The Physics of Complex Networks

Piet Van Mieghem*

Delft University of Technology
July, 2011 (living document, updated on
http://www.nas.ewi.tudelft.nl/people/Piet/TUDelftReports.html)

Abstract

What are complex networks and why are they so important? We briefly sketch a contour landscape with current state of the art, intertwined with new challenging questions. We have freely chosen to emphasize some topics and to neglect or omit others. Our purpose was not to provide a comprehensive review, but rather to provoke enthousiasm for the huge potential of complex networking.

1 Introduction

Our society depends more strongly than ever before on large, complex networks such as transportation networks, telephony networks, the Internet, power grids, financial networks and social networks, while our health needs the understanding of biological networks (metabolic, DNA, brain networks). Surely, we are surrounded by complex networks, but, do we understand how they really operate?

Around 2000, several remarkable, quite universal phenomena observed in many different complex[1] networks gave birth to a new wave of network research that is still continuing and expanding from physics, mathematics to engineering, biology, medical sciences, social sciences, and even finance. The most important universal complex network characteristics are (1) a power-law or scale-free degree distribution (first reported via Internet measurements by Faloutsos *et al.* [13]), (2) small-world structure proposed by Watts and Strogatz [50], (3) preferential attachment as a simple driver to explain the power-law degree distribution of scale-free graphs [4], (4) clustering and community structure since most complex networks are networks of networks and (5) a high robustness against random failures, but a vulnerability against targeted attacks of mainly the hubs (high-degree nodes). For over 10 years, this new wave has led to innovative research with high potential impact and the wave is still increasing in activity (measured in the number of papers published in various domains). We believe that the beginning of the third millennium will be typified by a transfer of knowledge of dynamic processes in living material to self-made and engineered structures, based on the principles of complex networks.

*Faculty of Electrical Engineering, Mathematics and Computer Science, P.O Box 5031, 2600 GA Delft, The Netherlands; *email*: P.F.A.VanMieghem@tudelft.nl.

[1]Complexity of systems is a rather well established topic and studied for over 50 years (see e.g. [24]) and that has created system's theory (or control theory). The novelty in Network Science lies in the recognition that the underlying topology is crucial for many properties in complex systems.

As mentioned in [34, p. 1], a generally accepted, all-encompassing definition of a complex network does not seem to be available. Instead, complex networks are understood by an enumeration of examples as given above. But, if even clear definitions are lacking, how can we obtain precise knowledge? What tools and underlying theories can be applied? Beside descriptive languages, extensive computer simulations and measurements, complex network theory mainly relies on graph theory for its topological structure, on probability theory to express characteristic properties such as the degree and eigenvalue distributions and on dynamic systems theory to describe the processes on the network (such as e.g. virus spread [41] and synchronization [21]). In these fields, progress is still being made: (a) new bounds on spectral and topology metrics, (b) asymptotic scaling laws, (c) new extremal graphs, (d) physical phenomena (coupling, synchronization, percolation, self-criticality, emergent and collective behavior and other, generally, non-linear processes). Although the main ambition is to understand networks via mathematical deductions, very often computer simulations are needed to scan the first order behavior, in order to direct analysis towards the correct path. Finally, measurements of real-world networks benchmark the quality of a new theory.

However, we have only set a tiny step in really understanding dynamic complex networks. One of the high-level main drivers is the construction of a complex networking framework that combines several disciplines and application domains (as exemplified below in Section 3) and that targets the right level of abstraction to find coherent and universal processes in these complex networks. On a small scale, the details are overwhelmingly different: nodes in a complex network such as molecules in biology, computers or routers or hand-held devices (such as the Iphone) in the Internet, neurons in the brain, companies in an economic network, etc. Yet, the art is to "look" at the right scale of aggregation to cope with the complexity, surmount the distracting origin of microscopic differences and to "see" the beauty of invariants (such as the universal characteristics mentioned above). In mathematical terms of group theory, we may translate the "right level of abstraction" to find the group properties that are isomorphic between several complex networks. In engineering language, as an example, the OSI model in telecommunication [32] is a framework to structure and divide the functionality of any telecom network into various layers: can we benefit from the OSI framework and apply it to other complex networks to describe dynamic processes?

2 Understanding complex networks

In this section, we overview some of the general themes that appear in the field of complex networks.

