Network Science
&
Telecommunications

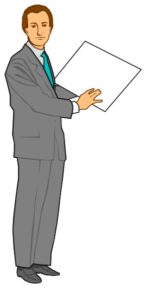
Piet Van Mieghem

ITC30 Networking Science Vision Day
5 September 2018, Vienna

NAS

TU Delft

Outline



Networks

Birth of Network Science

Function and graph

Outlook

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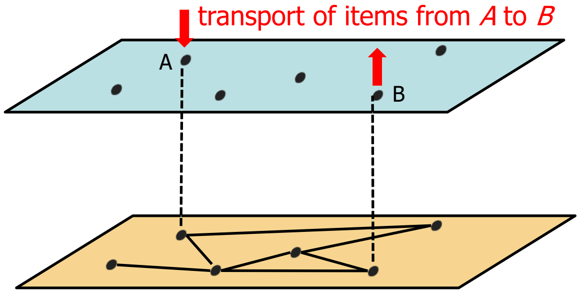
Network: service(s) + topology



Service (function)

software, processes

Topology (graph)

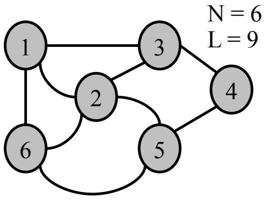
hardware, structure

Service and topology

- own specifications
- both are, generally, time-variant
- service is often designed independently of the topology
- often more than 1 service on a same topology

Three representations of a graph

Topology domain



$$A_{N\times N} = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

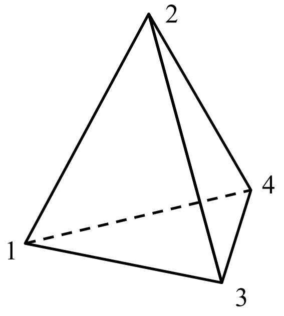
Spectral domain

$$A = A^T = X\Lambda X^T$$

$X_{N\times N}$: orthogonal eigenvector matrix

$\Lambda_{N\times N}$: diagonal eigenvalue matrix

Geometric domain



Simplex in Euclidean *(N-1)*-dimensional space

Devriendt, K. and P. Van Mieghem, 2018, "The Simplex Geometry of Graphs", Delft University of Technology, report20180717. (http://arxiv.org/abs/1807.06475).

