Robustness of Complex Networks

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Network Architecture and Services (NAS)

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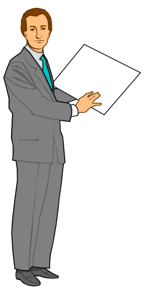
Outline



Framework for robustness

Computations

Conclusions

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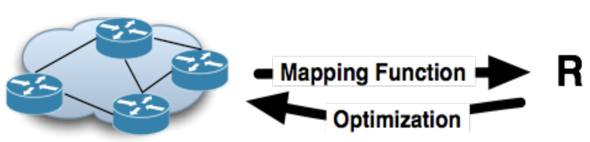
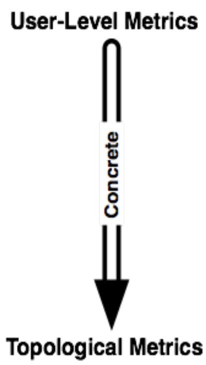
Network: topology + service(s)

- **Topology** (or network infrastructure):
  - graph *G* with *N* nodes and *L* links
  - link weights
  - “hardware”
- **Service**:
  - more abstract and less clearly defined
  - uses the network infrastructure to transport items (e.g. email service, telephony, video, cars on roads, neurons in brain, etc.)
  - “software”
- **Topology and service**
  - own specifications
  - service is often designed independently of the topology
  - often more than 1 service on a same topology

High-Level Goal: Express Network Robustness in a Number *R*



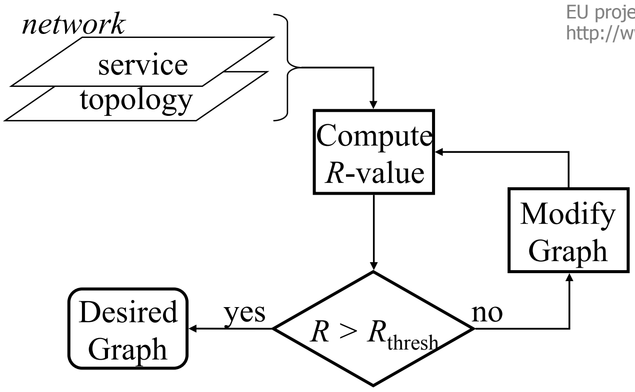


$$R \in [0,1]$$

R = 0: absence of robustness
R = 1: ideally robust

Simple framework: R-model

EU project FP7 – 224619: http://www.resumenet.eu/



Goals:

1. define “*R*-value” that characterizes the level of robustness in *any* network
2. compute the *R*-value
3. robustness classes (understanding): which *R* is desirable and what is $R_{\text{threshold}}$

P. Van Mieghem, C. Doerr, H. Wang, J. Martin Hernandez, D. Hutchison, M. Karaliopoulos and R. E. Kooij, 2010, "A Framework for Computing Topological Network Robustness", Delft University of Technology, report20101218.

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R-model

$$R = \sum_{k=1}^{m} s_k t_k = s.t \qquad (0 \le R \le 1)$$

s: the service vector with *m* components (interpreted as weights)
t: the topology vector where each component is a metric
(e.g. average degree, clustering coefficient, algebraic connectivity, minimum degree, diameter/hopcount, betweenness, etc...)

- Normalization: R = 0 (absense of network robustness)
  R = 1 (perfect robustness)
- **Linear**:
  - simplest *m*-dim expression & geometric interpretation (vector)
  - expectation E[R] easy
- no constraints on component values (else linear programming model)

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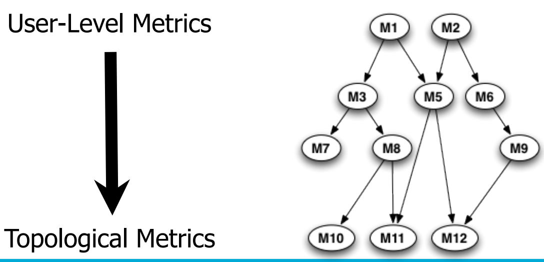
Issues with R-model: $R = \sum_{k=1}^{m} s_k t_k = s.t$