2.1 Describing complex networks and metrics

The topology of complex networks can be represented by a graph, denoted by G, consisting of a set $\mathcal{N}$ of N nodes connected by a set $\mathcal{L}$ of L links. Graphs with N nodes, in turn, are completely described by an $N \times N$ adjacency matrix A, in which the element $a_{ij} = 1$ if there is a link between node i and node j, otherwise $a_{ij} = 0$. All elements a_{ij} of the adjacency matrix A are thus either 1 or 0 and A is symmetric for undirected graphs, but not symmetric for directed graphs, where there can be a link from $i \to j$, but not in the opposite direction $j \to i$. Undirected graphs are considerably simpler to analyse because symmetry of A leads to nice and powerful properties such as real eigenvalues and eigenvectors. The eigenvalues of a non-symmetric matrix are, in general, complex.

Often the importance of links in a complex network are different and weights can be assigned to links in order to differentiate or specify additional properties such as the monetary cost when using the link, the delay, distance, capacity, etc. The weighted adjacency matrix W has the same structure as A, except that the element w_{ij} is a real number[2], that characterizes a certain property of the link. The set of all link weights is called the link weight structure of a graph. In general, and quite remarkably, there are few networks for which the link weight structure is known. The distances in transportation networks (e.g. in the road map of Europe) are the best example, but the delay in the Internet, the strength of the interaction in the human brain, the amount of money transfer between banks, reaction quantifiers in metabolic networks, and so on, are difficult to obtain. The understanding of the effect of the link weights on the shortest paths, how link weights are tunable to control flows, what the observability of a network is (which fraction of the links can be measured both in normal and failure situations) are key questions, but currently still insufficiently well understood. In my first short review [36] of the kind here presented, link weights in networks are overviewed in more detail, stochastic models of the shortest path problem in weighted graphs [33, Chapter 16] are discussed and the sensitivity of shortest paths to changes in the link weights was put forward as a robustness aspect of networks. Finally, we pointed to the remarkable phase transition [44, 40] that occurs when the extreme value index of the link weight distribution changes. This link weight phase transition separates a network into a superconductive state where all transport flows over a minimum spanning tree backbone and into a normal resistive state, where more paths between two node pairs are operational.

From the adjacency matrix, topological graph metrics can be computed such as the degree of a node, the betweenness of a link, the hopcount of a shortest path from i to j, the clustering coefficient, the edge/link connectivity, etc. We refer to [2, 10] for a quite extensive discussion and comprehensive list of graph metrics and to [33] for additional properties. Another type of metrics are spectral metrics [34] such as the largest eigenvalue of the adjacency matrix (spectral radius), the spectral gap, the algebraic connectivity, etc. The interest of spectral metrics originates from the fact that the eigenvalues of a graph are a unique fingerprint [29]: the spectral domain consisting of the eigenvalues and eigenvectors of the adjacency matrix A possesses the same information as the topology domain specified by A. However, in contrast to topological metrics, spectral metrics require more mathematics and lack to provide intuition: for example, what does an eigenvalue of a graph or/and its corresponding eigenvector mean? Metrics that need more information than the (weighted) adjacency matrix, such as a usage pattern or traffic flows, are called service metrics. Examples of service metrics are quality of service measures in ATM networks [32] such as packet loss and jitter, but more general service metrics are reliability, security, dependability [1], trustworthiness, etc. The latter class of service metrics suffers from a generally accepted definition and from the fact that they are, even with a clear definition, difficult to compute for a network. When studying metrics as characterizers of a network, the decomposition of service metrics into a number of more easily manageable ones is a reoccurring problem.

Most complex networks are hierarchical structures, where nodes can again be networks. Simon [24]

[2] Instead of assigning a single number to a link, we can specify a link by a link weight vector as in QoS routing [39] so that each non-zero element in A is replaced by an $m \times 1$ vector with real numbers.

even considers the hierarchy in complex systems as the characterizing property, as illustrated in his nice parable about two watchmackers, Hora and Tempus. Hierarchy and modularity [38] are strongly related. For example, the Internet consists of over 20 000 autonomous systems (domains) and each autonomous domain contains several IP routers [32]. In brain networks, one may focus on the neural level or on the functional level. While the network hierarchy can be represented by restructuring the adjacency matrix into a block adjacency matrix, only little studies have appeared on hierarchical structures, of which ATM's PNNI protocol [32] is perhaps the most detailed creation so far. Issues are the representation of a subnetwork as a "complex" node on higher hierarchical level, which involves node and/or link condensation [31] – i.e. the loss of information and detail –, and finding the best clusters or subnets (via e.g. modularity optimization [34]).