Network Science

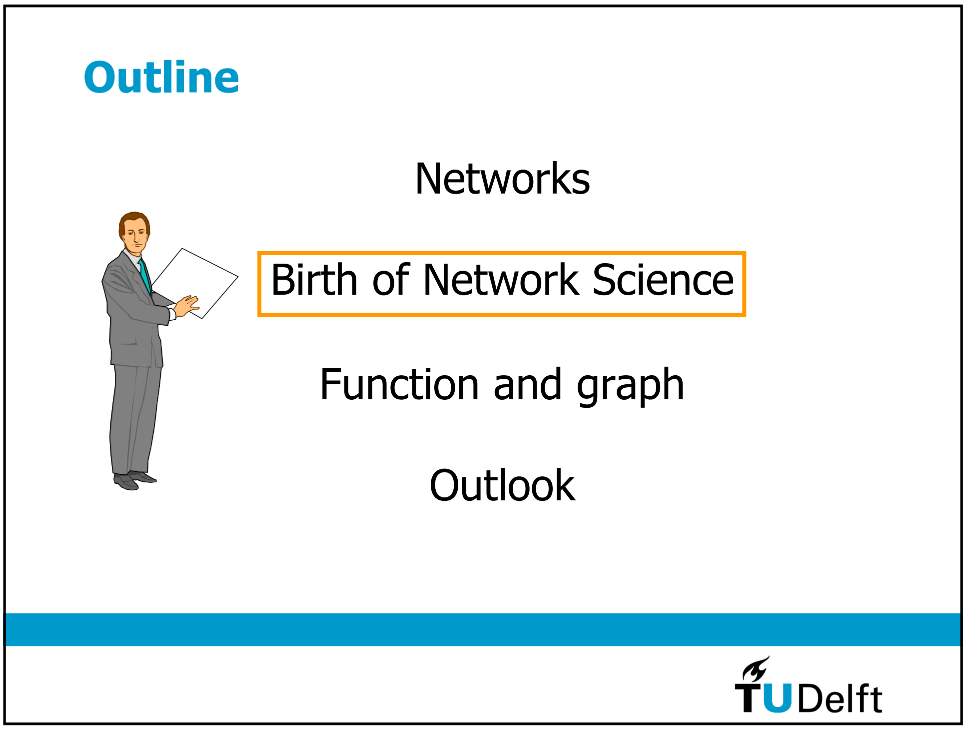
Theory of processes on graphs

What is new? → **Duality**: both process & graph

processes "proportional" to the graph
- electrical & water flow
- Laplacian relation between flow and potential
- conservation & physical laws

↕

services independently designed of the graph
- Internet
- actor networks



Outline

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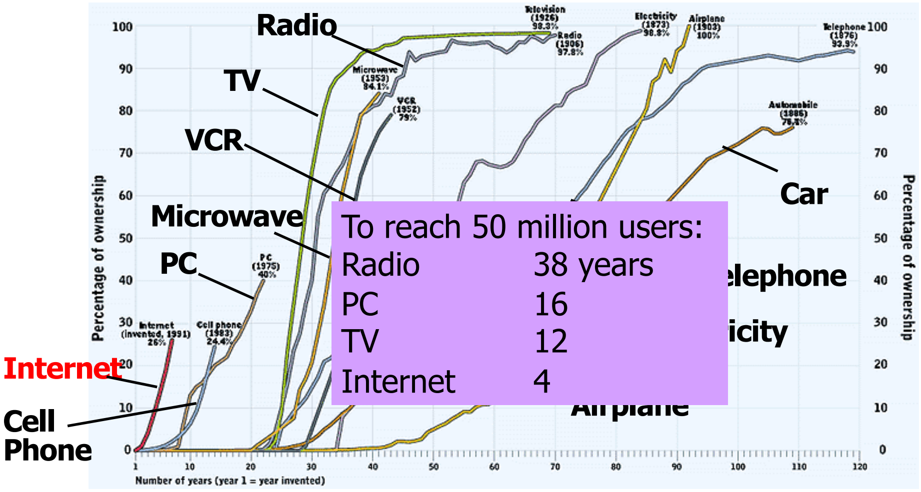
Technology Adoption (1997)



To reach 50 million users:	
Radio	38 years
PC	16
TV	12
Internet	4

Forbes Magazine July 7th, 1997

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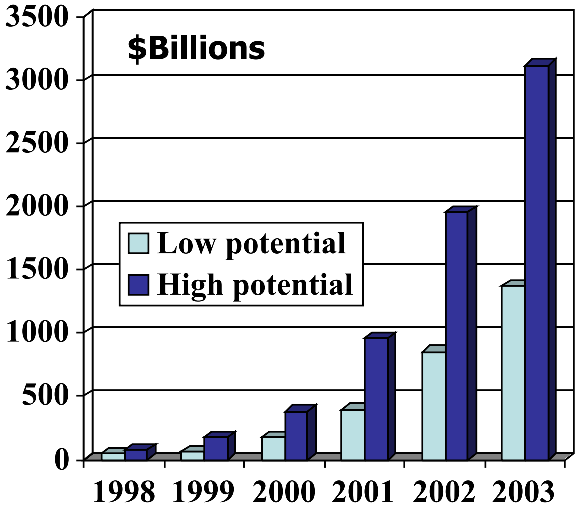
World ICommerce Sales (1999)



Internet:
a Hype or Fact?

