Study of Connectivity in Wireless Ad-hoc Networks with an Improved Radio Model

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Abstract. In this paper we study connectivity in wireless ad-hoc networks by modeling the network as an undirected geometric random graph. The novel aspect in our study is that for finding the link probability between nodes we use a radio model that takes into account statistical variations of the radio signal power around its mean value. We show that these variations, that are unavoidably caused by the obstructions and irregularities in the surroundings of the transmitting and the receiving antennas, have two distinct effects on the network. Firstly, they reduce the amount of correlation between links causing the geometric random graph tend to behave like a random graph with uncorrelated links. Secondly, these variations increase the probability of long links, which enhances the probability of connectivity for the network.
Another new result in our paper is an equation found for the calculation of the giant component size in wireless ad-hoc networks, that takes into account the level of radio signal power variations. With simulations we show that for the planning and design of wireless ad-hoc networks or sensor networks the giant component size is a good measure for "connectivity".

1 Introduction

Wireless multi-hop ad-hoc networks are formed by a group of nodes that communicate with each other over wireless channels. In this network, any node may have direct radio links with some other nodes in its vicinity. The nodes in a wireless ad-hoc network can be mobile. Each node can, if needed, function as a relay station for routing traffic to its final destination. Ad-hoc networks are decentralized, self-organizing networks and are capable of forming a communication network without relying on any fixed infrastructure.

Many aspects of wireless ad-hoc networks have been studied or are under investigation by the international research community. For example, extensive work has been done in the development and optimization of ad-hoc routing protocols ([1] and [2]). Other valuable papers have investigated the capacity and scalability of wireless ad-hoc networks (see for example [3] and [4]). Study of recent literature reveals that reliable mathematical modeling of ad-hoc networks is gaining increased attention ([5], [6], [7]). Good modeling of ad-hoc networks is essential to investigate fundamental properties of ad-hoc networks like connectivity and degree distribution. This paper is a contribution to mathematical modeling and better understanding of one of these fundamental properties, the connectivity. From a practical point of view, connectivity is a prerequisite to providing reliable applications to the users of a wireless ad-hoc network. To achieve a fully connected ad-hoc network there must be a path from any node to any other node. The path between the source and the destination may consist of one hop (when source and destination are neighbors) or several hops. When there is no path between at least one source-destination pair the network is said to be disconnected. A disconnected network may consist of several disconnected islands or clusters. The largest cluster in the network is called the giant component [8].

Connectivity in ad-hoc networks has been studied previously in various papers (see for example [9]). However, in our paper we have used for the first time the so called log-normal shadowing radio propagation model to study connectivity. This radio model takes statistically into account the dynamics of radio signal power variations. These variations are unavoidably caused by obstructions and irregularities in the surroundings of the transmitting and the receiving antennas. Therefore, this radio model is more realistic than the static and solely on distance dependent models that are commonly used to model wireless ad-hoc networks. We show here that these variations strongly affect the behavior of the network. We realize that radio channel modeling is a very complicated topic and by no means we claim to have used the most suitable model in this paper. As a matter of fact, we believe that radio modelling for better understanding of ad-hoc network characteristics is a research area where a lot needs to be done yet. Our approach here should therefore be seen as an attempt towards this goal.

Results presented in this paper also demonstrate that full connectivity is achieved at relatively high values of the mean nodal degree, while at far lower values of the mean degree a very large portion of the network could already be connected. Therefore we argue that for practical use of ad-hoc networks 1-connectivity (full connectivity) is too stringent a condition, and suggest to use the giant component size as a measure for connectivity in wireless ad-hoc networks.

The structure of this paper is as follows. Section 2 describes the radio model used in our study. Our mathematical presentation of wireless ad-hoc networks, which is based on a general model for geometric random graphs in combination with the aforementioned radio model, is described in Section 3. In Section 4 we explain two theoretical theorems introduced in the literature for connectivity in ad-hoc networks and examine them based on simulations using our radio model. We will see that our mathematical modeling of wireless ad-hoc networks in combination with our simulation results allows us to refine the existing theorems. In Section 5 we focus on the giant component and provide an equation for finding the giant component size in a wireless ad-hoc network as function of the mean degree. Our main conclusions are summarized in Section 6.

2 The radio model

In radio communications, the received signal levels decrease as the distance between the transmitter and the receiver increases. This phenomenon is called pathloss. Attenuation of radio signals due to the pathloss effect has been modeled by averaging the measured signal powers over long times and over various locations with the same distances to the transmitter. The mean value of signal power found in this way is referred to as the *area mean power,* $\mathbf{P}_a$ (in Watts or milli-Watts). The pathloss model states that $\mathbf{P}_a$ is a decreasing function of distance r between transmitter and the receiver and can be represented by a power law [10] as $\mathbf{P}_a(r) = c \cdot r^{-\eta}$. In this formula c is a constant[1] and η is the pathloss exponent. The pathloss exponent depends on the environment and terrain structure and can vary between 2 in free space to 6 in heavily built urban areas [11].

