Topological Characteristics of the Dutch Road Infrastructure

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***Abstract*— In this paper we study an example of a complex network, namely the road infrastructure of the Netherlands. In order to investigate the factors influencing the robustness of the complex network under consideration, we calculate a set of generic topological characteristics, related to the connectivity, degree, clustering and the shortest path length. Along with these widely considered topological characteristics, we also study the spectrum of the road graph, i.e. the collection of all eigenvalues of the associated adjacency matrix and the Laplacian matrix. The topological characteristics show that the road infrastructure differs substantially from many other real-world networks. However, there are complex structures, as for instance the power grid, that resemble topological properties of the Dutch road infrastructure.**

I. Introduction

Complex networks characterize a wide range of natural and man-made systems, e.g. the Internet, the WWW, networks of food web, social acquaintances, citation between papers, as well as many others. Traditionally, the topology of a complex network has been modeled as the Erdős-Rényi random graph [11], [12], [13]. However, the growing interest in complex networks has prompted many researchers to propose other more complex models, e.g. small-world [26] and scale-free networks [2]. Besides the modeling, considerable attention has also been given to the problem of capturing, in quantitative terms, the underlying complex principles (see e.g. [3], [10], [27]). In fact, each complex network is classified by a set of distinguishing topological characteristics, which in part define its robustness [1], [23].

In this paper we analyze a set of generic topological characteristics for the complex network of the Dutch road infrastructure. In the Netherlands, transportation is of special importance because the country functions as a gateway for the traffic of goods between western Europe and the rest of the world. Moreover, the rapid grow in the use of private cars [18] motivates an analysis of the underlying structure to better understand the ongoing traffic problem.

For the complex network of the road infrastructure we analyze the characteristics related to the connectivity, degree, clustering and the shortest path length. Along with these widely considered topological characteristics we also consider the spectrum of the road graph, i.e. the collection of all eigenvalues of the associated adjacency matrix [5], [7], [8] and the Laplacian matrix [19], [20], [21], [22]. Furthermore, we compare most of the considered characteristics to those in the Erdős-Rényi random graph. The advantage of the Erdős-Rényi random graph is that most of the topological characteristics can be analytically expressed in contrast to many others models where computations are hardly possible [4], [25].

The paper is organized as follows. Section 2 explains how the Dutch road infrastructure can be represented by a graph. Section 3 presents the analysis of the set of generic topological characteristics, related to the connectivity, degree, clustering, shortest path length and the eigenvalue spectrum in Sections 3.1-3.5, respectively. Section 4 summarizes our main results on the topological characteristics of the road graph.

II. Constructing the Road Graph

We have used the data from the National Road Database, provided by the AVV, i.e. the Dutch transport research centre, which is a part of the Rijkswaterstaat organization. The National Road Database is a digital database of nearly all roads in the Netherlands. In the National Road Database there are approximately 825 000 roadsections, each characterized by a unique roadsection-ID. The road graph has been created by including only the roadsections being part of the motorway or the provincial road. Here, we must make one important remark: each roadsection is assigned with a unique ID, even if it belongs to the same road. A road, on the other hand, is also characterized with a unique roadnumber. For instance, a road separated in two lanes, each having a unique roadsection-ID, is also having the same roadnumber for both lanes. This being the case, we take into account only the roadsections having different roadnumbers. As a consequence, the roadsection which represent a node in the road graph, is connected to other nodes by an undirected link, resulting in the road graph $G_R = (N_R, L_R)$, consisting of $N_R = 14756$ nodes and $L_R = 19253$ links, respectively.

III. Topological Characteristics

In this section, we quantitatively analyze the graph of the road infrastructure in the terms of various topological characteristics. The set of topological characteristics we compute here includes most of the characteristics discussed in the networking literature. Along with these widely considered topology characteristics, we also analyze the graph's spectrum of the associated adjacency matrix and the Laplacian matrix.