Internet Advertising
$0 in 1993
$300 million in 1996
$3.7 billion in 1999

Internet Revolution?
growth in GDP: 2.8%
Internet economy: 175%
from 1995 to 1998

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KPN stock over time

150 Euro
March 2000

Internet bubble

2.3 Euro
29/8/2018

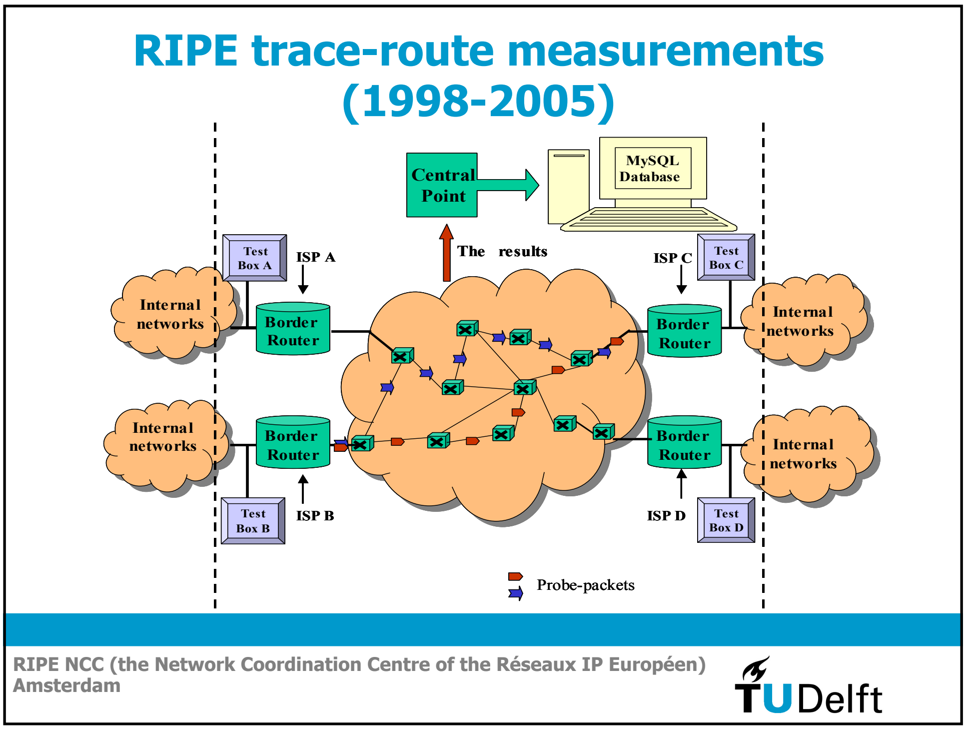
KPN: largest Dutch telecom provider (incumbent)

Google finance

Focal *technical* question in the telecom world before 2000

Can a time-ignorant protocol as the Internet provide real-time services such as telephony and real-time video?

Is the end-to-end delay below roughly 100ms between any pair of communicating nodes?



RIPE trace-route measurements (1998-2005)

RIPE NCC (the Network Coordination Centre of the Réseaux IP Européen) Amsterdam

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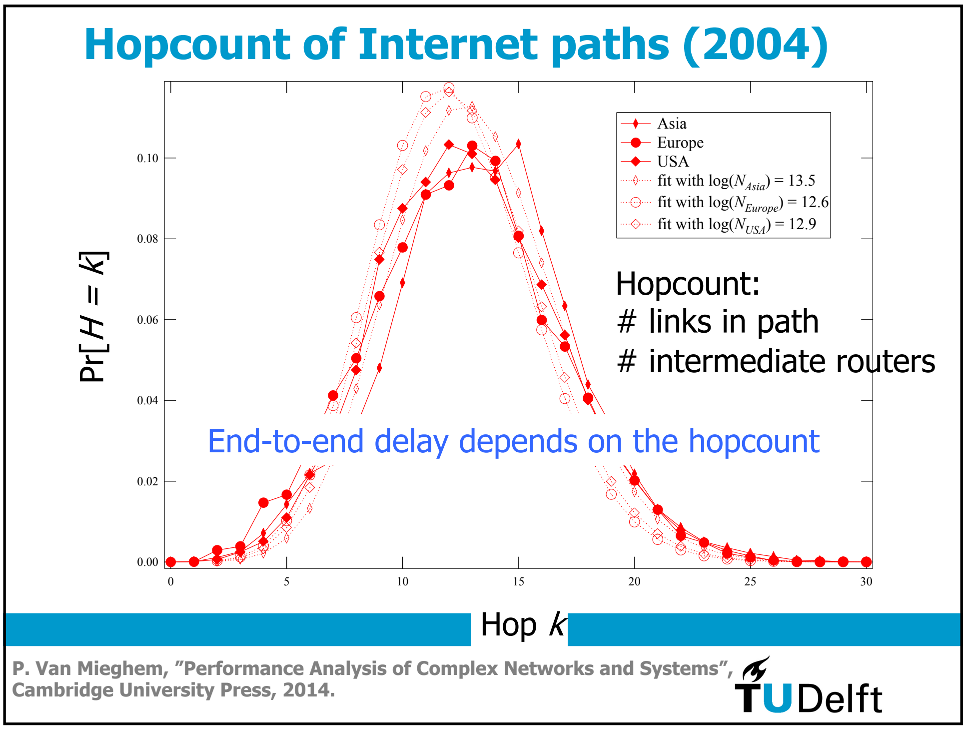