On a more philosophical level of abstraction, a network hierarchy is related to the concept of a holon, first introduced by Koestler [16]. Holons[3] are logical entities that distinguish themselves from their environment and are both a whole and a part; they can be regarded as generalizations of network nodes. Fig. 1 sketches the ordering and organization of holons into a holarchy, where four levels are distinguished. Holons on different levels of a holarchy can interact so that the holarchy evolves. If a

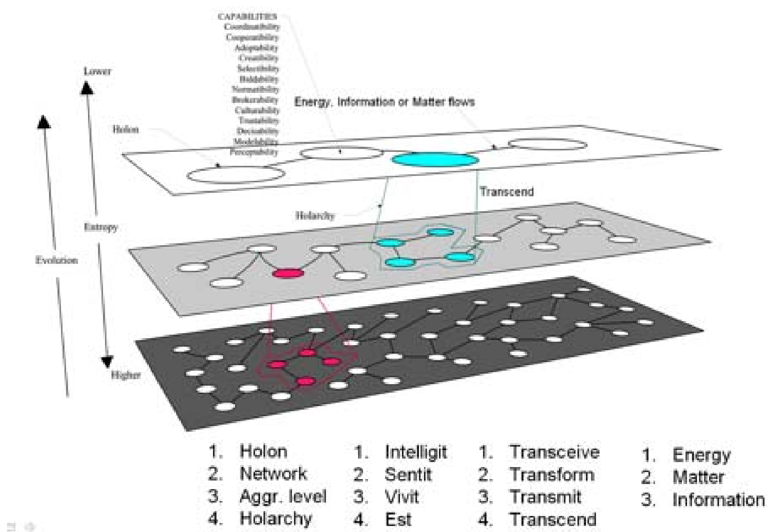

Figure 1: A sketch (*drawn by N. Baken*) of a holarchy with four different levels: (1) individual holons, (2) networks of isomorphic holons, (3) aggregation layers of isomorphic networks, (4) complete holarchies of isomorphic aggregation layers.

holarchy is recognized by its environment, the holarchy can recursively be coined as a holon in itself. The question is "Can we influence or control the complex operation and evolution of the holarchy?"

2.2 Dynamic processes on networks

Apart from the topological structure of a complex network, the network processes constitute the heart of a complex network: they determine why the network is built or created and they give value to a complex network. Examples of network processes or services are the transfer of IP packets in the

[3] From the Greek, $\tau o\ o\lambda o\nu$ meaning "the whole".

Internet, the transport of cars in a road network, the interaction between functional brain regions, the spread of rumors and news in a social network, etc. A general topic in complex network theory is the dynamics of processes on the graph, of which virus spread [41] and synchronization [27, 21] are reasonably well-understood examples. In most cases, we are interested to know whether the process is stable, whether phase-transitions [12] or forms of self-organization occur and how the process behaves when the network grows (scaling laws) or is modified (removal or adding of subgraphs). In summary, the effect of the topology (graph) on the functioning (process) of network is an important theme. Immediately related to this theme is the association of relevant topology metrics to the function of the network.

A considerably more difficult class of problems is the study of the interaction between the processes on the network and the underlying network itself. For example, a virus spreads in a network and the protection against the virus can consist of installing anti-virus software in computer networks or of vaccinations or medicines in a human social network. These actions do not change the underlying topology. However, adjusting the topology by avoiding contact with infected nodes (e.g. computers or humans) leads to another type of protection that requires the understanding of the coupling between graph and process (or function of the complex network). The last type of dynamics also receives increasingly more interest as most of our infrastructures are coupled [6]. For example, nearly all complex networks need energy, while the influence of digital communication to control these infrastructures increases. A failure occurring in the electricity distribution or in the control communication network may introduce failures or undesired behavior in the functioning of the complex network. Such cascade effects are poorly understood.

Another theme is to understand how biological processes in nature achieve such an amazing adaptivity and resilience against external factors. For example, the Alzheimer disease is only diagnosed with certainty when over 80% of the links in the brains are destroyed. What is the way biological networks evolve? Which topological processes (such as rewiring, creation, deletion of links) are determining the network structure during the lifetime of an organism?

2.3 Robustness of complex networks

Fig. 2 illustrates a general question in the field of complex networks: "Given a network at a certain time, is that network *appropriate* or *good* for our purposes?" For example, an Internet service provider may ask whether his current network is "good", a neurologist may want to know whether the functional network of the brain of a patient is "normal". Of course, the above question is ill-posed and not clearly stated because *appropriate* or *good* need to be defined.

