A brief tour through
Network Science

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GameSec 2017
Oct. 23-25, 2017, Vienna

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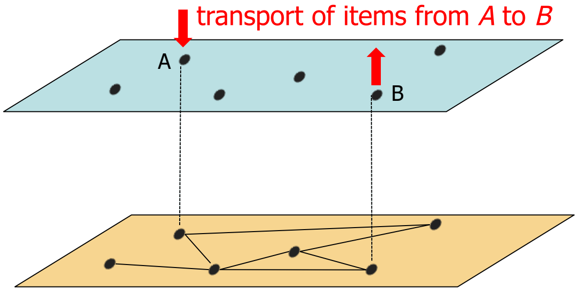
Network: service(s) + topology



Service (function)

software, algorithms

Topology (graph)

hardware, structure

Service and topology

- own specifications
- both are, generally, time-variant
- service is often designed independently of the topology
- often more than 1 service on a same topology

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Network Science

Theory of processes on graphs

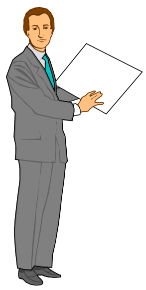
What is new? → **Duality**: both process & graph

Basic question: relation between process & graph

processes "proportional" to the graph
- electrical & water flow
- Laplacian relation between flow and potential
- conservation & physical laws

services independently designed of the graph
- Internet
- actor networks

Outline

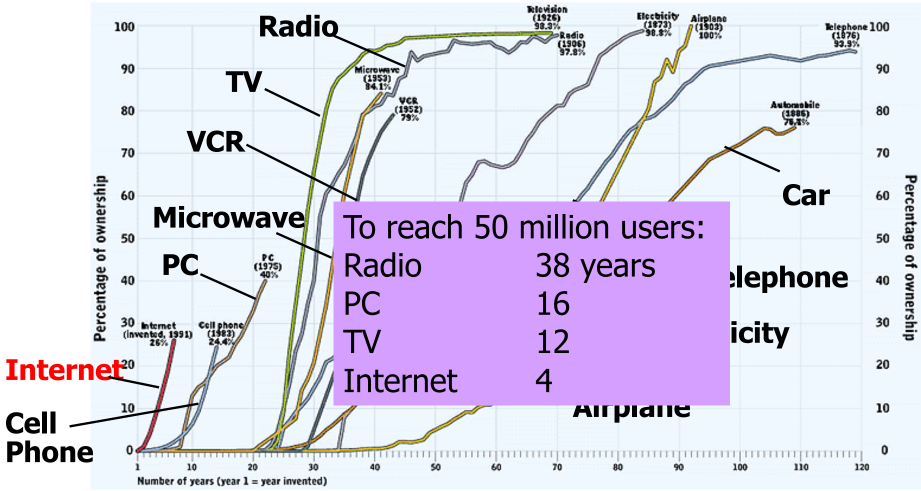


Birth of Network Science

Function and graph

Outlook

Technology Adoption (1997)



Forbes Magazine July 7th, 1997

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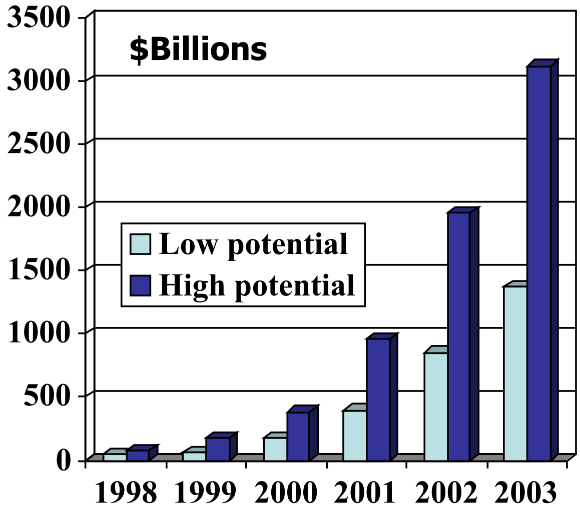
World ICommerce Sales (1999)



Internet:
a Hype or Fact?