The most commonly used radio model in ad-hoc networks is based on the pathloss phenomenon alone and assumes that the received power at any distance to the transmitter is equal to the area mean power (see e.g. [12]). If we assume that transmitted signals are received correctly when the received signal power is more than a minimum required threshold value[2], this model result into a circular coverage area around the transmitting node. All nodes and only those nodes within this circle are connected to the center node. We indicate the radius of the circular coverage area by R. If we normalize the distance between the transmitter and the receiver to R, the probability of connectivity as function of the normalized distance $\widehat{r} \triangleq r/R$ between two nodes, $p(\widehat{r})$, is a simple step function: $p(\widehat{r}) = 1$ if $\widehat{r} < 1$ and $p(\widehat{r}) = 0$ otherwise.

The pathloss model could be inaccurate because in reality the received power levels may show significant variations around the area mean power value. Due to these variations, short links could disappear while long links could merge (see Figure 1). In this paper we use a more realistic radio model for the study of wireless ad-hoc networks. This model is based on the log-normal shadowing radio propagation model[3] and allows for random power variations around the area mean power. In the lognormal radio model the mean received power taken over all possible locations that are at distance r to the transmitter is equal to the area mean power, similar to the pathloss model. However it is further assumed that the time averaged received power varies from location to location in an apparently random manner [13]. More precisely, the logarithmic value of the mean power at different locations is normally distributed (with standard deviation σ) around the logarithmic value of the area mean power. The standard deviation is

[1] The value of this constant depends on the transmitted power, the receiver and the transmitter antenna gains and the wavelength [11].

[2] From a communication theory point of view, this threshold is chosen such that the signal-to-noise ratio at the receiver is sufficiently large to support the desired data communication speed over the channel.

[3] The term "shadowing" used in the name of this model is somehow confusing because shadowing may imply that the model considers correlated fading in the received power at two locations blocked from the transmitter by means of e.g. a single wall. This however is not the case. Variations in signal powers at different location with the same distance to the receiver are assumed to be random and independent. The dependent reduction in radio signal powers due to obstruction by buildings is better referred to by the term "blocking" and is not included in the model.

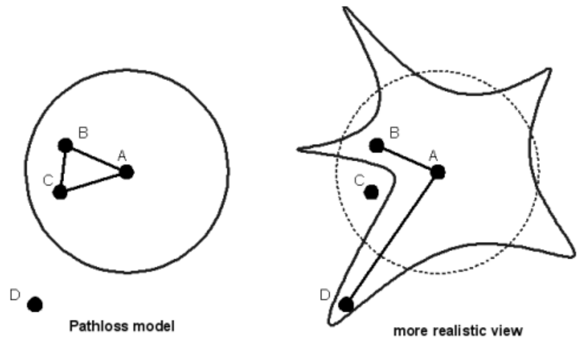

Figure 1. Abstract view showing links between nodes with and without random power variations.

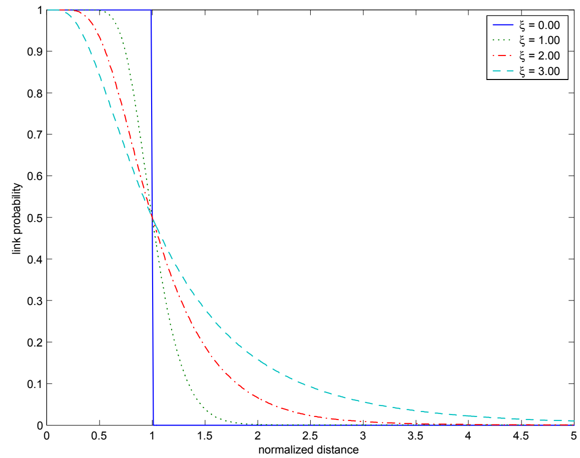

Figure 2. Link probability with log-normal shadowing radio model as function of the normalized distance between two nodes and for different values of ξ. The connected line for $\xi = 0$ is the same as the step function based on the the pathloss model.

larger than zero and, in the case of severe signal fluctuations due to irregularities in the surroundings of the receiving and transmitting antennas, measurements [11] indicate[4] that it can be as high as 12 dB. This model is described in detail in our previous work [14]. The link probability found based on this model in [14] is:

$$p(\widehat{r}) = \frac{1}{2}\left[1 - \operatorname{erf}\left(\alpha \frac{\log(\widehat{r})}{\xi}\right)\right], \quad \xi \triangleq \sigma/\eta. \tag{1}$$

In this formula α is a constant $\alpha = 10/(\sqrt{2}\log 10)$, and $\widehat{r}$ is the normalized distance between the transmitter and the receiver. The parameter ξ is defined as the ratio between the standard deviation of shadowing, σ, and the pathloss exponent, η. Low values of ξ correspond to small variations of signal power around the area mean power and high values of ξ correspond to stronger shadowing effects. In the case of $\xi \to 0$, there is no shadowing effect and our model is equivalent to the pathloss model (a simple step function). The best way to determine the most probable value range for ξ is through extensive measurements. To our knowledge this type of measurements for typical wireless ad-hoc network environments are not available yet. However, based on the aforementioned range of possible values for η and σ, we note that empirically ξ may vary between 0 and 6.

