Advances in Complex Systems, Vol. 14, No. 5 (2011) 691–710

DOI: 10.1142/S021952591100330X

MULTI-WEIGHTED MONETARY TRANSACTION NETWORK

H. WANG[*,§], E. VAN BOVEN[*,†,¶], A. KRISHNAKUMAR[†],
M. HOSSEINI[*,†], H. VAN HOOFF[‡], T. TAKEMA[‡],
N. BAKEN[*,†] and P. VAN MIEGHEM[*]

[*]*Faculty of Electrical Engineering,*
Mathematics, and Computer Science,
Delft University of Technology,
P. O. Box 5031, 2600 GA Delft, The Netherlands

[†]*KPN Royal, P. O. Box 30.000,*
2500 GA The Hague, The Netherlands

[‡]*Statistics Netherlands,*
P. O. Box 24.500, 2490 HA,
The Hague, The Netherlands
[§]*h.wang@tudelft.nl*
[¶]*e.f.m.vanboven@tudelft.nl*

Received 25 January 2011
Revised 10 May 2011

This paper aims to both develop and apply advances from the field of complex networks to large economic systems and explore the (dis)similarities between economic systems and other real-world complex networks. For the first time, the nature and evolution of the Dutch economy are captured by means of a data set analysis that describes the monetary transactions among 105 economical activity clusters over the period 1987–2007. We propose to represent this data set as a multi-weighted network, called the monetary transaction network. Each node represents a unique activity cluster. Nodes are interconnected via monetary transactions. The millions of euros that traverse the links and that circulate inside each activity cluster are denoted by a link weight and a node weight respectively. By applying innovative methodologies from network theory, we observe important features of the monetary transaction network as well as its evolution: (a) Activity clusters with a large internal flow tend to cooperate with many other clusters via high volume monetary transactions. (b) Activity clusters with a lower internal transaction volume prefer to transact with fewer neighboring nodes that have a higher internal flow. (c) The node weights seem to follow a power law distribution. Surprisingly, (b) and (c) have been observed in community structures of many real-world networks as well. (d) Activity clusters tend to balance the monetary volume of their transactions with their neighbors, reflected by a positive link weight correlation around each node. This correlation becomes stronger over time while the number of links increases over time as well.

Keywords: Complex network; monetary transaction network; weighted network; mode weight.

1. Introduction

For decades, many research initiatives have been devoted to studying economic systems by means of e.g. input–output matrices, representing the interdependencies between different branches of a national economy [1, 2]. This paper aims to contribute to a more in-depth understanding of economic systems by applying advances from the field of complex networks.

Large systems of elements (nodes) and their interactions or relations (links) can be represented as complex networks. Examples of complex networks range from biological networks and communication networks to social networks. The characterization of networks has been extensively investigated for classification purposes and for understanding the effect of the network structure on its functioning [3, 4]. Most studied complex networks are represented as unweighted networks. Many of these networks show the small-world property [5] characterized by dense local connections and short average distance, as well as a power-law degree distribution [6]. These topological features affect the functioning of a network, such as its robustness or vulnerability. Furthermore, many real-world networks employ link weights to quantify properties of links such as distance, cost, capacity and bit rate of traffic flows. Weighted analysis has been widely applied to characterize the world trade web [7, 8]. Measures that characterize weighted networks and that explore the correlations between topology and link weight structure, have recently been introduced [9–11]. While link weights are used to describe properties of links, we propose to further add node weights to capture node related features.

We propose to study the Dutch economy as a monetary transaction network that can be represented as a multi-weighted network. At the highest hierarchical level, society and economy are decomposed into a number of sectors, such as construction, education, finance, healthcare, manufacture and transport [12]. Sectors are interlinked via monetary transactions. Each sector can be further decomposed into activity clusters.[a] For example, the transport sector comprises railway transportation, airline transportation, etc. As a result, there are in total 105 activity clusters, assigned in a non-overlapping way to 20 sectors. Each activity cluster transacts money with other activity clusters. For its internal production, activity clusters require a certain amount of money that circulates within that cluster. Statistics