Internet Advertising
$0 in 1993
$300 million in 1996
$3.7 billion in 1999

Internet Revolution?
growth in GDP: 2.8%
Internet economy: 175%
from 1995 to 1998

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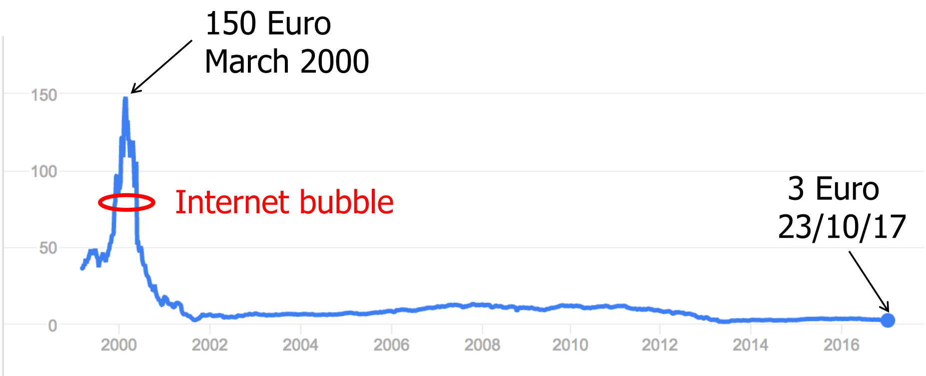
KPN stock over time



KPN: largest Dutch telecom provider (incumbent)

Google finance

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Focal *technical* question in the telecom world before 2000

Can a time-ignorant protocol as the Internet provide real-time services such as telephony and real-time video?

Is the end-to-end delay below roughly 100ms between any pair of communicating nodes?

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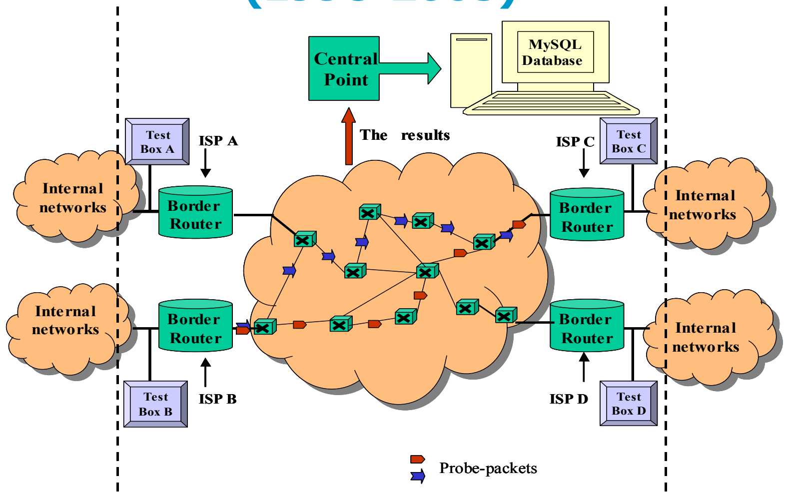
RIPE trace-route measurements (1998-2005)



RIPE NCC (the Network Coordination Centre of the Réseaux IP Européen) Amsterdam

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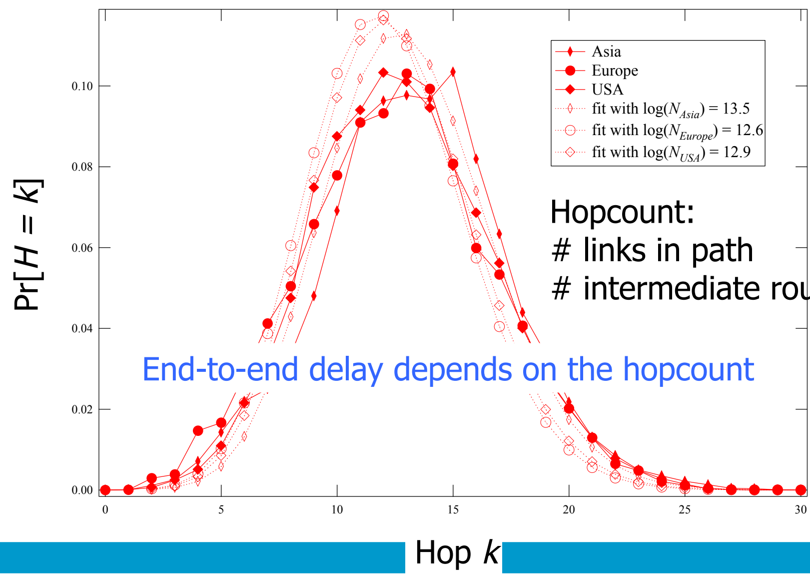
Hopcount of Internet paths (2004)