Hopcount of Internet paths (2004)

Hopcount:
links in path
intermediate routers

End-to-end delay depends on the hopcount

P. Van Mieghem, "Performance Analysis of Complex Networks and Systems", Cambridge University Press, 2014.

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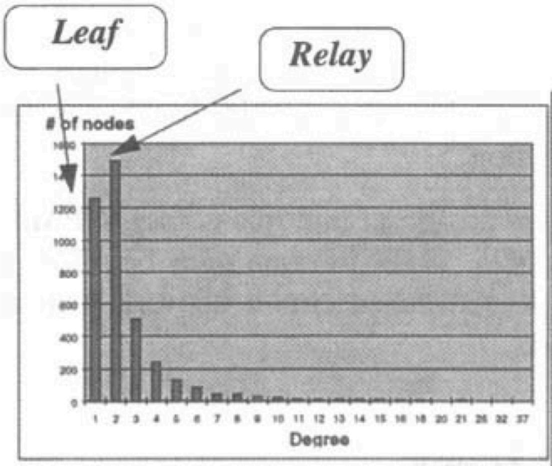
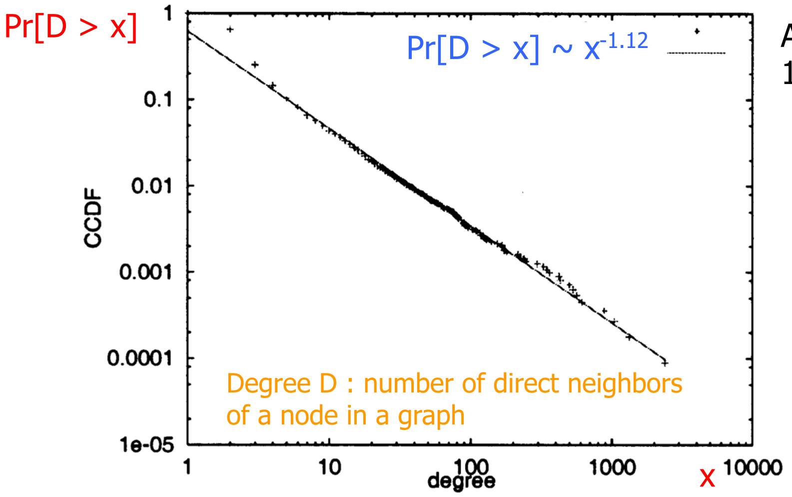
Trace-routes in the Internet 1995



Degree	# of nodes
9	25
10	17
11	11
12	8
13	11
14	11
15	7
16	3
18	3
20	1
21	2
25	1
32	1
37	1

J.-J. Pansiot and D. Grad, "On Routes and Multicast Trees in the Internet", ACM Computer Communication Review, Vol. 28, pp. 41-50, 1998.