A given network at a certain time, defined by a service and a topology as in Fig. 2, is translated into a mathematical object, on which computations can be performed such as the computation of a "goodness" value or robustness value, called the R-value. In general terms, the R-value is a performance measure that is relevant for the service and normalized to the interval $[0, 1]$. Thus, $R = 0$ corresponds to absence of network "goodness" and $R = 1$ reflects perfect "goodness". An example of a performance measure is a graph metric such as the average hopcount, the average betweenness, etc. One of the main purposes of a network robustness framework (see e.g. [37]) is to propose a methodology to define and compute an R-value that characterizes a level of robustness and to interpret R-values

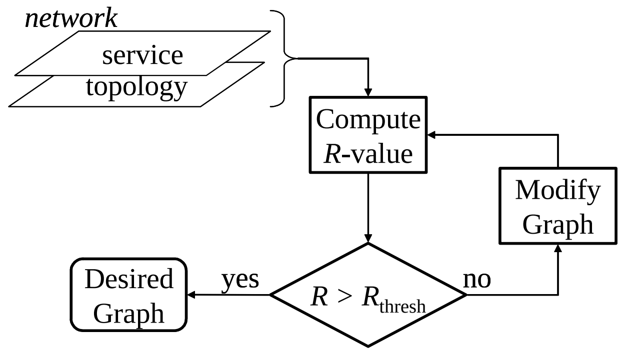

Figure 2: The organigram or flow chart of the high level goal to achieve network robustness.

such that classes of desired values can be determined. Next, as shown in Fig. 2, the current R-value is compared with the minimal desired one, R_{thresh}. The computation of the R-value can be part of periodic or event-triggered network maintenance/management operations. Either the R-value is sufficient in which case we refrain from taking any corrective action, or the R-value is too low, in which case a modification to improve the graph is required. A second goal of a network robustness framework is to propose efficient – possibly optimal – strategies how a graph can be modified to increase its R-value subject to some cost criterion. For example, is a patient with headache seriously ill (tumor, brain defect) or is an "aspirin" sufficient to help him? Based on MEG and EEG, the functional brain network is measured and "robustness" needs to be defined in terms of measurable quantities. Similarly, the Internet, on which we all rely, should be robust against "external perturbations" such as congestion, denial of service attacks, malware, power failures, etc. In summary, a robustness analysis studies under which circumstances (usage patterns, external influences) a network breaks down [22]. The interaction between (malicious) usage patterns and algorithms that are designed to cope with them (rerouting algorithms, congestion avoidance, anti-virus programs etc.) plays a major role.

However, even comparing two networks based on their topology only, is far more difficult than that it seems at first encounter. In [37], we show that most graph metrics are not mutually independent, and that their degree of correlation is graph specific. Thus, which set of graph metrics is sufficient and "orthogonal" enough to specify a graph is still an open question. Fig. 3 illustrates three different graphs that possess same values for a set of metrics. Hence, that set of metrics alone is not sufficient to distinguish between them. Moreover, scaling a graph by increasing the number of nodes N or link L induces changes in the metrics, which makes it difficult to compare networks of different size.

A special branch of network robustness is network security. In spite of the regular news reporting of intrusion and attacks of important computer systems, ranging from banks to governments, it is fair to say that little systematic network theory is available to secure a network, apart from some network immunization strategies [8, 20]. Most efforts today are heuristic, or unknown, since privacy and secrecy play an obscuring role in the domain of (network) security. A huge potential to employ the devices of complex network theory to design and construct (more) secure networks, subject to privacy and low cost constraints, is lurking, but know-how seems missing.

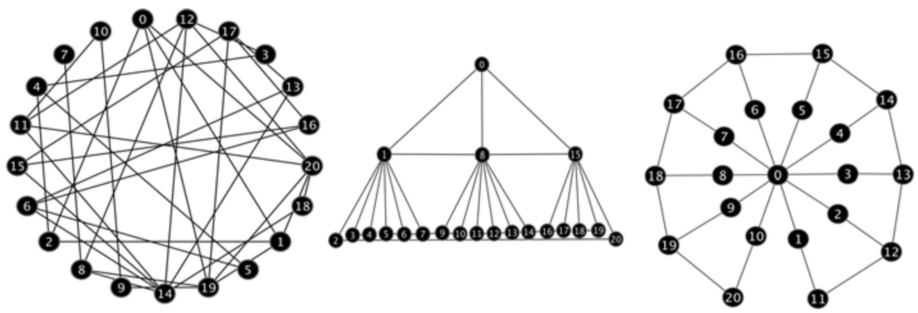

Figure 3: Three different graphs with $N = 20$ node and $L = 29$ links that possess the same average hopcount $E[H] = 2.4$, the same diameter $\rho = 4$ and the same algebraic connectivity $\mu_{N-1} = 0.4$. *The figure was created by C. Doerr.*