Hopcount:
links in path
intermediate routers

End-to-end delay depends on the hopcount

P. Van Mieghem, "Performance Analysis of Complex Networks and Systems", Cambridge University Press, 2014.

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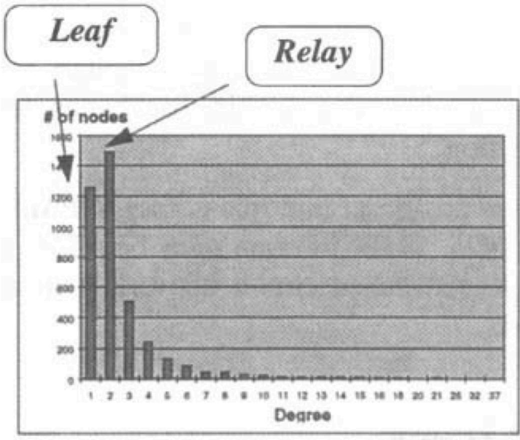
Trace-routes in the Internet 1995



Degree	# of nodes
9	25
10	17
11	11
12	8
13	11
14	11
15	7
16	3
18	3
20	1
21	2
25	1
32	1
37	1

J.-J. Pansiot and D. Grad, "On Routes and Multicast Trees in the Internet", ACM Computer Communication Review, Vol. 28, pp. 41-50, 1998.

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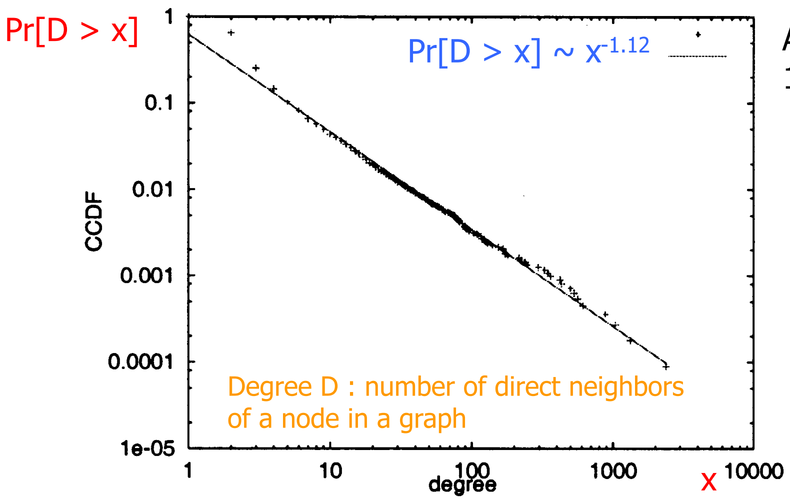
Internet: Power law degree distribution



AS data 1997-2002

G. Siganos, M. Faloutsos, P. Faloutsos & C. Faloutsos , "Power Laws and the AS-Level Internet Topology", IEEE Trans. On Networking, Vol. 11, No. 4, pp. 514- 524, 2003.

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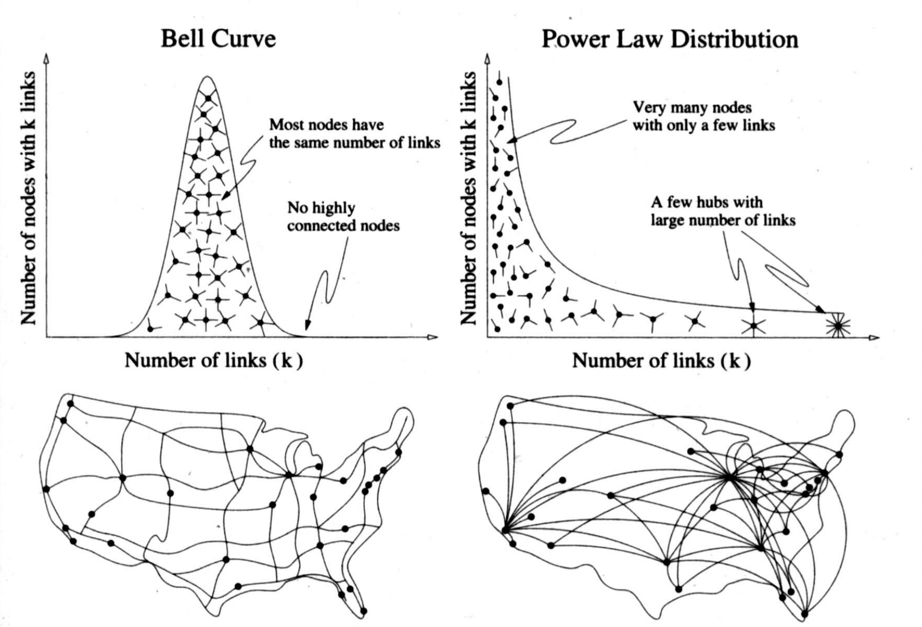