A. Connectivity

A graph is connected if there exists a path between each pair of nodes. When there is no path between at least one pair of nodes, a graph is said to be disconnected. A disconnected graph consist of several independent components or clusters. We have used two different procedures to check the number of components the road graph has: the Prim's algorithm[1] and the number of zero eigenvalues of the Laplacian matrix. In fact, the multiplicity of 0 as an eigenvalue of the Laplacian matrix is equal to the number of components of a graph [15]. The graph of the road infrastructure has exactly 170 components: the graph of the largest component (LC), defined as $G_{LC} = (N_{LC}, L_{LC})$, contains $N_{LC} = 14098$ nodes and $L_{LC} = 18689$ links, respectively. The second largest component has 24 nodes, which makes the largest component to be 'the giant one', since it clearly dominates all the other components. Table I depicts number of components, defined as S_{N_C}, of the size N_C contained in the road graph. Henceforth, along with the graph of the largest cluster G_{LC}, we also consider the graph formed by the remaining clusters (RC), denoted as $G_{RC} = (N_{RC}, L_{RC})$, where $N_{RC} = 658$ and $L_{RC} = 568$, respectively.

[1] Prim's algorithm is an algorithm that finds a minimum spanning tree for a connected weighted graph. This means that it finds a subset of the links that forms a tree that includes every node, where the total weight of all the links in the tree is minimized. When a graph is unweighted, any spanning tree is a minimum spanning tree [6].

TABLE I

Number of clusters S_{N_c} with size N_c in the road infrastructure graph G_R. In the analysis of G_R, we also consider two subgraphs: the graph G_{LC} of the largest component with $N_{LC} = 14098$ nodes and the graph G_{RC} of the remaining part with $N_{RC} = 658$ nodes.

S_{N_c}	124	11	7	6	4
N_c	2	4	3	6	8

S_{N_c}	2	2	2	2	2
N_c	10	12	14	17	24

S_{N_c}	1	1	1	1	1	1	1	1
N_c	5	15	18	19	21	22	23	14098

B. Degree Distribution

The two most basic characteristics of a graph are the number of nodes N and the number of links L. They define the mean nodal degree $E[D] = \frac{2L}{N}$, which is the coarsest connectivity characteristics of a graph. Networks with higher $E[D]$ are better connected on average and, consequently, are likely to be more robust. The graph of the road infrastructure results in the mean nodal degree of $E[D_R] = 2.6$ whereas the largest cluster, and the graph of the remaining clusters, have a mean nodal degree of $E[D_{LC}] = 2.7$ and $E[D_{RC}] = 1.7$ respectively.

The topological characteristics, given in Subsection III-A, shows why the graph of G_{LC} has similar mean nodal degree as G_R: the two graphs have almost the same number of nodes while G_{LC} has slightly more links in relative sense. However, the mean nodal degree of G_{LC}, in contrast to the mean nodal degree of the connected Erdős-Rényi random graph[2] $E[D_{ER}] = p(N_{LC} - 1) = \log(N_{LC}) = 9.55$, is rather low. This is due to

[2] The value of the link probability p above which a random graph almost surely becomes connected tends, for large N, to $p_c \sim \frac{\log N}{N}$ (for details see [17]).

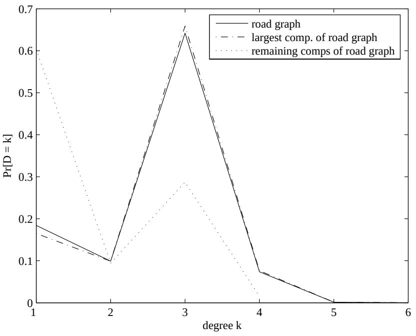

Fig. 1. The degree distribution of G_R, along with G_{LC} and G_{RC}.