2.4 Computations and network algorithms

An unfortunate feature is that most optimization problems in graphs are NP-hard, which means that an optimal solution cannot be computed for large complex networks and that heuristics are often the only resort. For example, in both virus spread and synchronization of oscillators, the onset[4] of an infected network or of a coupling between oscillators is inversely proportional to the spectral radius $\lambda_1(A)$ of the graph. In order to enlarge the threshold, links or nodes may be removed. The removal of those links that maximally lower the spectral radius or the minimum number of links to shift $\lambda_1(A)$ below some desired level is NP-hard [42]. The problem of finding a best path subject to several constraints (such as distance, time and energy-consumption) is known as QoS-routing [18], which is also NP-hard. The optimal placement of regenerators in optical networks [17] is another example of a NP-hard problem. For NP-complete problems, exact algorithms consume prohibitively long computation time so that heuristics are designed to trade-off computation time for accuracy. Much effort has been devoted to heuristic algorithms that provide a solution that is guaranteed to deviate no more than $x\%$ of the global optimal solution. Sometimes exact algorithms work surprisingly well in a certain range of the state space [18]. Often constraints can be chosen to enable efficient solutions [43].

Another inherent challenge in large networks is the mere extent of its size, which prevents that all details of the graph can be loaded into a normal computer. Large complex network have spurred the research on local computations, where only a certain surrounding of a node is taken into account, rather than the global topology information. Examples are "gossiping" algorithms [28], breath-first search of social networks and web-crawling. The massive growth in data and network sizes has led to a new breed of "sub-linear" algorithms, that only consider part of the input and consequently run in sub-linear time complexity. Since not all information is considered, exactness cannot be guaranteed. But through careful sampling, sub-linear algorithms might be able to give acceptable bounds. Recently [9], some classical optimization problems have been approximated in sub-linear time.

Beside the classical distributed computation, where a large problem is split over several computers, a new flavor of this concept has been introduced as "cloud-computing", where tasks are assigned to a

[4] Threshold of the phase transition.

combination ("cloud") of servers and computing devices, accessed over a network.

3 Applications domains

A non-comprehensive list of applications of complex networking is presented. We would not be surprised to see that the number of the listed applications can be doubled.

3.1 Infrastructural, man-made networks

Examples of this type of complex networks are the Internet, the world-wide web, transportation and energy networks, waste and water networks. Especially, due to energy considerations, the combination of ICT with electrical networks receives a lot of attention (e.g. smart homes & villages). The EU commission[5] has launched the theme "Internet Science". The latter is a combination of traditional telecommunications with new "complex network inspired flavor": how can the robust and self-adaptive behavior in nature be transferred to man-made infrastructures, how can other complex networks benefit from the Internet's communication structure, what is the role of non-technical drivers or stake-holders such as governments, financial interests of large players as Google and the "Power to the People" (a forum for democracy and freedom, based on free production and exchange of knowledge).

3.2 Biological networks

A tremendous potential for complex networking theory lies to be exploited and to be understood in the area of biological networks. Many triggering challenges arise such as understanding evolution based on networking (assortativity [45, 49], modularity [38], degree-preserving rewiring), the structure of metabolite reactions (which molecules are crucial for growth? [51]), how to compute the largest common graph structure in species? Understanding the key aspect of how complex systems operate inside cells using complex network theory (see e.g. the recent review [46]).

3.3 Human brain networks

One of the most complex networks is our brain network, consisting of over $N = 10^{11}$ neurons and $L = 10^{14}$ interconnections, with an estimated total of 500 000 km wiring[6]. Fig. 4 illustrates two main approaches in which complex network analysis can be employed to understand the human brain [26], of which the functional brain network is easier to measure. Understanding functional brain networks via topological metrics such as path length, degree, clustering, assortativity, modularity [25] and/or via spectral metrics is still a hot topic of research[7]. The application domain is huge: beside monitoring healthy people to understand emotion, memory and IQ [30], the major social and financial impact lies in understanding illnesses such as Alzheimer's disease and other types of dementia, brain tumors [47],

[5] The study that led the EU commission to fund a Network of Excellence on Internet Science is "Towards a Future Internet. Interrelation between Technological, Social and Economic Trends", Final Report for DG Information Society and Media, European Commission DG INFSO Project SMART 2008/0049, http://cordis.europa.eu/fp7/ict/fire/docs/agenda-04-apr-2011.pdf

[6] The moon-earth distance is 384 405 km.