Reprinted from Linked: The New Science of Networks by Albert-Laszlo Barabasi

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Birth of Network Science (around 2000)

- Observation of power law degree in many more networks:
  - World-Wide Web
  - movie actor collaboration network,
  - the human respiratory system
  - the size and location of earthquakes,
  - stock-price fluctuations
  - the web of human sexual contacts
  - biological cellular networks
  - scientific citation network
  - ...
- Pareto (1896), Zipf (1949), Bak (1996), Barabasi (1999)

A.-L. Barabasi, Network Science, Cambridge University Press, 2016

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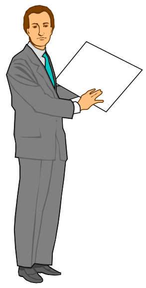
Outline



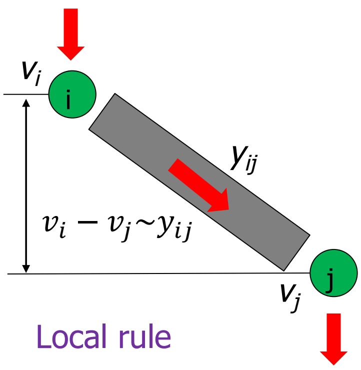
Birth of Network Science

$\boxed{\text{Function and graph}}$

Outlook

Simple dynamics on networks

We study a dynamic process that is "proportional to" or "linear in" in graph of the network

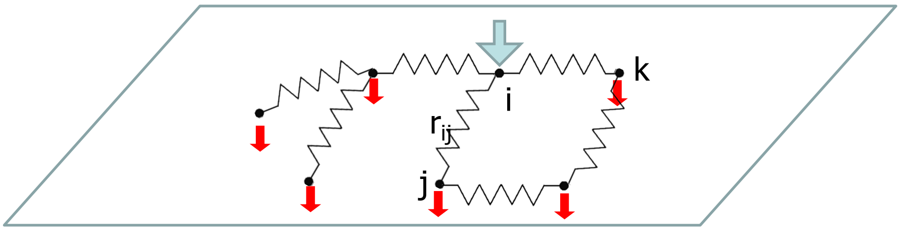


Examples:

- water (or gas) flow is proportional to pressure
- displacement (in spring) is proportional to force
- heat flow is proportional to temperature
- electrical current is proportional to voltage

Linear systems & the concept of superposition

R. P. Feynman, R. B. Leighton and M. Sands, *The Feynman Lectures on Physics* Vol.1, Chapt. 25, 1963



Inverses: $\boldsymbol{x} = \boldsymbol{Qv}$ and $\boldsymbol{v}=Q^{\dagger}\boldsymbol{x}$ with the convention for voltages $u^T v=0$

Q : weighted Laplacian of the graph
$Q^{\dagger}$: pseudo-inverse of the weighted Laplacian: $QQ^{\dagger} = Q^{\dagger}Q = I - \frac{1}{N}J$
x: vector with external nodal current
v: vector with nodal potentials

$v_i = Q^{\dagger}_{ii}$ If $x = e_i - 1/N\, u$

The best spreader is the node *k* with minimum $Q^{\dagger}_{kk}$

P. Van Mieghem, K. Devriendt and H. Cetinay, 2017, "Pseudo-inverse of the Laplacian and best spreader node in a network", Physical Review E, vol. 96, No. 3, p 032311.