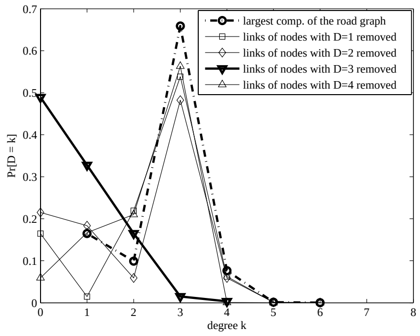

Fig. 2. The degree distribution of G_{LC} when the links of nodes with degrees $D_{LC} = 1, 2, 3$ and 4 are removed. Clearly, the removal of nodes with $D_{LC} = 3$ has the largest impact on the structure of G_{LC}

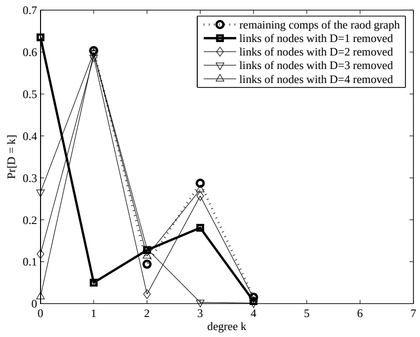

Fig. 3. The degree distribution of G_{RC} when the links of nodes with degrees $D_{RC} = 1, 2, 3$ and 4 are removed. Clearly, the removal of nodes with $D_{RC} = 1$ has the largest impact on the structure of G_{RC}

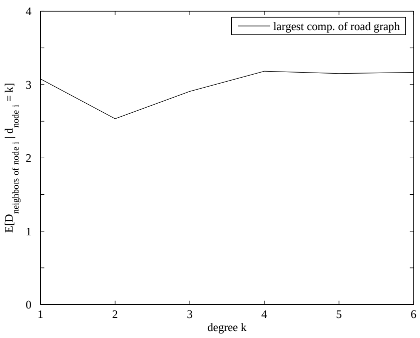

Fig. 4. The average degree of neighbors of a node in G_{LC}, given that this node has degree k.

the fact that in G_{LC} the number of links and nodes are comparable, creating the topology with low-degree nodes. To examine this in more detail, we have calculated the nodal degree distribution.

The nodal degree distribution $\Pr[D = k]$ is the probability that a randomly selected node has a degree k. The nodal degree distribution contains more information about connectivity in a given graph than the mean nodal degree: given a specific form of $\Pr[D = k]$ we can always restore the mean nodal degree by $E[D] = \sum_{k=1}^{D_{\max}} k \Pr[D = k]$, where $D_{\max}$ is the maximum degree in a given graph. From the degree distribution of the three considered graphs, i.e. G_R, G_{LC} and G_{RC}, shown in Figure 1, we can deduce the following.

A node in G_R is at least connected to one and at most to six neighboring nodes. Similarly, in G_{LC}, whereas in G_{RC} a node is at most connected to four neighboring nodes. Moreover, in G_R and G_{LC}, the probability $\Pr[D = k]$ is largest for degree 3, in contrast to G_{RC} where most of the nodes have degree 1 (see also Table I). In fact, nodes with degree 1 and 3 dominate more than 82% of the structure of G_{LC} and G_{RC}. This observation is also very evident from Figures 2 and 3, which show that the removal of links of nodes with degree 3 and 1 has the largest impact on the robustness of G_{LC} and G_{RC}, respectively.

To examine the correlation between the degrees within the neighborhood of a node i, we have calculated the average degree of the neighbors of a node i, given that this node has degree k. For G_{LC}, this average degree, as depicted in Figure 4, is nearly constant. As the majority of nodes has degree 3, $E[D_{neighbors\ of\ node\ i} | d_{node\ i} = k]$ follows the prediction of being constant. However, for a node with degree 2 this expected value is showing that a node more likely connects to a lower degree node. Hence, these nodes constitute the weakest part of the road infrastructure. Recall that for G_{LC}, we observed in Figure 1 that the probability for a node to have degree 2 is relatively small but still greater than the probability $\Pr[D \geq 4]$.