- dimension *m*: trade-off between accuracy and computational complexity (*not problematic, consensus*)
- orthogonality of the metrics (*fundamental problem*)
  - each metric should ideally be a basis vector in m-dim space
  - almost all topology metrics are dependent
  - degree of dependence depends on the graph
  - *Solution*: no metrics, but matrices (adjacency *A*, incidence *B*, Laplacian *Q*) or spectra (graph theory)?
- Normalization of graphs: how to compare graphs with different number of nodes and links?
- unclear how to map a service onto a service vector (recall s_k is projection of *s* on *k*-th metric)

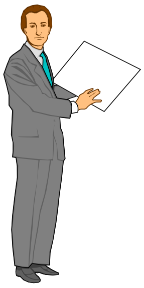
Which metrics to choose?

- Which metrics to choose is still an open question
  - Decomposition problem
  - Dependency problem
  - Normalization problem



Li, C., H. Wang, W. de Haan, C. J. Stam, P. Van Mieghem, "The Correlation of Metrics in Complex Networks with Applications in Functional Brain Networks", Journal of Statistical Mechanics: Theory and Experiment (JSTAT), issue November 2011, P11018.

Outline



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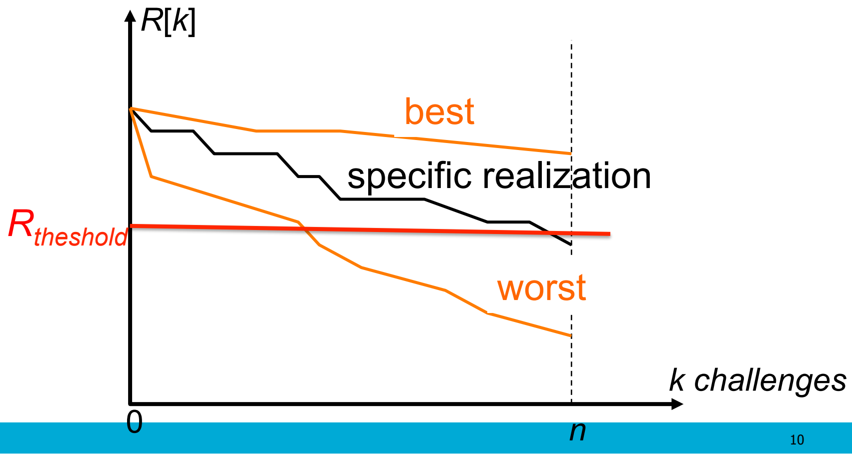
R as a function of "challenges"

- resilience is related to the network's capability to withstand perturbations from the outside during a given time interval



Realizations of a challenge are sequences of R-values: *R[0],R[1],R[2],...,R[n]*

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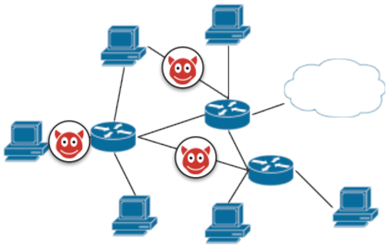
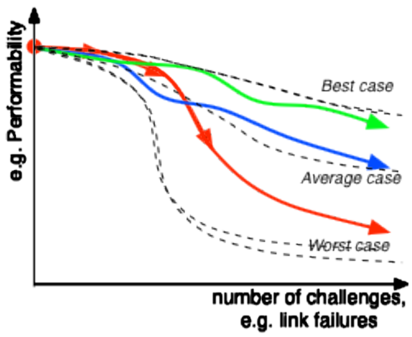
Understanding the Series of Events: R-Envelopes