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Interpretation: best spreader node

Inverses: $\boldsymbol{x} = \boldsymbol{Qv}$ and $\boldsymbol{v}= Q^{\dagger}\boldsymbol{x}$ Clearly: $u^T x=0$ and $u^T v=0$

$v_i = Q^{\dagger}_{ii}$ is same as $Q^{\dagger}_{ii} = v_i - u^T v = \frac{1}{N}\sum_{k=1}^{N}(v_i - v_k)$

The best spreader minimizes the sum of potential differences *between its own and all other node potentials*

→ "closeness" minimization of average distance to all other nodes

→ best spreader lies in center of "gravity"

$\zeta = (Q^{\dagger}_{11}, Q^{\dagger}_{22}, \cdots, Q^{\dagger}_{NN})$

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Contemplation: best spreader node

- *Valid for any weighted Laplacian*
  - Not heuristic, but based on the **law of conservation**
  - If resistances r_{ij} = 1: pure graph focus
  - The infinitesimal generator of a continuous-time Markov chain (MC) is minus a weighted Laplacian (which is not necessarily symmetric!)
    - Huge potential as nearly all processes can be approximated by a MC, provided the state space is sufficiently large
- *Interpretations*:
  - Ranking of nodes according to "diffusive centrality" or dynamic connectivity to all others
  - Resilience/Robustness: Safe-guarding nodes in this ranking to secure dynamic processes

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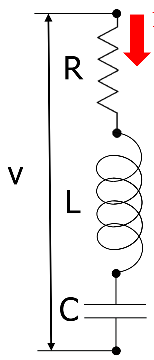
General impedance



$i = \frac{dq}{dt}$

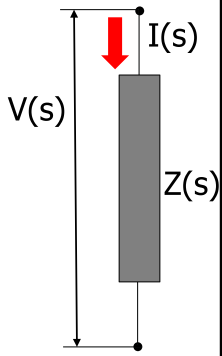
$v_R = Ri$

$v_L = L\frac{di}{dt}$

$v_C = \frac{q}{C}$

Laplace transform

$F(s) = \int_0^\infty e^{-st} f(t) dt$



$v = v_R + v_L + v_C$

$$\boxed{L\frac{di}{dt} + Ri + \frac{1}{C}\int_0^t i(u)du + v_C(0) = v}$$

$$\boxed{V(s) = Z(s)I(s) + T_0(s)}$$

with

$$Z(s) = R + sL + \frac{1}{sC}$$

$$T_0(s) = \frac{v_C(0)}{s} - Li(0)$$

Generalizing to an impedance network

The Laplace-transformed weighted Laplacian $x(s) = Q(s)v(s)$

- Formally, extend the concept of effective resistance matrix towards an N x N effective impedance matrix $\Omega(s)$
- Spectral decomposition of $Q(s) = \sum_{k=1}^{N-1} \mu_k(s) z_k(s) z_k^T(s)$
- Effective Graph Impedance

$$Z_G(s) = N trace\left(Q^\dagger(s)\right) = N \sum_{k=1}^{N-1} \frac{1}{\mu_k(s)}$$

Suggested Problem:

- Analyze the poles of $Z_G(s)$; compute the inverse Laplace transform towards the time-domain; deduce stability of the impedance network
- modify the graph G to enhance or change the stability region
- game ?

21

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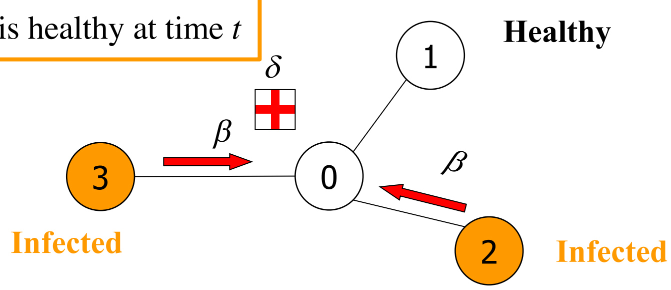
Continuous-time SIS model on networks

- Constant infection rate β on all links
- Constant curing rate δ for all nodes

  $\tau = \beta / \delta$: effective spreading rate

$$\boxed{\begin{array}{ll} X_j(t) = 1 & \text{node } j \text{ is infected at time } t \\ X_j(t) = 0 & \text{node } j \text{ is healthy at time } t \end{array}}$$



Infection and curing are independent Poisson processes

P. Van Mieghem, J. Omic, R. E. Kooij, "Virus Spread in Networks", IEEE/ACM Transaction on Networking, Vol. 17, No. 1, pp. 1-14, (2009).