[a]Currently, the most prevalent standard is the International Standard Industrial Classification (ISIC) [13] provided by the United Nations to all member states, enabling them to transparently classify their (national) economic activities. Most national statistics departments use (and comply to) the ISIC standard as a general framework to define and structure their national statistical data sets. The most recent ISIC version [13] comprises 21 sections, subdivided into 86 divisions. Users can translate these sections and divisions into a list of sectors, sub-sectors etc. The network, studied in this paper, is based on the Dutch Input Output table provided by the National Accounts department of Statistics Netherlands. This table contains a financial transaction matrix that reflects the yearly produced and delivered value inside and between 104 economic activity clusters thus addressing at least all 86 ISIC divisions. As the role of the household is increasingly important, we added consumer spending and salaries to the studied data set being the 105th node in the sector network.

Netherlands recorded all transactions in monetary terms observed in each year over the 21 years period (1987–2007) in the Netherlands among 105 activity clusters. The activity clusters (nodes) are, thus, interconnected via monetary transactions. The money flows that traverse the links account for the goods and services exchanged between the nodes. The amount of money flow traversing a link is represented by a link weight and the volume of the internal transactions within each activity cluster is denoted by a node weight.[b] Such a multi-weighted network representation provides a clear view of the network with respect to the nature of these transactions: the presence or absence of a transaction between activity clusters and the amounts transacted between as well as within activity clusters.

Characterizing multi-weighted networks has not yet been studied to our best knowledge. In this paper, properties of the multi-weighted monetary transaction network are systematically investigated via diverse measures/metrics. These measures reveal intrinsic properties of the monetary transaction network as well as traces of major trends. Moreover, the monetary transaction network is shown to share similar features with the community structures of other real-world networks. As the weights can be different in nature, a multi-weighted network can even incorporate multiple link weights and/or node weights, where our methodology can still be applied.

2. Monetary Transaction Network

Our data set aggregates all monetary transactions observed in each year over a 21 year period (1987–2007) in the Netherlands among 105 different activity clusters. Figure 1 is a conceptual visualization of the Dutch sector network at two hierarchical levels. At the sector and activity cluster level, the network describes monetary transactions between sectors and between activity clusters respectively. Throughout this work, we mainly examine the monetary transaction network at the activity cluster level, because at the sector level, the network is almost fully meshed close to a complete graph.

A network topology, denoted by a graph $G(N, L)$, consists of a set $\mathcal{N}$ of N nodes interconnected by a set $\mathcal{L}$ of L links. It can be represented by an adjacency matrix A, an $N \times N$ matrix consisting of elements a_{ij} that are either one or zero depending on whether there is a link between node i and j or not. A weighted adjacency matrix $W^{\tilde{}}$ may further incorporate the link weight structure by letting $w_{ij}^{\tilde{}}$ denote the weight of link $i \to j$.

[b]When a transaction in between clusters is defined as a link, the monetary transaction within an activity cluster can be regarded as a self-loop. In this example, node weight can be understood as the weight of a self-loop. The term node weight, however, can be used in more general cases where the node weight and link weight may capture different properties of links and nodes. For example, the number of employees in a cluster, can be as well-denoted by node weight which has no evident association to self-loops.