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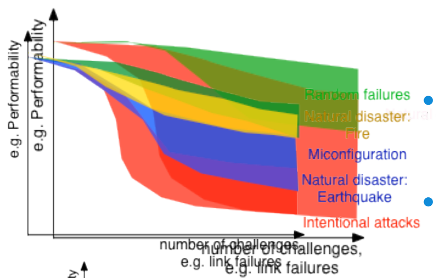
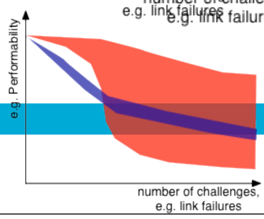
Understanding the Series of Events: Comparing R-Envelopes



- Comparing resilience based on metric envelopes give a visual explanation of the network degradation process
- Depending on the application domain a more bounded envelope might be preferable
- The effect of various failure sources on the evaluated metric can be revealed



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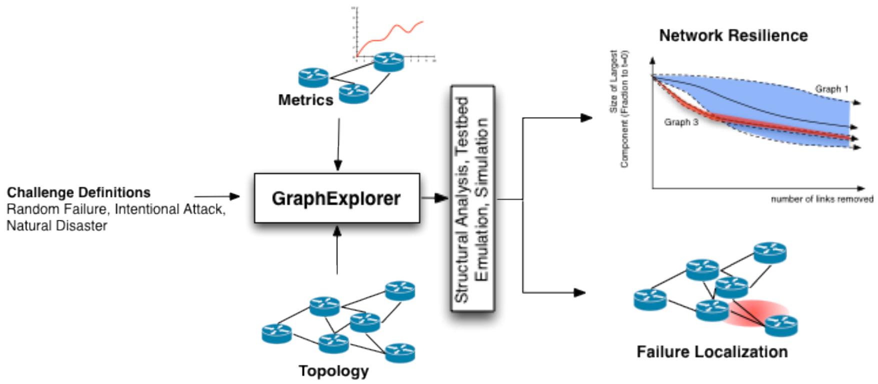
Computational Approach to Measuring Resilience



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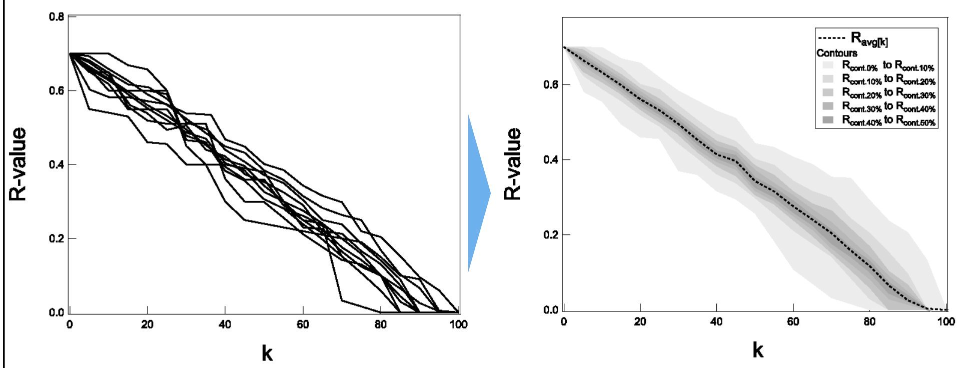
C. Doerr and J. Martin-Hernandez, "A Computational Approach to Multi-Level Analysis of Network Resilience," Proc. 3rd Int. Conf. Dependability, Venice, Italy, July 2010.

Multiple realizations: stochastic approach

10 random realizations

density function Pr*[R[k]=x]*



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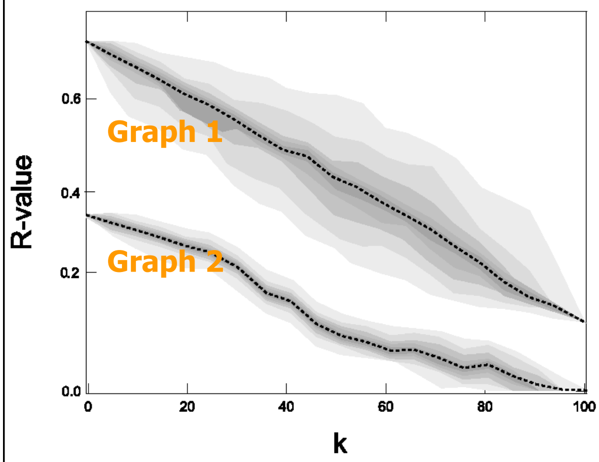
How to compare different graphs?