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Governing SIS equation for node j

$$\boxed{\frac{dE[X_j]}{dt} = E\left[-\delta X_j + (1-X_j)\beta\sum_{k=1}^{N} a_{kj}X_k\right]}$$

time-change of E[X_j] = Pr[X_j = 1], probability that node j is infected

if *infected*: probability of curing per unit time

if *not infected (healthy)*: probability of infection per unit time

$$\frac{dE[X_j]}{dt} = -\delta E\left[X_j\right] + \beta\sum_{k=1}^{N} a_{kj}E\left[X_k\right] - \beta\sum_{k=1}^{N} a_{kj}E\left[X_jX_k\right]$$

R. Pastor-Satorras, C. Castellano, P. Van Mieghem and A. Vespignani, "Epidemic processes in complex networks", Review of Modern Physics, 2015

SIS Prevalence

- Fraction of infected nodes in the graph G

$$S(t) = \frac{1}{N}\sum_{j=1}^{N} X_j(t) \qquad \text{(random variable!)}$$

- Prevalence: Expected fraction of infected nodes in G

$$y(t) = E\left[S(t)\right] = \frac{1}{N}\sum_{j=1}^{N} \Pr\left[X_j(t) = 1\right]$$

also called order parameter in statistical physics

P. Van Mieghem, F. Darabi Sahneh and C. Scoglio, 2014, "Exact Markovian SIR and SIS epidemics on networks and an upper bound for the epidemic threshold", Proceedings of the 53rd IEEE Conference on Decision and Control (CDC'14), December 15-17, Los Angeles, CA, USA (also on http://arxiv.org/abs/1402.1731).

"Local rule - global emergent properties" class

$$\boxed{\frac{dE[X_j(t)]}{dt} = E\left[-\delta X_j(t) + (1 - X_j(t))\beta \sum_{k=1}^{N} a_{kj} X_k(t)\right]}$$

Local SIS rule

Global emergent SIS spread

$$\boxed{\frac{dy(t^*)}{dt^*} = -y(t^*) + \frac{\tau}{N} E\left[w^T(t^*) Q w(t^*)\right]}$$

The Laplacian $Q = \Delta - A$
The normalized time t* = δ t
Bernoulli state vector
$w(t^*) = (X_1(t^*), X_2(t^*), \ldots, X_N(t^*))$

P. Van Mieghem, 2016, "Approximate formula and bounds for the time-varying SIS prevalence in networks", Physical Review E, Vol. 93 No. 5, p. 052312.

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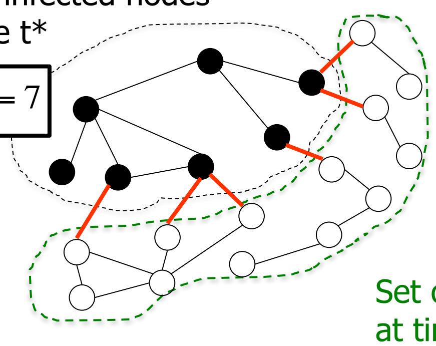
SIS prevalence dynamics

$$\boxed{\frac{dy(t^*)}{dt^*} = -y(t^*) + \frac{\tau}{N} E\left[w^T(t^*) Q w(t^*)\right]}$$

Set of infected nodes at time t*

$\boxed{NS(t^*) = 7}$



$\boxed{w^T(t^*) Q w(t^*) = 6}$

Cut-Set: set of links with 1 infected node at time t*

Set of susceptible nodes at time t*

P. Van Mieghem, 2016, "Universality of the SIS prevalence in networks", Delft University of Technology, report20161006 (http://arxiv.org/abs/1612.01386).