Figure 2 shows for different values of ξ the link probability calculated with (1). For $\xi > 0$ there is a nonzero probability that nodes at a normalized distance larger than 1 are connected. Also, there is a nonzero probability that nodes at normalized distances less than 1 are disconnected. In this figure we see that as shadowing becomes more severe, the link probability at short distances reduces, while at large distances the link probability increases. We mention here briefly that especially the long-distance connectivity probability will affect the hopcount and connectivity in the network; similar to the small world networks extended with a few "long" links [15]. This matter will be investigated extensively in Section 4.

3 The geometric random graph model

We denote a random graph by $G_p(n)$, where n is the number of nodes in the graph and p is the probability of having a link (edge) between any two nodes [8]. The fundamental assumption in random graphs is that the presence of a link between two nodes is *independent* of the presence of any other link. The degree of a node is defined as the number of nodes connected directly to that node. In other words, the degree

[4] It should be noted that the measurements that we refer to have been done on lower frequencies than frequencies used in WLAN networks. If a wireless ad-hoc network is making use of WLAN radio modules, the range of variation in σ could be different.

of a node is the number of neighbors of that node. In a random graph the degree, d, has a binomial distribution [8]:

$$\Pr\left[d=k\right]=\binom{n-1}{k}p^{k}\left(1-p\right)^{n-1-k}\simeq\frac{z^{k}e^{-z}}{k!},$$

where z is the mean (average) nodal degree: $z = E\left[d\right] = (n-1)p$. The second expression above is the Poisson approximation for large n.

A wireless ad-hoc network consists of a number of nodes (radio devices) spread over a certain geographic area. Every node may be connected to other nodes in its vicinity. We assume that connections between nodes are two-way, undirected links. Because of node movements and radio signal fluctuations, the topology of the network can change over time. At any instant in time an ad-hoc network can be considered as a graph with a fixed topology, but it cannot be modeled as a random graph. The reason is that in a wireless ad-hoc network the actual set of connections, in contrast to random graphs, depends on the geometric distance between nodes. A direct consequence of the dependency of the links on the distance between nodes is that in wireless ad-hoc networks there is an increased probability of two nodes to be connected when they have a common neighbor. In other words, in a wireless ad-hoc network links are *correlated*. This effect is also called clustering [16] and has been observed and studied extensively for other network types like social networks [5]. In the literature, graphs with distance-dependent links between nodes and correlated links are referred to as geometric random graphs (see e.g. [6]). An undirected geometric random graph with n nodes is denoted as $G_{p(r_{ij})}(n)$, where $p(r_{ij})$ is the probability of having a link between two nodes i and j (or j and i) at distance r_{ij}.

Geometric random graphs models proposed so far to model wireless ad-hoc networks (see e.g. [9], [12] and [17]) are based on the pathloss radio model. This means that the necessary and sufficient condition for two nodes to be connected is that the distance between them is less than a certain threshold value[5]. In all these models, due to the identical circular coverage area around all nodes, the network behavior resembles a highly clustered regular lattice.

In this paper for the study of connectivity in wireless ad-hoc networks we also use a geometric random graph model. However, for the link probability between nodes we use the radio model explained in Section 2. We denote our geometric random graph as $G_{p(\widehat{r}_{ij})}(n)$, where $\widehat{r}_{ij}$ is the normalized distance between nodes i and j, and $p(\widehat{r}_{ij})$ is specified by (1). There is a substantial difference between our geometric random graph model and the forms proposed till now in the literature. Our model can take into account variations in the received radio signal power. Due to these variations the strict distance dependency of the link probability is blurred and correlation between links is reduced. Correlation between links decreases with increasing ξ. Therefore we expect our geometric random graph model to shift between a highly clustered regular lattice (for $\xi \to 0$) and a random graph (for highest values of ξ). In Section 4 we focus deeper into this point and will verify this statement.

4 Connectivity

In this Section we will first provide an overview of theoretical published results for the connectivity in random graphs and in geometric random graphs. Subsequently, based on our simulation results we will show that our geometric random graph model allows us to refine these connectivity theorems for wireless ad-hoc networks.