Internet: Power law degree distribution

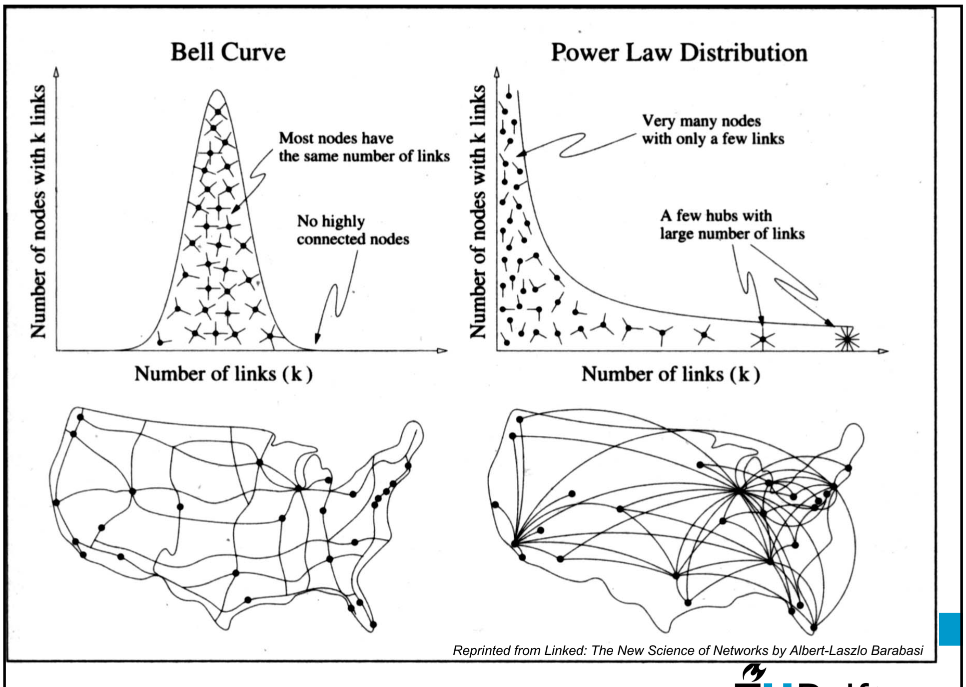


$\Pr[D > x] \sim x^{-1.12}$

Degree D : number of direct neighbors of a node in a graph

AS data
1997-2002

G. Siganos, M. Faloutsos, P. Faloutsos & C. Faloutsos , "Power Laws and the AS-Level Internet Topology", IEEE Trans. On Networking, Vol. 11, No. 4, pp. 514- 524, 2003.



Reprinted from Linked: The New Science of Networks by Albert-Laszlo Barabasi

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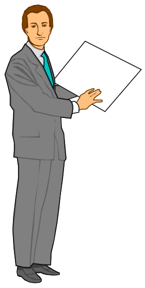
Birth of Network Science (around 2000)

- Observation of power law degree in many more networks:
    - World-Wide Web
    - movie actor collaboration network,
    - the human respiratory system
    - the size and location of earthquakes,
    - stock-price fluctuations
    - biological cellular networks
    - scientific citation network
    - …
- Pareto (1896), Zipf (1949), Bak (1996), Barabasi (1999)

A.-L. Barabasi, Network Science, Cambridge University Press, 2016

Outline



Networks

Birth of Network Science

Function and graph

Process is "linear" in the graph

"Local rule – global emergent" process

Outlook

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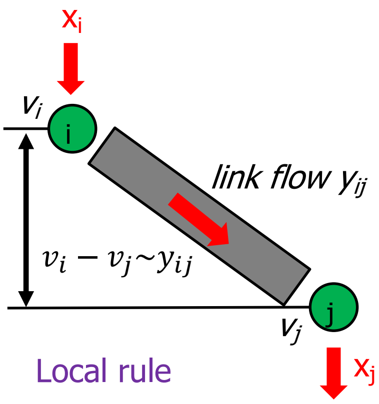
Linear dynamics on networks

Linear dynamic process: "proportional to" (~) graph of network

Examples:

- water (or gas) flow ~ pressure
- displacement (in spring) ~ force
- heat flow ~ temperature
- electrical current ~ voltage



Local rule

$$x = Q \cdot v$$

x	=	Q	.	v
injected nodal current vector		weighted Laplacian of the graph		nodal potential vector

P. Van Mieghem, K. Devriendt and H. Cetinay, 2017, "Pseudoinverse of the Laplacian and best spreader node in a network", Physical Review E, vol. 96, No. 3, p 032311.