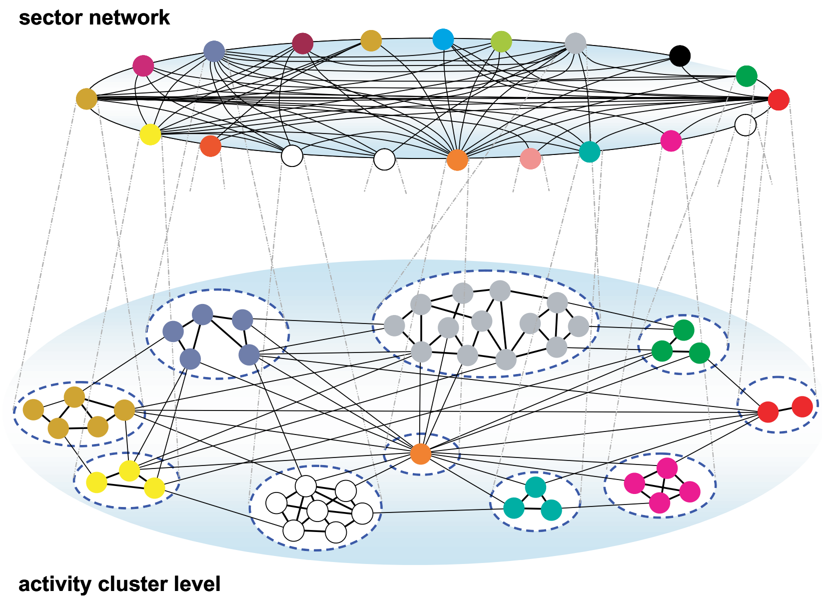


Fig. 1. A conceptual visualization of the monetary transaction network at the sector level (above), where each node represents a sector, defined as a group of activity clusters, and at the activity cluster level (bottom), where each node represents an activity cluster.

Each of the 21 network instances contains $N = 105$ nodes, which correspond to the activity clusters. We studied the monetary transaction network in an undirected way due to the complexity of weighted and directed network analysis. Thus, we adapted the original directed matrix $W^{\tilde{}}$ into W by defining the link weight $w_{ij} = w_{ij}^{\tilde{}} + w_{ji}^{\tilde{}}$ being the sum of the original money flows from activity cluster i to j and from j to i. Thus, the link weight w_{ij} represents how active two clusters are mutually involved in monetary transactions. We further use the node weight w_i to denote the money flow within an activity cluster i. Node weight w_i and link weight w_{ij}, which can generally describe distinct properties of a node and link respectively, are not necessarily related.

As shown in Fig. 2, the total money flow $\sum_w = \sum_{i=1}^{N} w_i + \sum_{i<j} w_{ij}$ increases over time. After correcting for inflation,[c] the total money flow still increases due to the improved production over the observed period. Thus, two factors influence the increase of the total money flow: (a) inflation and (b) the improved production over the observed period. Being aware of the effects of the two factors, we

[c]If we take the year 1987 as a reference, due to inflation, 1 euro in 1988 is valued $\frac{1}{1+z_1}$ euro and in 1989 $\frac{1}{(1+z_1)(1+z_2)}$ euro, where z_1 and z_2 is the inflation rate in 1988 and 1989, respectively. Thus, the value of 1 euro in i years after 1987 is equivalent to $\frac{1}{\prod_{j=1}^{i}(1+z_j)}$ euro in 1987. In this way, we convert the monetary value in each year to the equivalent amount in 1987 to reduce the effect of inflation.

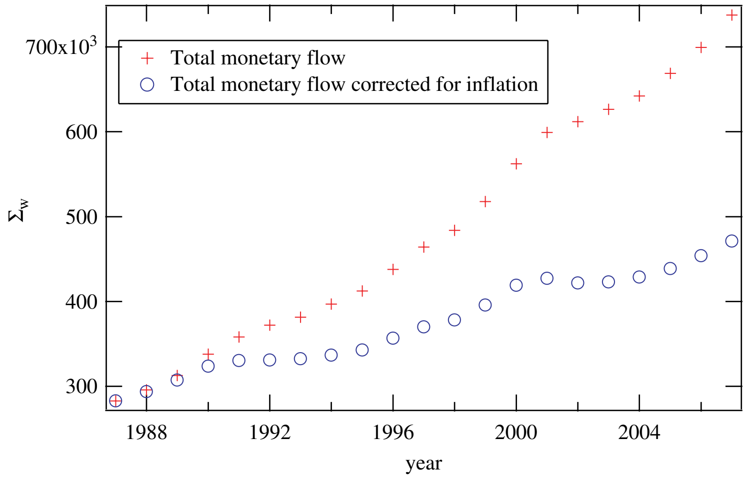


Fig. 2. The monetary flow $\sum_w$ in millions of euros of each year from 1987 to 2007 with and without inflation correction.

normalize for each individual year all the link weights and node weights of the sector network by the total money flow $\sum_w$ in that year/network instance such that $\sum_{i=1}^{N} w_i + \sum_{i<j} w_{ij} = 1$ and $w_i, w_{ij} \in [0,1]$ hold in each network. The normalization is essential to compare and to discover collective features of the class of 21 sector network instances excluding the influence of inflation. As depicted in Fig. 3, the ratio $R = \frac{\sum_{i<j} w_{ij}}{\sum_{i=1}^{N} w_i + \sum_{i<j} w_{ij}}$ of the total of all link weights divided by the total of all link weights and all node weights of each year proved to be surprisingly constant within the range $[0.892, 0.905]$ over the period from 1987 to 2007.