[7] See e.g. the website Connected Brains at http://home.kpn.nl/stam7883/

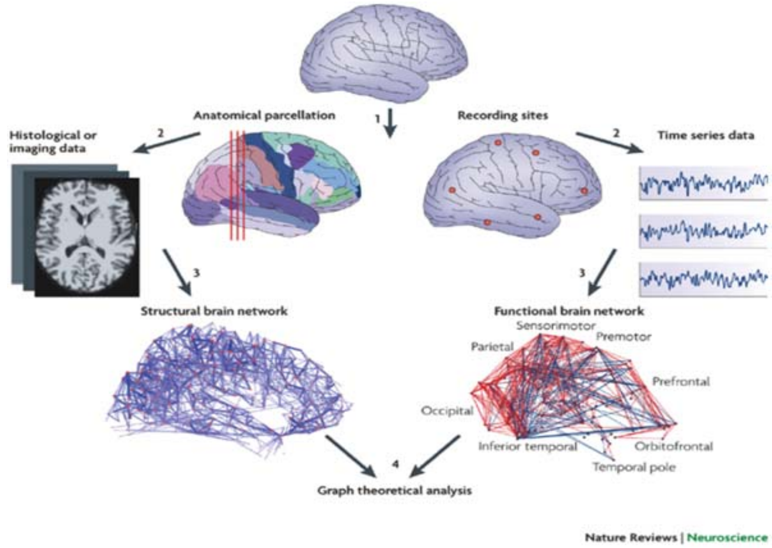

Figure 4: The structural and functional brain network (from [7]).

epilepsy and schizophrenia. For example, dynamic synchronization processes and phase transitions are related to the onset of epileptic seasures, which may be understood via the spectral radius (as explained in Sec. 2.2).

3.4 On-line Social networks

Examples of on-line social networks are Facebook, Hi5, LinkedIn and Orkut, MySpace and Twitter, Flickr, Last.fm and YouTube, Digg, Delicious and Reddit. Complex networking can shed light on the growth of the social network, its degree distribution, assortativity (rich talk to rich), epidemic information dissemination. Relevant questions and topics in social networking are "Are friends overrated?" [11], "Who are the influential people in an OSN?" [15], "What is the best way to make your story/message popular?", "collaborative voting [35]", "How can social networking enhance the marketing of new policies or products?"

3.5 Financial networks

Economic systems have been studied based on input-output matrices [19] and economic activity classification systems[8]. Blending the disciplines of economic systems and complex networks is relatively new and is coined as economic networks [23]. This emergent field of economic networks contributes to the understanding of the vulnerability of economic systems and touches upon its evolution, properties, interdependencies and correlations in comparison with other real networks. Money flows in an economy and the number of financial transactions between sector networks are correlated to social welfare [48].

[8] United Nations Statistics Department, International Standard Industrial Classification rev. 4, New York, 2008.

3.6 Bio-psycho-social networks

Bio-psycho-social networking studies the influence of psychic diseases on social interactions. A person's state (see [14] and many ref. therein) such as sense of happiness, loneliness, political ideology, obesity, alcohol consumption and smoking behavior are all determined by his position within a social network as measured by centrality metrics, clustering coefficients, etc. Conversely, a person's state may affect the position within the social network, as observed in psychiatric disorders such as major depression, psychosis and mania, which interfere strongly with activities of daily living and lead to a loss of partners, friends, jobs and social roles [5]. To date, no comprehensive study has been performed in the social sciences and psychiatry that examines the relationships between the topological positions of nodes within social networks and the states of these nodes. Barabasi [3] has created a multi-level network representation that examines the relationships between biological factors, disease scores and social factors.

Network science is expected to provide clinical psychiatry with a solid mathematical framework, to allow prediction of optimal treatment strategies including farmacotherapy and psychotherapy, and to cause a significant reduction of the costs of mental healthcare.

4 Summary

We have touched upon a few themes of complex networking, briefly explaining the concept, the motivation and the challenge, while referring to articles where more details can be found.

Along the way towards understanding and exploiting complex networks, we hope to increase our understanding of the real big issues like "artificial intelligence" and "what features need a complex (biological or man-made) network to possess in order to live (behave entirely autonomous)"?

Acknowledgements We would like to thank Nico Baken, Edgar van Boven, Rutger Goekoop, Fernando Kuipers, Huijuan Wang and those whose comments and input improved the early manuscript.

Since the sketched overview is by no means complete and since we consider the document temporarily as a "living document", we would be most grateful to receiving additional input to enlarge the coverage of the domain of complex networking.

References

[1] A. Aviziensis, J.-C. Laprie, B. Randell, and C. Landwehr. Basic concepts and taxonomy of dependable and secure computing. *IEEE Transactions on Dependable and Secure Computing*, 1(1):11–33, January-March 2004.

[2] S. Baccaletti, V. Latora, Y. Moreno, M. Chavez, and D.-U. Hwang. Complex networks: Structure and dynamics. *Physics Reports*, 424:175–308, 2006.

[3] A. L. Barabási. Network medicine – from obesity to the diseasome. *New England Journal of Medicine*, 357(4):404–407, July 2007.