To elaborate our definition of connectivity, we regard connectivity to be independent from traffic load in the network. On the physical layer connectivity between nodes is predicted by the radio model explained in Section 2. Whether two connected nodes can communicate with each other at any given moment in time depends (a.o.) on the interference condition which is directly linked to simultaneous communication between other nodes in the network. Due to interference, communication between two connected nodes may drop to lower speeds or even become impossible at certain times. However, in these cases we say that the link capacity is reduced, instead of saying that the probability of connectivity between these two nodes is decreased. In other words, we consider interference as a capacity-affecting factor.

[5] In [3] the effects of interference are considered and it is assumed that a link can only be used when the ratio of the wanted signal power to the sum of the noise and interference power caused by other node's communications is more than a certain threshold value. However, here too it is assumed that signal power loss depends only on the distance between nodes.

4.1 Analytical derivations

For the study of connectivity we consider a wireless ad-hoc network at any instant in time as a graph with fixed topology. Two paths in a graph are said to be independent if any node common to both paths is an end-node of both paths. A graph is said to be k-connected if for each pair of nodes there exist at least k mutually independent paths connecting them [9]. An other equivalent definition [8] is that a graph is k-connected if and only if there is no set of $k-1$ nodes whose removal would disconnect the graph. The connectivity of a graph is the maximum k such that the graph is k-connected. In the literature connectivity has been studied for random graphs as well as geometric random graphs. Here we give an overview of two main theorems with relation to connectivity.

Theorem 1. *If we start with a graph on n vertices and an empty edge set and add edges randomly and independently one by one until having m edges, the graph almost surely*[6] *becomes 1-connected when $m \geq \frac{n \log n}{2} + O(n)$. Considering that $p \triangleq m/\binom{n}{2}$, we can say that for a random graph to be 1-connected there must hold:*

$$p \geq \frac{\log(n)}{n} \qquad a.s. \tag{2}$$

Theorem 1 dates from the pioneering work of Erdös and Rényi [18] on random graphs where they considered the $G_p(n)$ model to study the threshold for connectivity in graphs. While (2) holds for random graphs, in [17] and [9] it is shown that this result is also valid for geometric random graphs with the pathloss radio model, in any dimension higher than one (but not for one-dimensional graphs).

Intuitively one may see that the connectivity in wireless ad-hoc networks depends on the number of nodes per unit area and on the transmission range of wireless devices. Increasing the density of nodes or increasing the transmission power of a radio node will increase the nodal degree. Based on this deduction, it is not surprising to see that the second theorem of connectivity relates connectivity to the nodal degree.

Theorem 2. *In a random graph of n nodes if edges are added one by one to the empty graph in an order chosen uniformly at random form the $\binom{n}{2}!$ possibilities, then almost surly the resulting graph becomes k-connected when it achieves a minimum degree of k. In other words, for large n,*

$$\Pr\left[G \text{ is } k\text{-connected}\right] = \Pr\left[d_{\min} \geq k\right] \qquad a.s. \tag{3}$$

where $d_{\min}$ is the minimum degree per node.

Theorem 2 is proved for random graphs in [8]. In [12] and [9] it is proved that this theorem is also valid for geometric random graphs with the pathloss radio model, in any dimension higher than one *when* $\Pr\left[d_{\min} \geq k\right]$ *is almost 1.*

These two theorems of connectivity are not conflicting theorems for random graphs. It can be proved (see appendix) for random graphs, that for large n, $\Pr\left[G_p(n) \text{ is 1-connected}\right] \simeq 1$ if $p > \log(n)/n$, and $\Pr\left[G_p(n) \text{ is 1-connected}\right] \simeq 0$ if $p < \log(n)/n$.

In the remainder of this section we will investigate connectivity in wireless ad-hoc networks by using our geometric random graph model explained in Section 3. As mentioned before, this model is more realistic than geometric random graph models with the pathloss radio model. We present results obtained through simulations. As it will be summarized at the end of this section, we believe that our simulation results provide new insights into the theory of connectivity in wireless ad-hoc networks.

4.2 Simulations and discussion

Our focus will be on 1-connectivity. Higher orders of connectivity are not considered at this moment. The simulation program distributes n nodes uniformly over a square area and establishes links between node-pairs using the probability function (1). The *service area* of the ad-hoc network is the whole area where nodes are uniformly distributed. In the resulting graph for each simulation run we check the 1-connectivity and store information regarding the number of clusters (components) in the graph, the mean component size, the total number of components and the degree distribution. We have performed simulations with $n = 250$, 500 and 1000. For each value of n, results are gathered for $\xi = 0, 1, .., 6$ and

[6] We say that a graph has some property Q *almost surely* (a.s.) or *with high probability* (whp) if the probability it has Q tends to one as n tends to infinity.