3. The Monetary Transaction Network at Activity Cluster Level

The monetary transaction network at activity cluster level is well-represented as a multi-weighted network. A multi-weighted network generally incorporates three domains of information: (1) the topology, namely, the unweighted network structure, which solely describes the interconnections of nodes, (2) the link weight structure, which associates a weight to each link and (3) the node weight structure that assigns a weight to each node. In this section, we first introduce measures to capture the main features of the monetary transaction network in each of three domains respectively. Correlations among the three domains will also be investigated. The evolution of the network properties during a 21-year period will be studied in this paper as well. We aim to thoroughly explore whether or not evident course changes or turning points have occured for each network property over time. Such observations may reflect, explain or even help predicting developments of our economical system.

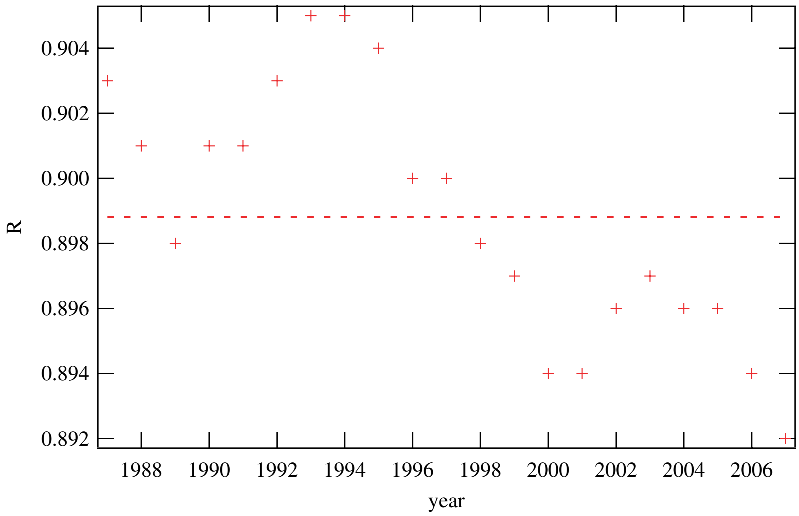


Fig. 3. The ratio $R = \frac{\sum_{i<j} w_{ij}}{\sum_{i=1}^{N} w_i + \sum_{i<j} w_{ij}}$ of the total of all link weights divided by the total of all link weights and all node weights of each year from 1987 to 2007. The dotted line is the average over the 21 years.

3.1. *Topology*

Over the past several years, a variety of measures has been proposed to describe different features of a network topology. Measures related to the node degree[d] and the clustering coefficient of a node[e] are widely studied in most complex networks. We refer to [4] for an extensive survey of metrics and examine here the most fundamental metrics.

The number of links L continuously increases during the period 1987–2001, as shown in Fig. 4, implying that the interactions among activity clusters become increasingly prevalent. The number of links decreases over the period 2002–2005, subsequently recovers from 2006 on. This observation seems to correspond in time and reflect the effect of the crash of the Internet bubble in the year 2001.

The power-law degree distribution $\Pr[D = k] = ck^{-\tau}$, where $c = \frac{1}{\sum_{k=1}^{N-1} k^{-\tau}}$ is a normalization constant, has been observed in many complex networks. It is characterized by a large number of low degree nodes and a small number of hubs which have a high degree. In contrast, the monetary transaction network possesses a large number of high degree nodes (see Fig. 5), which connect to almost every other node. During the entire period of 21 years, the activity cluster that represents the household proved out to have the highest degree 103 compared to all the others.

[d]The degree of a node is the number of its direct neighbors.
[e]The clustering coefficient of a node is the ratio of the number of links among its d_i direct neighbors divided by the total possible number of links $\binom{d_i}{2}$. It describes the density of connections among the direct neighbors of this node.