[4] A. L. Barabási and R. Albert. Emergence of scaling in random networks. *Science*, 286:509–512, 1999.

[5] M. H. Beauchamp and V. Anderson. SOCIAL: an integrative framework for the development of social skills. *Psychological Bulletin*, 136:39–64, January 2010.

[6] S. V. Buldyrev, R. Parshani, G. Paul, H. E. Stanley, and S. Havlin. Catastrophic cascade of failures in interdependent networks. *Nature Letters*, 464:1025–1028, April 2010.

[7] E. Bullmore and O Sporns. Complex brain networks: graph theoretical analysis of structural and functional systems. *Nature Reviews Neuroscience*, 10:186–198, March 2009.

[8] Y. Chen, G. Paul, S. Havlin, F. Liljeros, and H. E. Stanley. Finding a better immunization strategy. *Physical Review Letters*, 101:058701, August 2008.

[9] A. Czumaj and C. Sohler. Sublinear-time algorithms. *Bulletin of the European Association for Theoretical Computer Science (EATCS)*, 89:23–47, 2006.

[10] L. Da F. Costa, F. A. Rodrigues, G. Travieso, and P. R. Villas Boas. Characterization of complex networks: A survey of measurements. *Advances in Physics*, 56(1):167–242, Februari 2007.

[11] C. Doerr, S. Tang, N. Blenn, and P. Van Mieghem. Are friends overrated? a study for the social aggregator digg.com. *IFIP Networking 2011, Valencia, Spain*, May 9-13 2011.

[12] S. N Dorogovtsev and A. V. Goltsev. Critical phenomena in complex networks. *Reviews of Modern Physics*, 80:1275–1335, October-December 2008.

[13] M. Faloutsos, P. Faloutsos, and C. Faloutsos. On power-law relationships of the Internet Topology. *Proceedings of ACM SIGCOMM'99, Cambridge, MA*, pages 251–262, 1999.

[14] J. H. Fowler, J. E. Settle, and N. A. Christakis. Correlated genotypes in friendship networks. *Proceeding of the National Academy of Science (PNAS)*, 108(1):1993–1997, February 2011.

[15] M. Kitsak, L. K. Gallos, S. Havlin, F. Liljeros, L. Muchnik, H. E. Stanley, and H. A. Makse. Identification of influential spreaders in comlex networks. *Nature Physics*, 6:888–893, November 2010.

[16] A. Koestler. *The ghost in the machine*. Hutchinson & Co, London, 1967.

[17] F. A. Kuipers, A. Beshir, A. Orda, and P. Van Mieghem. Impairment-aware path selection and regenerator placement in translucent optical networks. *18th IEEE International Conference on Network Protocols (ICNP2010), Kyoto, Japan*, Oct. 5-8 2010.

[18] F. A. Kuipers and P. Van Mieghem. Conditions that impact the Complexity of QoS Routing. *IEEE/ACM Transaction on Networking*, 13(4):717–730, August 2005.

[19] R. E. Miller and P. D. Blair. *Input-Output Analysis*. Cambridge University press, second edition, 2009.

[20] J. Omic, J. Martin Hernandez, and P. Van Mieghem. Network protection against worms and cascading failures using modularity partitioning. *22nd International Teletraffic Congress (ITC 22), Amsterdam, Netherlands*, September 7-9 2010.

[21] J. G. Restrepo, E. Ott, and Brian R. Hunt. Onset of synchronization in large networks of coupled oscillators. *Physical Review E*, 71(036151):1–12, 2005.

[22] C. M. Schneider, A. A. Moreira, J. S. Andrade, S. Havlin, and H. J. Herrmann. Mitigation of malicious attacks on networks. *Proceeding of the National Academy of Science (PNAS)*, 108(10):3838–3841, 2011.

[23] F. Schweitzer, G. Fagiolo, D. Sornette, F. Vega-Redondo, and D. R. White. Economic networks: What do we know and what do we need to know? *Advances in Complex Systems*, 12(4), 2009.

[24] H. A. Simon. The architecture of complexity. *Proceedings of the American Philosophical Society*, 106(6):467–482, December 1962.

[25] C. J. Stam, A. Hillebrand, H. Wang, and P. Van Mieghem. Emergence of modular structure in a large-scale brain network with interactions between dynamics and connectivity. *Frontiers in Computational Neuroscience*, 4(133):1–13, September 2010.

[26] C. J. Stam and J. C. Reijneveld. Graph theoretical analysis of complex networks in the brain. *Nonlinear Biomedical Physics*, 1(3):1–19, July 2007.