C. Clustering

Clustering expresses local robustness in the graph and thus has practical implications: the higher the local clustering of a node, the more interconnected are its neighbors. Clustering in a graph is quantified by a clustering coefficient, which is defined by Watts and Strogatz [26], [27]. The clustering coefficient $c_G(i)$ of a node i is the proportion of links y between the nodes within the neighborhood of a node i, divided by the number of links that could possibly exist between them. For an undirected graph, a node i with degree d_i has at most $\frac{d_i(d_i-1)}{2}$ links among the nodes within its neighborhood. Thus, the clustering coefficient $c_G(i)$ for a node i is given by $c_G(i) = \frac{2y}{d_i(d_i-1)}$. In other words, the clustering coefficient is the ratio between the number of triangles that contain node i and the number of triangles that could possibly exist if all neighbors of i were interlinked. The clustering coefficient for the entire graph is the average of the clustering coefficient of nodes with degree larger than 1, given as $c_G = \frac{1}{N-|\mathcal{N}^{(1)}|}\sum_{i\in\mathcal{N}-\mathcal{N}^{(1)}} c_G(i)$, where $\mathcal{N}$ is the set of all nodes and $\mathcal{N}^{(1)}$ the set of degree 1 nodes in the graph.

In the graph of the largest cluster G_{LC}, the clustering coefficient for 7976 nodes is equal to 0, similar to lattice graphs where no links exist among the neighboring nodes, i.e. the neighbors of a node are not interlinked. Although, 7976 nodes have the clustering coefficient equal to 0, the average clustering coefficient for G_{LC}, $c_{LC} = 0.11$, is still relatively high. For the Erdős-Rényi random graph [25], with the same number of nodes as in the largest cluster, the clustering coefficient is $c_{ER} = \frac{E[D_{ER}]}{N_{LC}-1} = 0.00068$, showing indeed that the graph of the largest cluster graph has relatively high clustering coefficient.

D. Shortest Path Length Distribution

The shortest path length is important for many networking applications, the most important being routing. The basic characteristic associated with the shortest path length is the hopcount, i.e. the number of hops or links between a pair of nodes[3]. Correspondingly, the hopcount distribution $\Pr[H = h]$ is the probability that a random pair of nodes are at h hops from each other. From the hopcount distribution, the mean hopcount is derived as $E[H] = \sum_{h=1}^{H_{\max}} h \Pr[H = h]$, where $H_{\max}$ is the longest hopcount between any pair of nodes. $H_{\max}$ is also referred to as the diameter d of a graph.

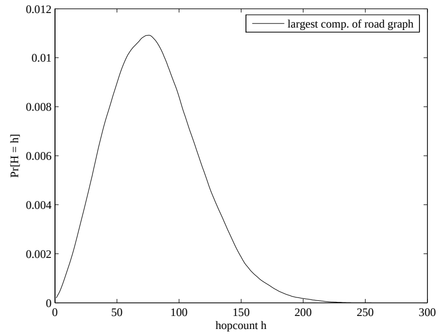

Fig. 5. The shortest path length (hopcount) distribution of G_{LC}.

The hopcount distribution for the largest cluster G_{LC} of the road graph, as shown in Figure 5, approximates the Gaussian distribution with the mean $E[H_{LC}] = 80.6$ and the standard deviation $\sigma[H_{LC}] = \sqrt{Var[H_{LC}]} = \sqrt{1324.8} \simeq 36$. According to Figure 5, the diameter is $d_{LC} = 255$. For the Erdős-Rényi random graph of the same size as the largest cluster, it is presented in [25] that the mean hopcount is $E[H_{ER}] = \frac{\ln(N_{LC})}{\ln(E[D_{ER}])} \simeq 5$ and the diameter is about two times the mean hopcount. These results indicate that the complex network of the road infrastructure does not follow the prediction of exponentially growing graphs, such as the Erdős-Rényi random graph, but it belongs to the class of $\mathcal{D}$-lattice graphs, where $E[H] \sim \frac{\mathcal{D}}{3}N^{\frac{1}{\mathcal{D}}}$ and $\mathcal{D}$ is the lattice dimension. Thus, the road infrastructure graph most likely is a subgraph of a two-dimensional lattice graph because the lattice dimension equals $\mathcal{D} = 1.99$.