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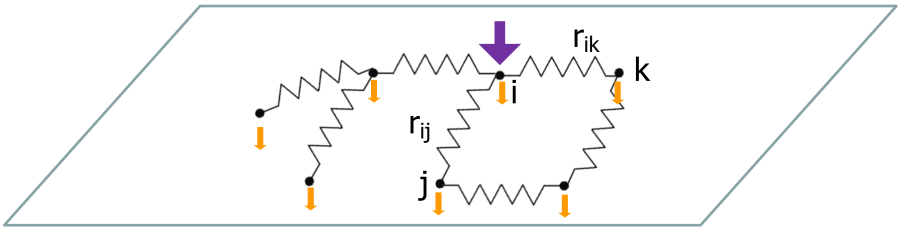


Inverses: $x = Qv \leftrightarrow v = Q^{\dagger}x$ with voltage reference $u^T v = 0$

$Q^{\dagger}$: pseudoinverse of the weighted Laplacian obeying $QQ^{\dagger} = Q^{\dagger}Q = I - \frac{1}{N}J$

$J = uu^T$: all-one matrix u : all-one vector

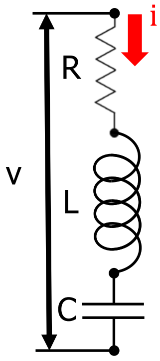
Unit current injected in node *i*
$x = e_i - 1/N\, u$

→

nodal potential of *i*
$v_i = Q^{\dagger}_{ii}$

→ The best spreader is the node *k* with minimum $Q^{\dagger}_{ii}$

Laplace-transformed $x(s) = Q(s)v(s)$



$i = \frac{dq}{dt}$

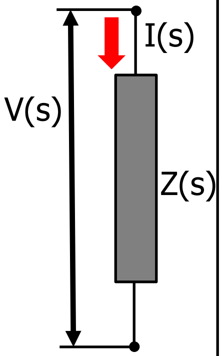
$v_R = Ri$

$v_L = L\frac{di}{dt}$

$v_C = \frac{q}{C}$

Laplace transform →

$$F(s) = \int_0^{\infty} e^{-st} f(t)dt$$



v = $v_R + v_L + v_C$

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int_0^t i(u)du + v_C(0) = v$$

$$V(s) = Z(s)I(s) + T_0(s)$$

with

$$Z(s) = R + sL + \frac{1}{sC}$$

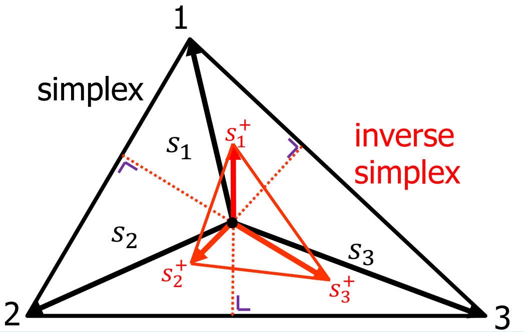
$$T_0(s) = \frac{v_C(0)}{s} - Li(0)$$

Geometry of a graph (dual representation)

Spectral decomposition: $Q^{\dagger} = ZM^{\dagger}Z^{T} = \left(Z\sqrt{M^{\dagger}}\right)\left(Z\sqrt{M^{\dagger}}\right)^{T}$

The matrix $S^{\dagger} = \left(Z\sqrt{M^{\dagger}}\right)^{T}$ has rank *N-1* and $Q^{\dagger} = \left(S^{\dagger}\right)^{T} S^{\dagger}$

The *i*-th column vector $s_i^{\dagger}$ obeys $\left\| s_i^{\dagger} - s_j^{\dagger} \right\|_2^2 = \omega_{ij}$



From $S^{T}S^{\dagger} = I - \frac{uu^{T}}{N}$:

$$\begin{cases} s_j^{T} s_i^{+} = -\dfrac{1}{N} \\ s_i^{T} s_i^{+} = 1 - \dfrac{1}{N} \end{cases}$$

ω_{ij} is the effective resistance between node *i* and *j*

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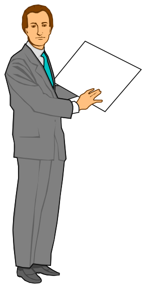
Outline



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"Local rule – global emergent" process