[27] S. H. Strogatz. From Kuramoto to Crawford: exploring the onset of synchronization in populations of coupled oscillators. *Physica D*, 143:1–20, 2000.

[28] S. Tang, E. Jaho, I. Stavrakakis, I. Koukoutsidis, and P. Van Mieghem. Modeling gossip-based content dissemination and search in distributed networking. *Computer Communications*, 34:765–779, 2011.

[29] E. R. van Dam and W. H. Haemers. Which graphs are determined by their spectrum? *Linear Algebra and its Applications*, 373:241–272, 2003.

[30] M. P. Van den Heuvel, C.J. Stam, R.S. Kahn, and H. E. Hulshoff Pol. Efficiency of functional brain networks and intellectual performance. *Journal of Neuroscience*, 29:7620–7625, 2009.

[31] P. Van Mieghem. Topology information condensation in hierarchical networks. *Computer networks*, 31(20):2115–2137, November 1999.

[32] P. Van Mieghem. *Data Communications Networking*. Techne Press, Amsterdam, 2006.

[33] P. Van Mieghem. *Performance Analysis of Communications Systems and Networks*. Cambridge University Press, Cambridge, U.K., 2006.

[34] P. Van Mieghem. *Graph Spectra for Complex Networks*. Cambridge University Press, Cambridge, U.K., 2011.

[35] P. Van Mieghem. Human psychology of common appraisal: the reddit score. *IEEE Transactions on Multimedia*, submitted 2011.

[36] P. Van Mieghem. Robustness of large networks. *International Conference on Systems, Man and Cybernetics (IEEE SMC 2005), Hawaii, USA*, October 10-12 2372-2377.

[37] P. Van Mieghem, C. Doerr, H. Wang, J. Martin Hernandez, D. Hutchison, M. Karaliopoulos, and R. E. Kooij. A framework for computing topological network robustness. Delft University of Technology, Report20101218 (www.nas.ewi.tudelft.nl/people/Piet/TUDelftReports), 2010.

[38] P. Van Mieghem, G. Ge, P. Schumm, S. Trajanovski, and H. Wang. Spectral graph analysis of modularity and assortativity. *Physical Review E*, 82:p. 056113, November 2010.

[39] P. Van Mieghem and F. A. Kuipers. Concepts of exact quality of service algorithms. *IEEE/ACM Transaction on Networking*, 12(5):851–864, October 2004.

[40] P. Van Mieghem and S. M. Magdalena. A phase transition in the link weight structure of networks. *Physical Review E*, 72:056138, November 2005.

[41] P. Van Mieghem, J. Omic, and R. E. Kooij. Virus spread in networks. *IEEE/ACM Transactions on Networking*, 17(1):1–14, February 2009.

[42] P. Van Mieghem, D. Stevanović, F. A. Kuipers, C. Li, R. van de Bovenkamp, D. Liu, and H. Wang. Decreasing the spectral radius of a graph by link removals. *Physical Review E*, 84(1):016101, July 2011.

[43] P. Van Mieghem and L. Vandenberghe. Trade-off curves for QoS routing. *IEEE INFOCOM06*, 2006.

[44] P. Van Mieghem and H. Wang. The Observable Part of a Network. *IEEE/ACM Transactions on Networking*, 17(1):93–105, 2009.

[45] P. Van Mieghem, H. Wang, X. Ge, S. Tang, and F. A. Kuipers. Influence of assortativity and degree-preserving rewiring on the spectra of networks. *The European Physical Journal B*, 76(4):643–652, 2010.

[46] M. Vidal, M. E. Cusick, and A.-L. Barabási. Interactome networks and human disease. *Cell*, 144:986–998, March 2011.

[47] H. Wang, L. Douw, J. M. Hernandez, J. C. Reijneveld, C. J. Stam, and P. Van Mieghem. Effect of tumor resection on the characteristics of functional brain networks. *Physical Review E*, 82(2):021924, August 2010.

[48] H. Wang, E. van Boven, A. Krishnakumar, M. Hosseini, T. Takema, H. van Hooff, N. Baken, and P. Van Mieghem. Multi-weighted monetary transaction network. *Journal Advances in Complex Systems*, to appear 2011.

[49] H. Wang, W. Winterbach, and P. Van Mieghem. Assortativity of complementary graphs. *The European Physical Journal B*, to appear 2011.

[50] D. J. Watts and S. H. Strogatz. Collective dynamics of "small-worlds" networks. *Nature*, 393:440–442, June 1998.

[51] W. Winterbach, H. Wang, M. Reinders, P. Van Mieghem, and D. de Ridder. Correlating the topology of a metabolic network with its growth capacity. *Bionetics 2010, Boston, USA*, December 1-3 2010.