E. Graph Spectra

We have calculated the spectrum of the adjacency matrix and the Laplacian matrix of the road infrastructure graph. First, we introduce the adjacency matrix and the Laplacian matrix that allow us to calculate the graph spectra.

The adjacency matrix A of a graph G with N nodes is an $N \times N$ matrix whose rows and columns are labeled by graph nodes, i.e. a 1 or 0 in position (i, j) according to whether node i and node j are adjacent or not. The Laplacian matrix of a graph G with N nodes is an $N \times N$ matrix[4] $Q = \Delta - A$, where $\Delta = diag(k_i)$ and k_i is the degree of the node $i \in \mathcal{N}$. The

[3]For many applications it is useful if links in a graph can be labelled with a weight. Such a graph is referred to as a weighted graph. Recall that in this analysis the link weights are not taken into account. Hence, the shortest path length equals the hopcount.

[4]Note that for k-regular graphs we have $Q = kI - A$. In contrast, for non-regular graphs there is no easy one-to-one correspondence between A and Q.

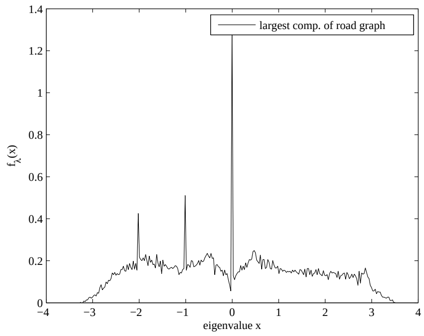

Fig. 6. The adjacency spectrum of G_{LC}.

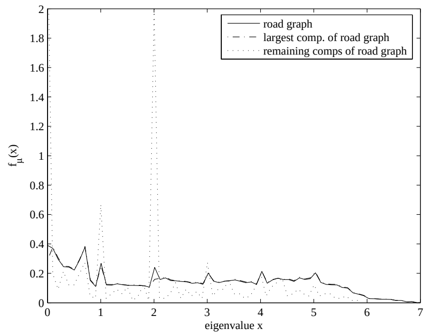

Fig. 7. The Laplacian spectrum of G_R, along with G_{LC} and G_{RC}.

eigenvalues of A and Q are called respectively the adjacency and the Laplacian eigenvalues.

The adjacency eigenvalues are all real, denoted by $\lambda_N \leq \lambda_{N-1} \leq ... \leq \lambda_1$. The spectral density of a graph is the density of the eigenvalues of its adjacency matrix A. For a finite system, it can be written as a sum of δ functions $\rho(\lambda) = \frac{1}{N}\sum_{i=1}^{N}\delta(\lambda - \lambda_i)$, which converges to a continuous function with $N \to \infty$. The spectral density of a graph can be directly related to the graph's topological characteristics [7]. In Figure 6, the spectral density of the largest component of the road graph is shown. Since no triangles exist in a subgraph of lattice, we are interested in whether triangles exist in the structure of G_{LC}. This can be derived from the characteristic polynomial [25]: the number of triangles in G_{LC} is $\frac{1}{6}\sum_{i=1}^{N}\lambda_i^3 = 1448$.

