that COVID-19 is substantially more dangerous for the elderly. Elderly people’s interactions with their non-elderly relationships are reduced to lower the risk of being infected and deceased. This work applies the two-population SIR model to study if the mortality can be efficiently reduced when the connections between elderly and non-elderly individuals are significantly reduced. It reveals that severing ties between two populations can postpone the pandemic but not effectively cut mortality. We further propose the

merged SIR model and find that reconnecting two populations at an appropriate time can significantly lessen the final mortality. Assuming that rare vaccines are available, this study recommends immunizing elderly hub individuals first to better decrease mortality. Future works may extend the merged SIR model to more complicated models considering more realistic scenarios, e.g., the time-varying infection rates, the exposed state and the situation that some recovered individuals return to being susceptible.

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