Graph 1:

- High average
- **but** greater range of values

Graph 2:

- Small range of values
- **but** lower average

S. Trajanovski, J. Martin-Hernandez, W. Winterbach and P. Van Mieghem, 2012, "A Framework for Computing Topological Network Robustness", submitted.

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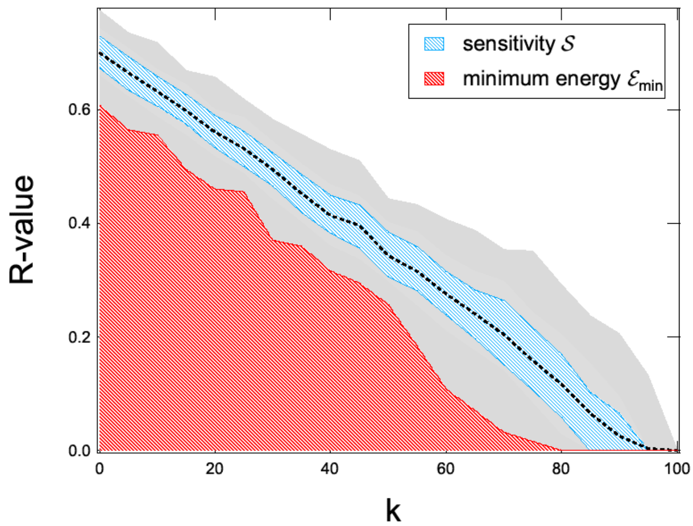
Proposed benchmarks



Higher **sensitivity** results in better stability.

Higher **energy** results in better average R-value.

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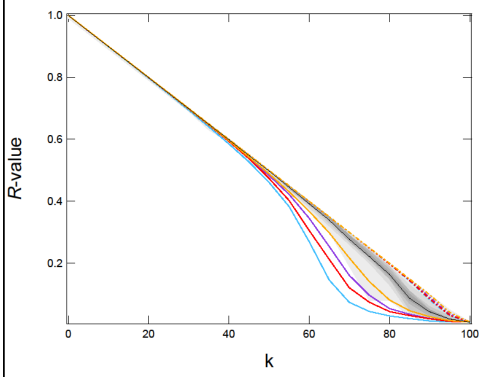
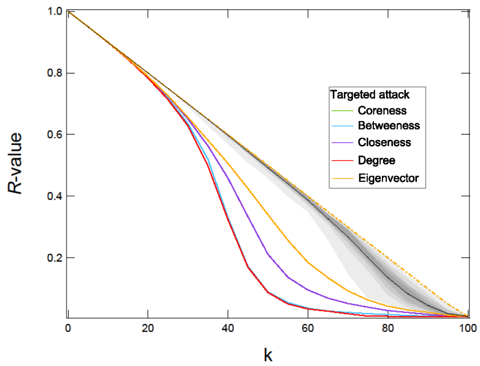
Graph models (example I)

Erdös-Rényi graph



Barabasi-Albert graph



...where R=efficiency = 1/H

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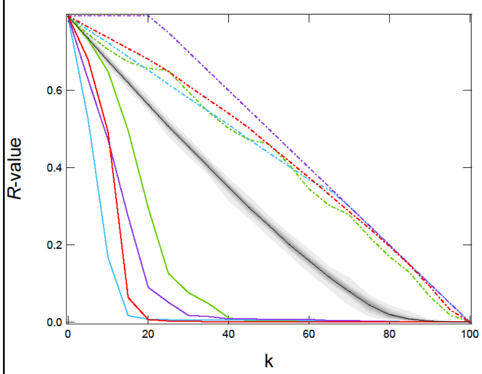
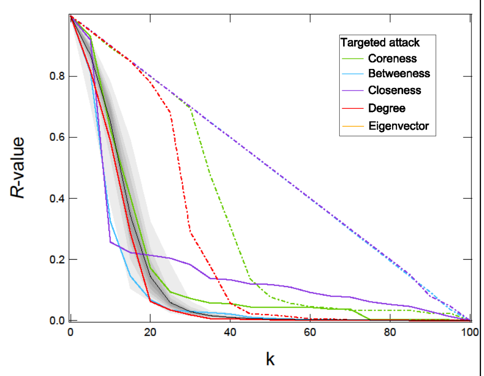
Real-world networks (example II)