Peaks in the spectrum reflect structure and regularity in the graph. Given that most of the nodes have degree 1 or 3, the regularity in the structure is expected. Hence, nodes with small degrees are responsible for the peak at $\lambda = 0$ [9], [14]. For instance, the local configurations with two and more dead-end nodes produce eigenstates $\lambda = 0$, where the dead-end node is a node with degree 1. The corresponding eigenvectors have non-zero components only at the dead-end nodes [16], [24]. More local configurations that produce zero eigenvalue are shown in Figure 8. In fact, each time when two rows in the adjacency matrix A are the same, the rank of A decreases with 1, which is equivalent to an increase in the multiplicity of the eigenvalues $\lambda = 0$. Furthermore, two connected nodes with the same neighbors result in the eigenvalue -1. This is due to the fact that the sum of the identity matrix I and the adjacency matrix A has identical rows, which correspond to the two connected nodes.

The Laplacian eigenvalues are all real and nonnegative [21]: they are contained in the interval $[0, N]$. The set of all N Laplacian eigenvalues $\mu_N = 0 \leq \mu_{N-1} \leq ... \leq \mu_1$ is called the Laplacian spectrum of a graph G. Figure 7 depicts the Laplacian spectrum of the entire graph G_R, along with the largest component G_{LC} and the remaining part G_{RC} of the road infrastructure.

The Laplacian spectrum of the road graph contains a peak at $\mu = 1$ and several smaller peaks at $\mu = 2, 3, 4$ and 5, respectively. A peak at $\mu = 1$ most likely originates from a significant amount of nodes with degree 1 [9], [14]. Taking this conjecture into account, the question we seek to answer is: "Does the specific spectral behavior of the road graph come from the majority of nodes with the corresponding degree?" or else, "To what extent are the basic topological structures, such as a path, cycle and a tree, responsible for it?" To answer this question we will study the Laplacian spectrum of two considered subgraphs, G_{LC} and G_{RC}. In particular, we will examine whether a specific spectral behavior of the road graph originates from the nodes located in G_{LC} and more importantly, which nodes are responsible for it?

From Figure 7 we observe that G_R has almost the same spectral behavior as G_{LC}. The only difference is that in the spectrum of G_{LC} there exists no peak at $\mu = 2$. Moreover, in Figure 7 we observe that the spectrum of G_{RC} has a peak at $\mu = 2$ and smaller ones at $\mu = 1, 3$ and 4, respectively. The Laplacian spectrum of a graph, which is a union of several disjoint components, is the addition of the spectra of each component [21]. The same holds true for G_{RC}, consisting of 169 components from which 124 are line graphs with only two nodes. In fact, the Laplacian spectrum of the path graph P_N with $N = 2$

nodes comprises the eigenvalues $\mu(P_2) = 0$ and 2. Thus, the majority of nodes with the degree 1 is also responsible for the peak in the spectrum of G_{RC} and, consequently, for the disappearance of the peak at $\mu = 2$ in the Laplacian spectrum of G_{LC}. Recall that besides this peak at $\mu = 2$, caused by the basic topological structure, nodes with small degrees, i.e. degree 1, are primarily responsible for the peak at the corresponding μ. In Figures 13, 14, 15 and 16, respectively, we plot the Laplacian spectrum of G_{RC} after sequential removal of nodes with degrees $1, 2, 3$ and 4, respectively. From these Figures we observe that although a peak at certain μ primarily originates from a significant amount of nodes with the corresponding degree, the specific spectral behavior of G_{RC} is above all influenced by eigenvalues being a consequence of basic topological structures. Thus, only the nodes of G_{RC} with degree 1 are responsible for the peak at $\mu = 2$ in the Laplacian spectrum of G_R.

In Figures 9, 10, 11 and 12, we plot the Laplacian spectrum of G_{RC} after sequential removal of nodes with degrees $D_{LG} = 1, 2, 3$ and 4, respectively. From Figure 9 it is obvious that the peak at $\mu = 1$ primarily originates from nodes with the corresponding degree whereas in Figure 11 we see that the removal of nodes with degree 3 completely changes the structure of the road graph. On the other hand, in Figures 10 and 12 it is visible that the removal of nodes with degree 2 and 4 does not influence the underlying structure.