Outlook

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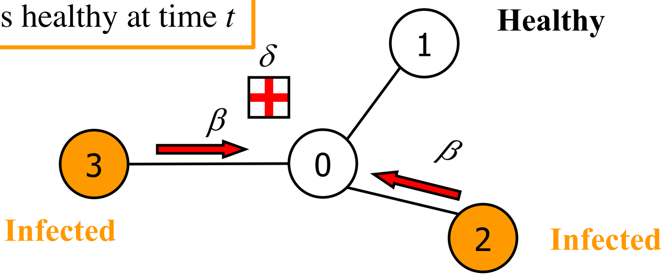
Continuous-time SIS model on networks

- Constant infection rate β on all links
- Constant curing rate δ for all nodes

$\tau = \beta / \delta$: effective spreading rate

$X_j(t) = 1$ node j is infected at time t

$X_j(t) = 0$ node j is healthy at time t



Infection and curing are independent Poisson processes

P. Van Mieghem, J. Omic, R. E. Kooij, "Virus Spread in Networks", IEEE/ACM Transaction on Networking, Vol. 17, No. 1, pp. 1-14, (2009).

Governing SIS equation for node *j*

$$\frac{dE[X_j]}{dt} = E\left[-\delta X_j + (1 - X_j)\beta\sum_{k=1}^{N} a_{kj} X_k\right]$$

time-change of E[X_j] = Pr[X_j = 1], probability that node j is infected

if *infected*: probability of curing per unit time

if *not infected (healthy)*: probability of infection per unit time

$$\frac{dE[X_j]}{dt} = -\delta E\left[X_j\right] + \beta\sum_{k=1}^{N} a_{kj} E\left[X_k\right] - \beta\sum_{k=1}^{N} a_{kj} E\left[X_j X_k\right]$$

R. Pastor-Satorras, C. Castellano, P. Van Mieghem and A. Vespignani, "Epidemic processes in complex networks", Review of Modern Physics, 2015

SIS Prevalence

- Fraction of infected nodes in the graph G

$$S(t) = \frac{1}{N}\sum_{j=1}^{N} X_j(t) \qquad \text{(random variable!)}$$

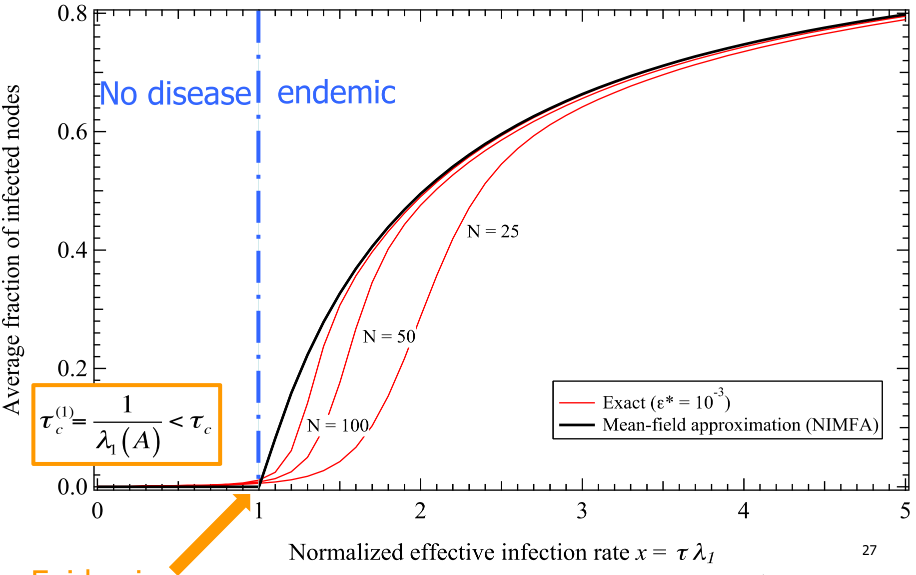
- Prevalence: Expected fraction of infected nodes in G

$$y(t) = E\left[S(t)\right] = \frac{1}{N}\sum_{j=1}^{N} \Pr\left[X_j(t) = 1\right]$$

also called order parameter in statistical physics

P. Van Mieghem, F. Darabi Sahneh and C. Scoglio, 2014, "Exact Markovian SIR and SIS epidemics on networks and an upper bound for the epidemic threshold", Proceedings of the 53rd IEEE Conference on Decision and Control (CDC'14), December 15-17, Los Angeles, CA, USA (also on http://arxiv.org/abs/1402.1731).