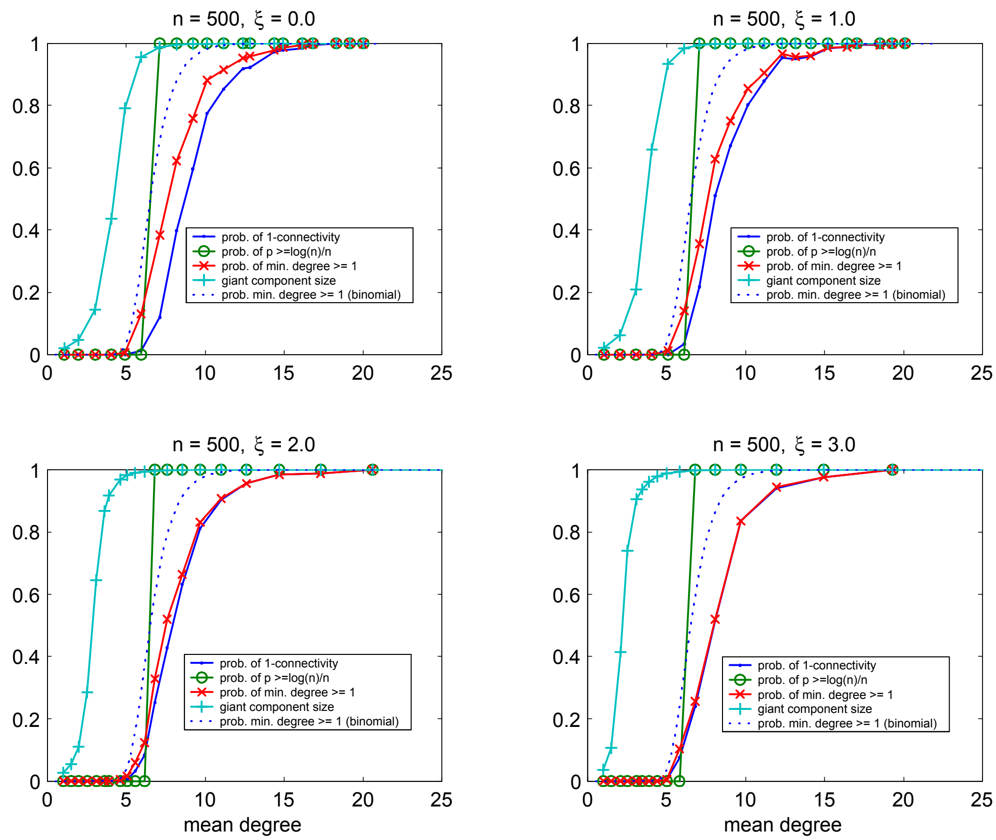

Figure 3. Simulated results for different values of ξ showing: the probability of 1-connectivity, the probability of p exceeding the $log(n)/n$ threshold, the probability of the minimum node degree being more than or equal to one, and the giant component size as fraction of the total number of nodes. For comparison reasons, we have drawn on each graph the probability of minimum degree being more than or equal to one for a binomial degree distribution.

different values of the area size. Changing the area size changes the expected values for the nodal degree and allows us to study connectivity as function of the mean degree. For each unique combination of the area size, ξ and n we have repeated simulations with 500 independent network configurations.

Two different procedures can be used for checking 1-connectivity [7]. The first procedure, based on Prim's algorithm for spanning trees, chooses a node at random and uses a simple flooding algorithm to tag all nodes belonging to the same cluster. This procedure is repeated for all untagged nodes until no untagged nodes remain in the graph. If the largest cluster found in this way contains all nodes, the network is 1-connected. In the process of checking for 1-connectivity, this procedure provides us the exact size of all clusters in the graph. By definition we will call the largest cluster in the graph the *giant component.* The *size* of each cluster is defined as the ratio of the number of nodes in that cluster to the total number of nodes in the network. Similarly, the *giant component size* is the ratio of the number of nodes in the giant component to the total number of nodes forming the network. The second procedure for checking 1-connectivity uses the $n \times n$ Laplacian of G. The Laplacian [19] is the difference between the diagonal node degree matrix, in which element (i, i) is degree of the node i; and the adjacency matrix, in which element (i, j) is one or zero depending on whether a link does or does not exist between nodes i and j (diagonal elements of the adjacency matrix are zeros). The number of zero eigenvalues of the Laplacian is equal to the number of cluster in G [19]. We have used the first procedure to gather simulation results, while the second procedure is applied consistently to verify reliability of the first procedure.

Figure 3 shows a part of the simulated results for 500 nodes. Each subplot corresponds to a different value of ξ. In each subplot in this figure we have shown as function of the node's mean degree the following data obtained through simulations:

1. The probability of 1-connectivity.
2. The probability of p exceeding the $\log(n)/n$ threshold, which allows us to check the accuracy of the first theorem of connectivity by comparing this data with the first set of data mentioned above.