The application of the Laplacian spectrum analysis to the three considered graphs leads to the following conclusion. Peaks appearing in the spectra are mainly due to the majority of nodes with the corresponding degree, since the removal of nodes with degree of lower

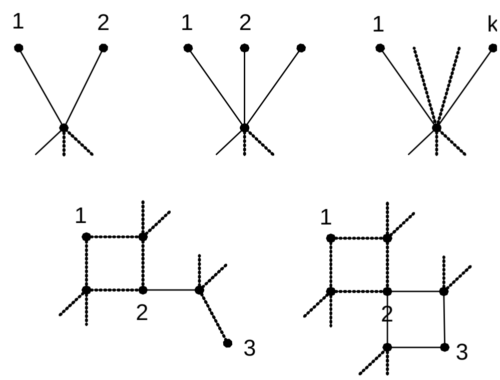

Fig. 8. Local configurations that produce zero eigenvalue in the spectrum of the adjacency matrix.

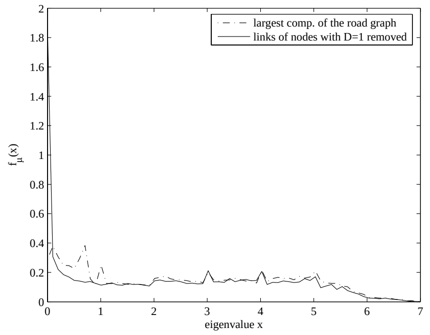

Fig. 9. The spectrum of the Laplacian matrix of G_{LC} when the links of nodes with degree $D_{LC} = 1$ are removed.

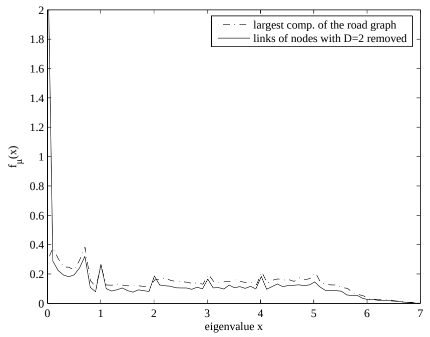

Fig. 10. The spectrum of the Laplacian matrix of G_{LC} when the links of nodes with degree $D_{LC} = 2$ are removed.

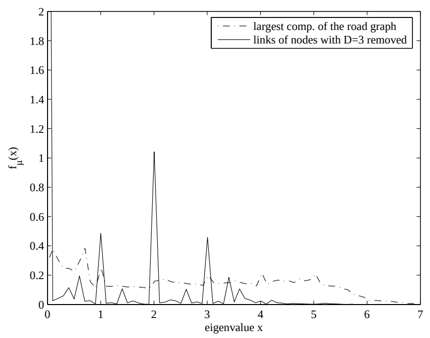

Fig. 11. The spectrum of the Laplacian matrix of G_{LC} when the links of nodes with degree $D_{LC} = 3$ are removed.

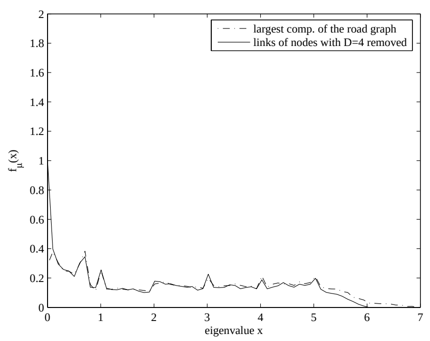

Fig. 12. The spectrum of the Laplacian matrix of G_{LC} when the links of nodes with degree $D_{LC} = 4$ are removed.

probability of appearance hardly does influence the underlying structure of the subgraph in consideration. Moreover, the basic topological structures are responsible for a particular spectral behavior but only if the majority of nodes to a large extent contributes to it.