SIS Prevalence versus viral infectiousness



$$\tau_c^{(1)} = \frac{1}{\lambda_1(A)} < \tau_c$$

Epidemic
Threshold strongly depends on the network!

"Local rule - global emergent properties" class

$$\frac{dE[X_j(t)]}{dt} = E\left[-\delta X_j(t) + (1 - X_j(t))\beta \sum_{k=1}^{N} a_{kj} X_k(t)\right]$$

Local SIS rule

Global emergent SIS spread

$$\frac{dy(t^*)}{dt^*} = -y(t^*) + \frac{\tau}{N} E\left[w^T(t^*) Q w(t^*)\right]$$

The Laplacian $Q = \Delta - A$
The normalized time t* = δ t
Bernoulli state vector
$w(t^*) = (X_1(t^*), X_2(t^*), \ldots, X_N(t^*))$

P. Van Mieghem, **2016, "Approximate formula and bounds for the time-varying SIS prevalence in networks", Physical Review E, Vol. 93 No. 5, p. 052312.**

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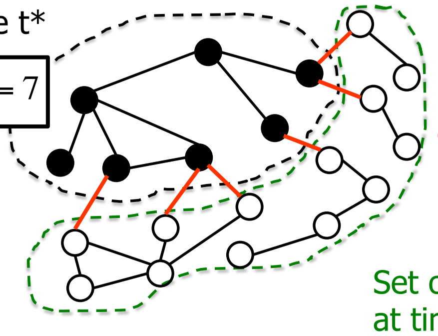
SIS prevalence dynamics

$$\frac{dy(t^*)}{dt^*} = -y(t^*) + \frac{\tau}{N} E\left[w^T(t^*) Q w(t^*)\right]$$

Set of infected nodes
at time t*

$NS(t^*) = 7$



$w^T(t^*) Q w(t^*) = 6$

Cut-Set: set of links with 1 infected node at time t*

Set of susceptible nodes
at time t*

P. Van Mieghem, 2016, "Universality of the SIS prevalence in networks", Delft University of Technology, report20161006 (arxiv1612.01386).
Van Mieghem, P. and K. Devriendt, 2018, "An Epidemic Perspective on the Cut Size in Networks", Delft University of Technology, report20180312.

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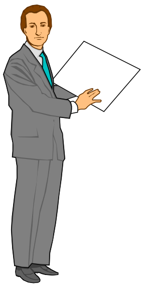

Networks

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Function and graph

Outlook

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Types of Networks

- Infrastructural networks (public, high value)
    - telecommunication, electricity, transport (train, air, car, ship), water, gas, waste
- Economic networks:
    - banks; flow of money, products, ...
- Biological networks:
    - organisms, metabolic, brain, ...
- Relational networks: private, graph clear; function?
    - Online social (Twitter, Facebook, ...), family trees, company/nation organization, collaboration, software structure, ...

Network isomorphisms: what is common in all those networks?

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Robust design of networks

- Robust/resilient networks against failures
  - major difficult: what is robustness?
  - ○ We need standardization to agree on a computable R-value
- Optimization problem?
  - ○ network science: local-rule, global emergent properties
  - ○ control/system theory: operational points around instabilities ('phase transition')
  - ○ game theory: discover the rules & strategy of the game

➡ autonomous networking
(i.e. with minimum human interference)

➡ self-adaptive networking (brain)

Books

Performance Analysis of Complex Networks and Systems
Piet Van Mieghem

Graph Spectra for Complex Networks
Piet Van Mieghem
CAMBRIDGE

Data Communications Networking
Piet Van Mieghem

Articles: **http://www.nas.ewi.tudelft.nl**

Open Postdoc position

Optimal scheduling and routing with stringent end-to-end delay requirements

(i.e. combining ‘service’ and ’topology’)

More info? Please, email me

Thank You

Piet Van Mieghem
NAS, TUDelft
P.F.A.VanMieghem@tudelft.nl