3. The probability of the minimum nodal degree being more than or equal to 1, which allows us to check the accuracy of the second theorem of connectivity by comparing this date with the first set of data mentioned above.
4. The giant component size.

The dotted line without markers in each subplot is added for comparison reasons and shows, as function of the mean nodal degree, the probability of 1-connectivity (or the probability of the minimum degree being more than or equal to 1) in a random graph with n nodes. Because of binomial degree distribution in random graphs and the independence of the links, this probability is computed as:

$$\Pr\left[\text{min. degree} \geq 1 \text{ in } G_p(n)\right] = \left[\sum_{k=1}^{n-1} \binom{n-1}{k} p^k (1-p)^{n-1-k}\right]^n = \left[1-(1-p)^{n-1}\right]^n. \tag{4}$$

The first conclusion we can draw after analyzing simulation data is that our results indeed comply with both theorems of connectivity for geometric random graphs with the pathloss radio model (in other words, when $\xi = 0$). However we can add more important additional details to refine these theorems:

- In all simulated cases, the first theorem of connectivity based on $\log(n)/n$ threshold predicts an almost surely connected network at those values of the mean degree where the probability of 1-connectivity is rather low (about 0.2 or less in subplots of Figure 3). Is this theorem too optimistic? We can examine this question by looking at the size of the giant component. For example, in Figure 3 for $\xi = 0$ the giant component size at the threshold where p exceeds $\log(n)/n$ is 0.987. In an other set of simulation with 1000 nodes (not shown in this paper), the giant component size at this threshold point for $\xi = 0$ was 0.998. This means that from the 1000 nodes, only 2 nodes did not belong to the giant component. The giant component size at the threshold point where this theorem predicts "almost surely" connectivity increase as $n \to \infty$, and this is exactly what the theorem stands for.
- In simulated cases with the low values of ξ the actual probability of connectivity coincides with the probability of $d_{\min} \geq 1$ only when $\Pr\left[d_{\min} \geq 1\right]$ is almost 1. This complies with the second theorem of connectivity for geometric random graphs. However, when the ξ increases, these two lines merge at lower values of $\Pr\left[d_{\min} \geq 1\right]$. For example, for $\xi = 3$ these two lines are overlapping each other virtually for the entire range of mean degrees. This behavior was expected from the second theorem of connectivity only for random graphs, but not for geometric random graphs. We can conclude that when ξ increases, the increase in the long-distance connectivity probability together with the reduction of the short-distance connectivity probability reduces the correlation between links. As a result, the geometric random graph approaches the generic random graph behavior, and the probability of 1-connectivity equals the probability $d_{\min} \geq 1$ for all values of $\Pr\left[d_{\min} \geq 1\right]$.
- For the same area size and for the same number of nodes the average nodal degree increases with increasing value of ξ (see Figure 4). From a radio propagation point of view, a higher value of ξ means a higher probability of having links with nodes at farther distances. This translates itself into a higher value of the mean node degree over the service area. This phenomenon was addressed previously in [14]. The increase in the mean nodal degree directly enhances the probability of connectivity.
- In all simulated cases we see that the giant component size is growing steeply towards 1 for those values of the mean degree that the probability of 1-connectivity is very low. For a relatively large span of the mean degree values the giant component is already covering most of the network but 1-connectivity is not achieved yet. This is due to only a few isolated nodes or small node clusters outside the giant component. This fact is demonstrated in Figure 5 that shows the mean size of components other than the giant component for different values of ξ. Starting from small values of the mean degree, as the mean degree increases, the mean size of the giant component as well as the mean size of other components increase. However, soon the giant component will "swallow" smaller clusters and causes their mean size to drop rapidly. In [20] it is proved that the size of the components other than the giant component is $O(\log n)$, to which our simulated results comply. We believe for practical use of ad-hoc networks 1-connectivity is a too stringent condition to satisfy. Therefore, we suggest to use the giant component size as a measure for connectivity in wireless ad-hoc networks. The giant component size not only provides information about the network being fully connect or not, but also it provides additional information about the fraction of the network which is fully connected. For practical use of ad-hoc networks it may suffice to provide conditions that, for example, only 99% of the network is connected.

This last point regarding the use of the giant component as a more suitable measure of connectivity is discussed in more details in the following section.

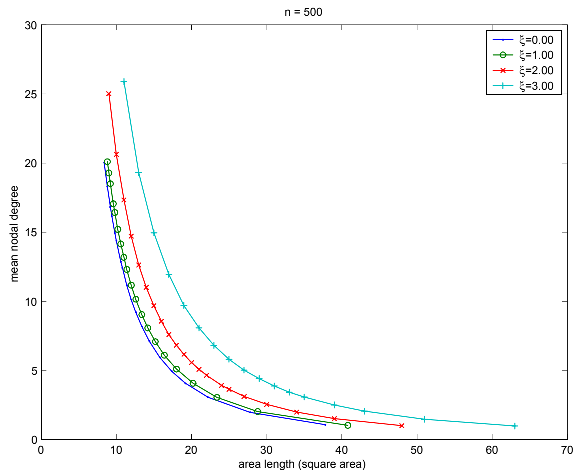

Figure 4. Mean nodal degree for 500 nodes uniformly distributed over areas of different sizes for different values of ξ.