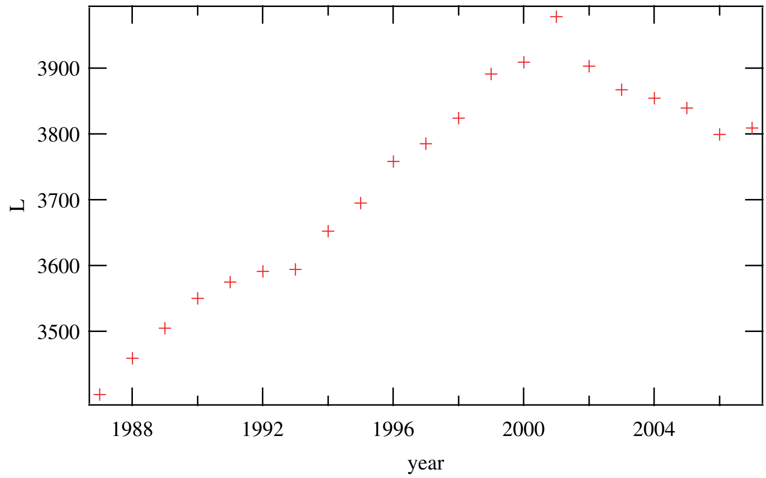


Fig. 4. The number of links L in the sector network measured from 1987 to 2007.

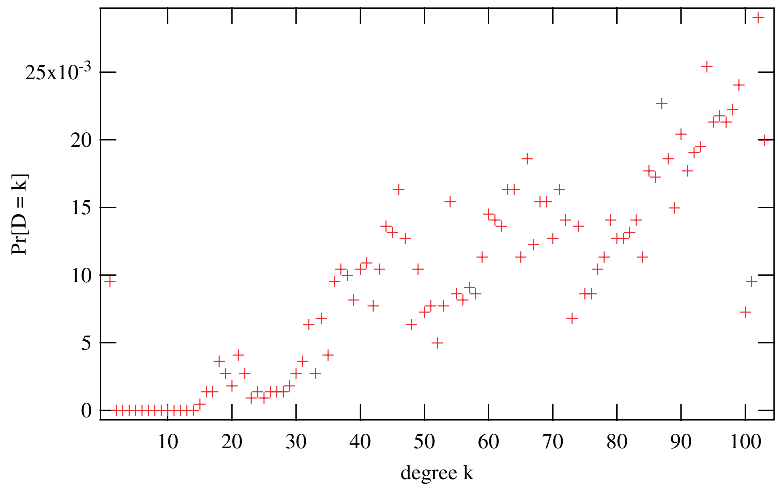


Fig. 5. Degree distribution $\Pr[D = k]$.

The existence of such hubs (or high degree nodes) and high link density leads to a large clustering coefficient and a small diameter, where the diameter is the maximum distance in terms of the number of hops or links over all pairs of nodes in a network. In each of the 21 network instances, the diameter is maximally 3 because the number of nodes is $N = 105$ and the maximum degree is 103.

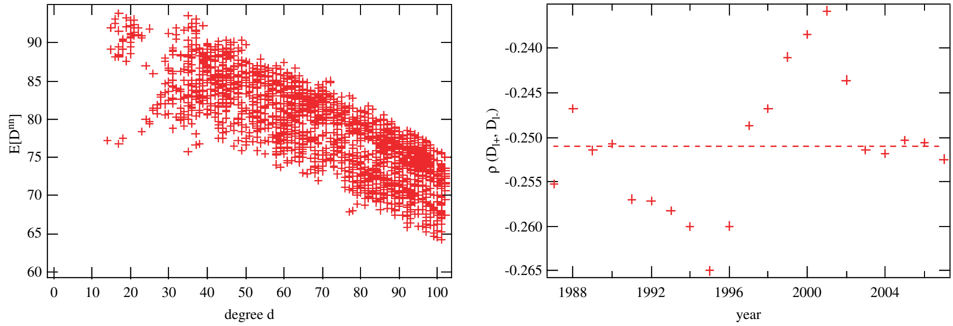
"Mixing" in complex networks [14, 15] refers to the tendency of network nodes to preferentially connect to other nodes with either similar or opposite properties. Networks, where nodes preferentially connect to nodes with (dis)similar property, are called (dis)assortative. When the property of interest is the degree of a node, we examine the degree correlation of any two nodes connected by a link in a network.