IV. Conclusion

This paper has focussed on the road infrastructure of the Netherlands, which is an example of a complex network. For this complex network we have calculated topological characteristics related to connectivity, degree, clustering and the shortest path length. Apart from these widely considered characteristics, we have investigated the graph's spectrum of the associated adjacency and the Laplacian matrix. In particular, we discussed how the underlying complex principles, captured in a wide range of topological characteristics, are related to the robustness of the road graph.

The topological characteristics we have analyzed, reveal the following conclusions. The graph of the road infrastructure is dominated by a single giant component. The degree distribution, derived from the entire as well as the largest component, differs substantially from that of many other real-world networks: in literature, e.g. [1], [23], it has been overwhelmingly shown that the degree distribution of many real-world graphs belongs to the class of scale-free networks. Furthermore, in [23], many real-world graphs have a small-world character similar to random graphs, but they have unusually large clustering coefficient like scale-free networks. Recall, in Section III-D we showed that the Dutch road infrastructure does not have short path lengths because it is most likely a subgraph of a two-dimensional lattice. Despite the awareness that the Dutch road infrastructure differs substantially from many other real-world networks, its topological characteristics do resemble specific complex structures, for instance the power grid, which is also found in [23].

In this paper, we used a set of generic topological characteristics to analyze the underlying topology of the road infrastructure graph. Consequently, a broader group of complex networks stands on the agenda for future work.

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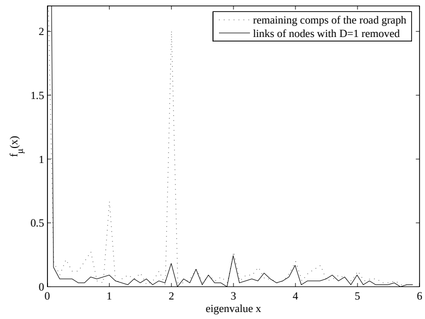

Fig. 13. The spectrum of the Laplacian matrix of G_{RC} when the links of nodes with degree $D_{RG} = 1$ are removed.

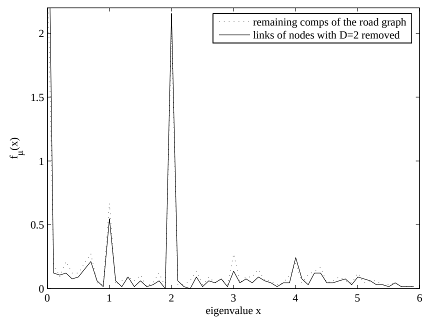

Fig. 14. The spectrum of the Laplacian matrix of G_{RC} when the links of nodes with degree $D_{RG} = 2$ are removed.

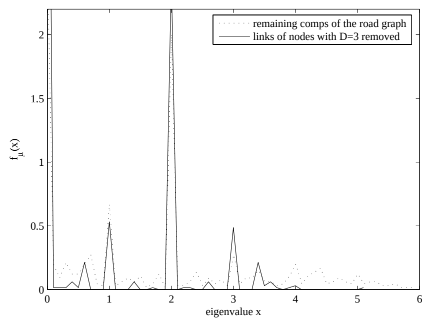

Fig. 15. The spectrum of the Laplacian matrix of G_{RC} when the links of nodes with degree $D_{RG} = 3$ are removed.

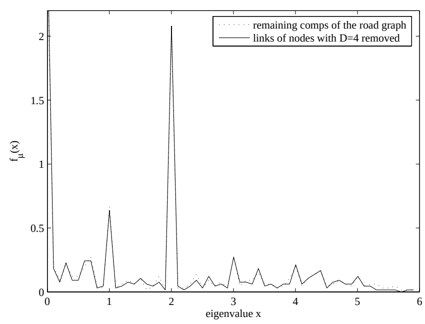

Fig. 16. The spectrum of the Laplacian matrix of G_{RC} when the links of nodes with degree $D_{RG} = 4$ are removed.

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