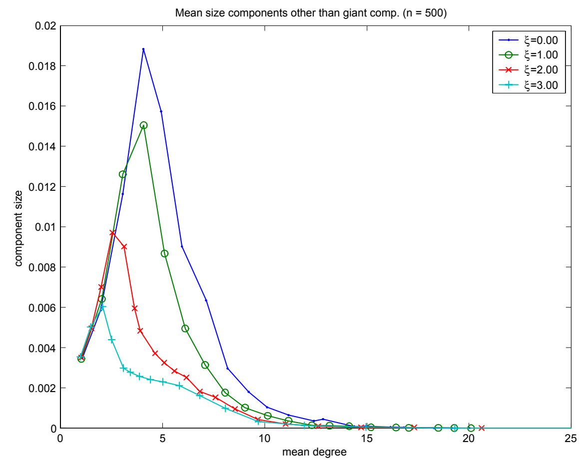

Figure 5. Mean size of components other than the giant component for different values of ξ.

5 Giant component size

In this section we provide first an analytical derivation for the giant component size in random graphs. Then we will show how this derivation can be extended to calculate the giant component size in geometric random graphs.

5.1 Analytical derivations

Let C_{rg} be the giant component size in a random graph (the fraction of a random graph occupied by the giant component). In [8] as well as [21] it is found that when n is large, C_{rg} is the non-zero solution to the following equation:

$$C_{rg} = 1 - \exp(-zC_{rg}). \tag{5}$$

Here, $z = E[d]$ is the mean degree of the graph. Although fast converging series exist [22] to solve (5), a standard zero finding algorithm like Newton-Raphson method can also be used to find C_{rg} as function of z. We have shown these values in Figure 6 (the dotted line).

5.2 Simulations and discussion

In the subplots of Figure 3 we already showed the giant component size found through simulations for $\xi = 0, 1, 2$ and 3. In Figure 6 we have plotted them next to each other (with an additional line for $\xi = 6$) and compared them[7] with the giant component size in a random graph, found using (5).

From Figure 6 we see that the lines representing the giant component size for high values of ξ (for example for $\xi = 6$) almost exactly match with the values predicted by (5) for random graphs. However, for low values of ξ the giant component size appears to be shifted along the mean degree axis. The amount of this shift is higher for lower values of ξ. We have tried several function forms to estimate this shift. A good approximation found for this shift is: $2.64\exp(-0.44\xi)$. Taking this into account, the size of the giant component in wireless ad-hoc networks by approximation, C_{ah}, is the non-zero solution to the following equation:

$$C_{ah} = 1 - \exp(-\widetilde{z}C_{ad}), \tag{6}$$

where $\widetilde{z} = z - 2.64\exp(-0.44\xi)$.

Figure 7 shows the in this way calculated giant component size in wireless ad-hoc networks for different values of ξ. For comparison, the giant component size in random graphs is drawn on each subplot of this figure. As visible in this figure, there is a good match between the simulated and the calculated values of the giant component size.

[7] The giant component sizes found through simulations in Figure 6 are found for $n = 500$. Other simulation results for $n = 250$ and $n = 1000$ indicated no noticeable difference with these values.

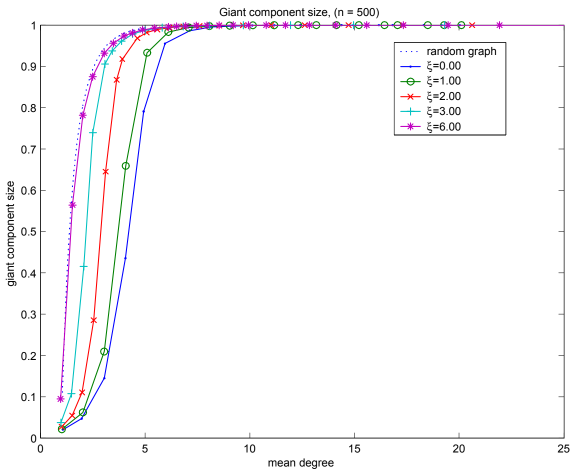

Figure 6. Comparison of the giant component size in a random graph with the values found for wireless ad-hoc networks.