The degree correlation can be measured by the relation between the degree d of a node and the average degree of its direct neighbors $E[D^{nn}]$. If there is no degree correlation in a network, $E[D^{nn}]$ is independent of d in the scatter plot of all the nodes. Figure 6(a) evidently shows that if a node has a large degree, the degree of its neighbors $E[D^{nn}]$ is small on average. Thus, at the activity cluster level the 21 network instances are disassortative in the degree.

The mixing property (or assortativity) in node degree of a network can be measured by means of another method: the linear degree correlation coefficient ρ_D, which is computed [16] as

$$\rho_D = 1 - \frac{\sum_{i\sim j}(d_i - d_j)^2}{\sum_{i=1}^N d_i^3 - \frac{1}{2L}\left(\sum_{i=1}^N d_i^2\right)^2}, \quad (1)$$

where d_j is the degree of node j and $i \sim j$ denotes that node i and j are linked. For example, networks, where high-degree nodes preferentially connect to other high-degree nodes, are assortative ($\rho(D_{l^+}, D_{l^-}) > 0$), whereas networks, where high-degree nodes connect to low-degree nodes, are disassortative ($\rho(D_{l^+}, D_{l^-}) < 0$). Figure 6(b) depicts the disassortativity of each monetary transaction network measured from 1987 and 2007. Evident transitions occur in the year 2001, where the maximal degree correlation/assortativity $\rho(D_{l^+}, D_{l^-})$ has been observed. A similar transition in the number of links in 2001 has also been observed in Fig. 4. Both observations may suggest the effect of the crash of the Internet bubble in the year 2001.



(a) (b)

Fig. 6. (a) Scatter plot of the degree d of a node and the average degree of its direct neighbors $E[D^{nn}]$. (b) The degree correlation $\rho(D_{l^+}, D_{l^-})$ in each monetary transaction network measured from 1987 and 2007, where the dotted line is the average over the 21 years.

3.2. *Link weight structure*

An important feature in the link weight domain is the probability density function of link weights. As shown in Fig. 7, the link weights distribution can be well-fitted with a power-law distribution $f_{w^l}(x) \sim x^{-1.6}$. Thus, a few links possess a large link weight, while the majority has a small link weight.

The link weight structure can be generally characterized by different types of link weight correlations, which, however, received much less attention in literature. The link weight correlation of links incident to a node examines whether links connected to a same node tend to possess similar or dissimilar link weights. Ramasco and Gonçalves [17] have proposed a measure that examines the ratio of the average variance $E_{\text{org}}[\sigma_w]$ of the link weights around each node divided by that $E_{\text{rand}}[\sigma_w]$ of an ensemble of weight-reshuffled instances of the original graph. For example, the variance of the link weight around a node i can be defined as

$$\sigma_w^2(i) = \Sigma_{j\in\mathcal{N}(i)} \left(w_{ij} - \frac{\Sigma_{j\in\mathcal{N}(i)} w_{ij}}{d_i} \right)^2 ,$$

where $\mathcal{N}(i)$ is the set of neighboring nodes of i, d_i is the degree of node i and $\frac{\Sigma_{j\in\mathcal{N}(i)} w_{ij}}{d_i}$ is, thus, the average link weight of the links arriving at i. The link weight correlation is then measured as

$$\Delta_w = \frac{E_{\text{org}}[\sigma_w]}{E_{\text{rand}}[\sigma_w]}, \tag{2}$$