Citation network



European power grid



...where R= size of giant component

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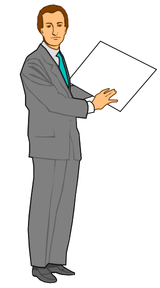

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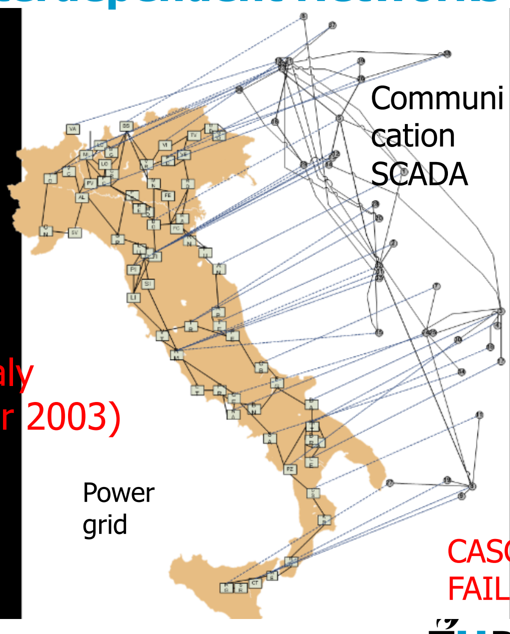
Interdependent Networks

Cyber Attacks- CNN Simulation (2010)

Rosato et al
Int. J. of Crit. Infrastruct. 4, 63 (2008)

Blackout in Italy
(28 September 2003)



With gratitude to Professor Shlomo Havlin

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Taxonomy "network challenges"

	single networks	interdependent networks
random attacks	Erdos-Renyi (1960) S: 2nd order phase transition	Buldyrev, Parshani, Paul, Stanley, Havlin; Nature (2010) S: 1st order phase transition
targeted attacks	Trajanovsky, Martin-Hernandez, Winterbach, Van Mieghem (2012) S: R-envelope highest degree & betweenness	open

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S: size of the giant component

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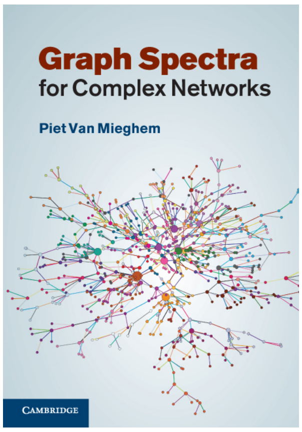
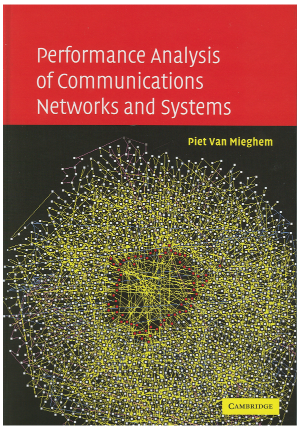
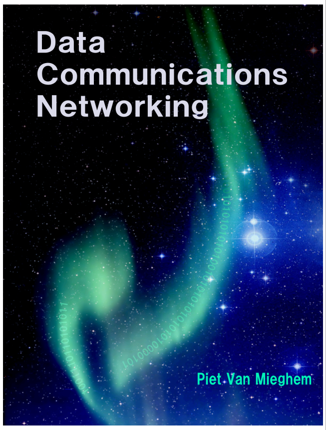
Books







Articles: **http://www.nas.ewi.tudelft.nl**

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