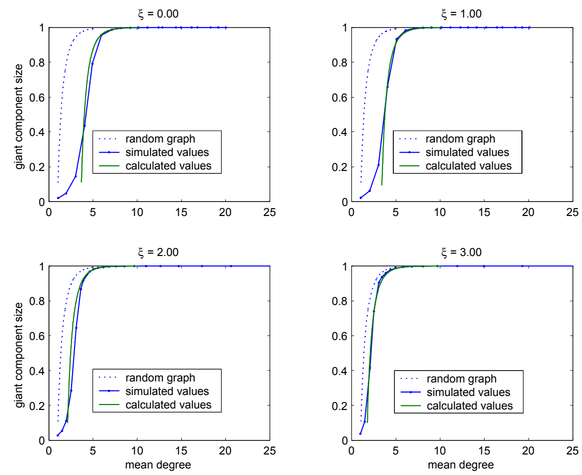

Figure 7. Simulated and calculated values for the giant component size in wireless ad-hoc networks for different values of ξ.

6 Conclusions

In this paper we have proposed to use the log-normal shadowing radio model for finding the link probability between nodes in wireless ad-hoc networks. This radio model takes into account the dynamics of radio signal power variations around the area mean power, and could therefore be more realistic than the commonly used static pathloss models. In Section 2 we have explained how fluctuations of radio signals can be indicated with a single parameter ξ. Low values of ξ correspond to small variations of the radio signal power around the area mean power and high values of ξ correspond to stronger fluctuations. We have studied connectivity in wireless ad-hoc networks by modeling the network as a geometric random graph with *tunable* link correlation. We call our model tunable because the correlation coefficient in the network is dependent on the value of ξ of the radio model.

Through extensive simulation, we have examined two general theorems of connectivity that so far have been formulated in the literature for random graphs and for geometric random graphs based on the pathloss radio model (see Section 4.1). Our main conclusions are:

1. Variations in ξ affect the behavior of the geometric random graphs. When $\xi \to 0$, the network behavior resembles a regular lattice network. However, as ξ increases the geometric random graph behaves more and more like a random graph. Apparently increased variations in the received signal powers reduces the correlation between links.
2. A higher value of ξ means a higher probability of having links with nodes at farther distances. This translates itself into a higher value of the mean nodal degree and increased probability for connectivity.
3. The giant component size can be used as a practical measure for "connectivity" in wireless ad-hoc networks. In this paper we have derived an equation for the calculation of the giant component size as function of the mean degree and ξ in wireless ad-hoc networks (see (6)). Our formula can be used to provide directives for the average required number of neighbors per node (average degree per node) to obtain connectivity over any desired percentage of the network. Average degree can be changed by adjusting the transmission power of nodes or by changing the density of nodes.

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Appendix

Denote by $f(p) = \Pr[G_p(n) \text{ is 1-connected}]$. According to (**??**), $f(p) = \left[1-(1-p)^{n-1}\right]^n$, which shows that $f(p)$ is always one for fixed $0 < p < 1$ and large n. Therefore, the asymptotic behavior of $\Pr[G_p(n) \text{ is 1-connected}]$ requires to investigate the influence of p as function of n. The order of $f(p_n)$ for large n is:

$$f(p_n) = \exp\left(n\log\left(1-(1-p_n)^{n-1}\right)\right) = \exp\left(-n\sum_{j=1}^{\infty}\frac{(1-p_n)^{jn-j}}{j}\right)$$

$$= \exp\left(-n(1-p_n)^{n-1} - n\sum_{j=2}^{\infty}\frac{(1-p_n)^{jn-j}}{j}\right) = e^{-n(1-p_n)^{n-1}}\left(1+O\left(n\sum_{j=2}^{\infty}\frac{(1-p_n)^{(n-1)j}}{j}\right)\right).$$

If we define $c_n \triangleq n\cdot(1-p_n)^{n-1}$, then the order term $O\left(n\sum_{j=2}^{\infty}\frac{(1-p_n)^{(n-1)j}}{j}\right) = O\left(n\sum_{j=2}^{\infty}\frac{c_n^j}{jn^j}\right)$ vanishes for large n provided we choose $c_n = O\left(n^\beta\right)$ with $\beta < \frac{1}{2}$. For large n, we thus have that $f(p_n) = e^{-c_n} \sim e^{-An^\beta}$ which tends to 0 for $0 < \beta < \frac{1}{2}$ and to 1 for $\beta < 0$. Hence, the critical exponent where a sharp transition occurs is $\beta = 0$. In that case, $c_n = c$ (a real positive constant) and

$$p_n = 1 - \exp\left(\frac{\log c}{n-1} - \frac{\log n}{n-1}\right) = \frac{\log n}{n} + O\left(\frac{\log c}{n}\right).$$

In summary,

$$f(p) \longrightarrow \begin{matrix} 0 \text{ if } & p < \log(n)/n \\ 1 \text{ if } & p > \log(n)/n \end{matrix},$$

with a transition region around $\frac{\log n}{n}$ of width of $O(\frac{1}{n})$.