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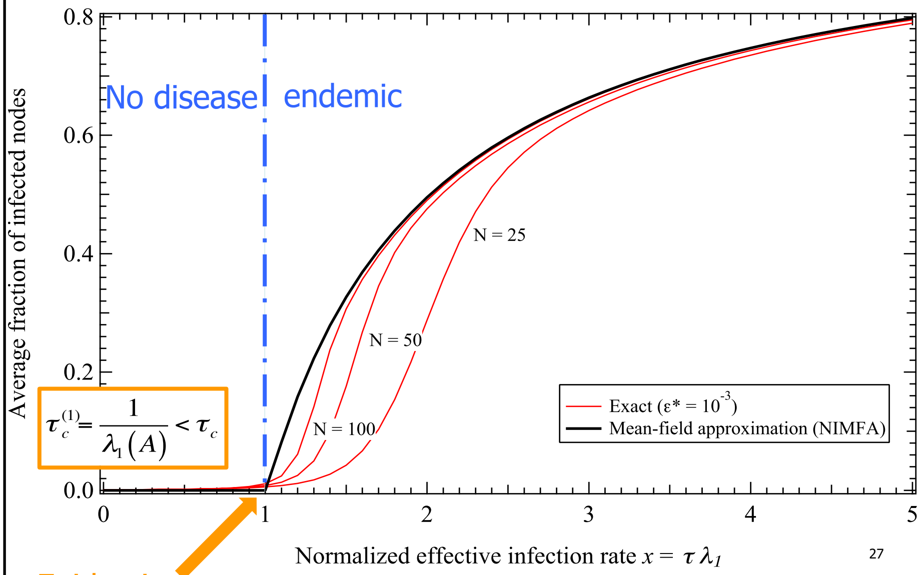
SIS Prevalence versus viral infectiousness



27

Epidemic
Threshold strongly depends on the network!

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Tragedy for large networks

$$\boxed{\tau_c > \tau_c^{(1)} = \frac{1}{\lambda_1(A)}}$$

For large power law networks, the maximum degree d_{max} → large

⬇

Mean-field epidemic threshold $\tau_c^{(1)}$ → zero

⬇

End of the world

!

$\lambda_1(A)$: spectral radius of the adjacency matrix A of graph = largest eigenvalue of A

28

$$\sqrt{d_{\max}} \le \lambda_1(A) \le d_{\max} \Rightarrow \frac{1}{d_{\max}} \le \tau_c^{(1)} \le \frac{1}{\sqrt{d_{\max}}}$$

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Is everything really so bad?

Most real contact networks have bounded degree (d_{max} < constant)

⬇

Mean-field epidemic threshold $\tau_c^{(1)} > 0$

⬇

But, **control of epidemics** is key

- ○ *Underlying graph*: resilience of networks (NAS, TUDelft)
- ○ *Dynamic epidemic process*: 'dynamic' control policies (cut-set, immunization,...)

⬇

Game theoretical approach?

$$\boxed{\tau_c > \tau_c^{(1)} = \frac{1}{\lambda_1(A)}}$$

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Applications to game theory

- several SIS studies in the past
  - ○ game theory together with Ariel Orda, then Eitan Altman & co-workers
- optimal control of cut (time-dependent rule)
- containing epidemic spread:
  - ○ modification of graph + process
  - ○ adaptive epidemics (graph-update rule)
- combination with 'social information' or 'awareness'
  - ○ multilayer view
  - ○ *what* is the best strategy (subject to *which* constraints)

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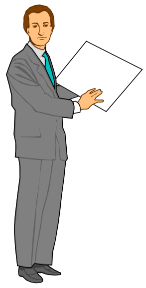
Outline



Birth of Network Science

Function and graph

Outlook

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Robust design of networks

- major difficult: what is robustness?
  - besides network & services, regulation & money
  - human actors (game theory)
- optimization problem?
  - network science: local-rule, global emergent properties
  - control/system theory: operational points around instabilities ('phase transition')
  - game theory: discover the rules & strategy of the game

→ autonomous networking
(i.e. with minimum human interference)

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Big data: *nihil novum sub sole*

Invention of the telescope (17th century)

Large: Galileo → Kepler → Newton → Einstein

Small: van Leeuwenhoek → viruses → quantum mechanics (QM)

Newton (genius!) saw order in the chaos of his time

TODAY: we encounter similar chaos and difficulties as in his time:

complexity of systems (too many variables and actors)

inconsistency between large (RT, relativity theory) and small (QM)

Amazingly:

- quantum gravity: the connection towards consistency between RT and QM seems to lie in "networks"
- new telescope: gravitational waves!

33

34