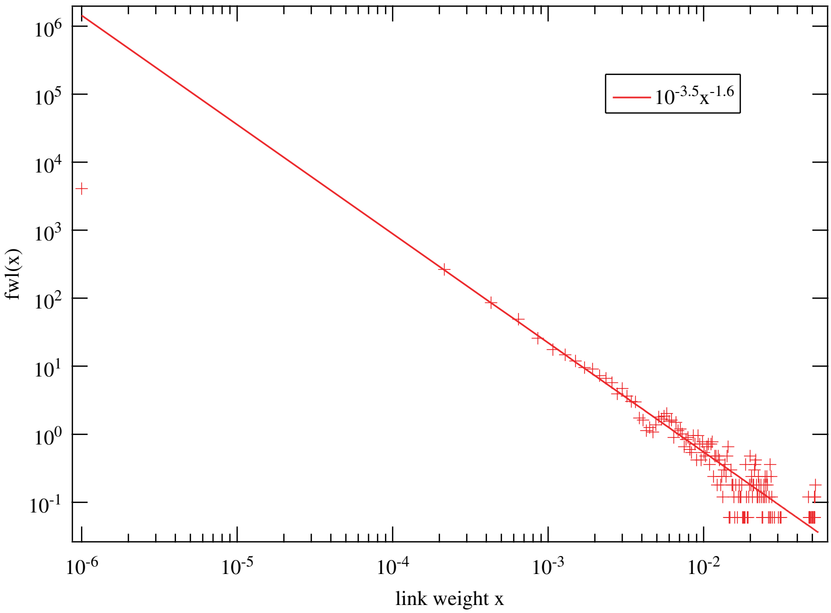


Fig. 7. The probability density function $f_{w^l}(x)$ of link weights.

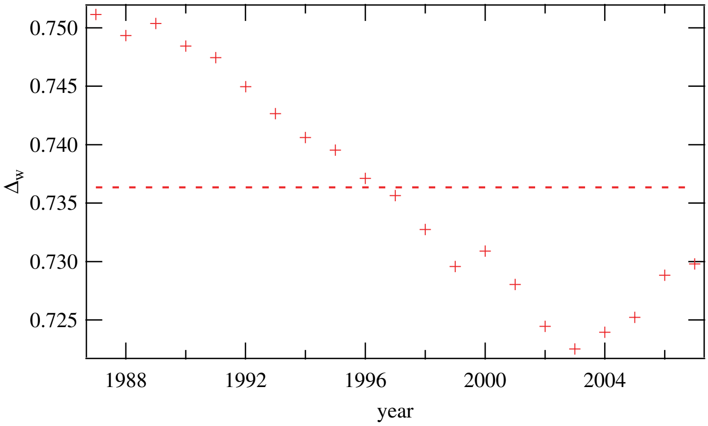


Fig. 8. The link weight correlation Δ_w around each node of the monetary transaction network measured from 1987 and 2007. The dotted line is the average over the 21 years.

where the average standard deviation of link weights around each node $E[\sigma_w]$ is estimated for the original graph and an ensemble of weight-reshuffled[f] instances. The type of link weight correlation around each node in a network is revealed by comparing with the randomized instances: positive ($\Delta_w < 1$), negative ($\Delta_w > 1$) or non-correlated ($\Delta_w = 1$). As depicted in Fig. 8, all the network instances measured between 1987 and 2007 are positively correlated in link weight surrounding a node since $\Delta_w < 1$ and the average link weight correlation is $E[\Delta_w] = 0.74$. Hence, the money flows between one cluster and its cooperative clusters tend to be similar in amount. This link weight correlation becomes stronger over time 1987–2003, as reflected by the decrease in Δ_w in Fig. 8. This may imply the trend that nowadays each activity cluster tends to balance its monetary transactions with its cooperative clusters, instead of exchanging far more money with some cluster than with the others.

We introduce another measure of link weight correlation around each node. Firstly, we define the node strength $s_i = \sum_{j \in \mathcal{N}(i)} w_{ij}$ as the total weight of all the links connected to a node i. Note that node strength is the total volume of external transactions of an activity cluster while node weight describes the total amount of internal transactions within an activity cluster. The average link weight incident to a node i is $\frac{s_i}{d_i}$. The relation between the weight w_{ij} of link (i, j) and the geometric mean $\sqrt{\frac{s_i s_j}{d_i d_j}}$ of the average link weight incident to node i and to node j is examined over all the links in a network. If $\sqrt{\frac{s_i s_j}{d_i d_j}}$ and w_{ij} are positively correlated, a high link weight w_{ij} implies a high $\sqrt{\frac{s_i s_j}{d_i d_j}}$. Other links connected to i and j have to possess relatively high link weights if the network is large and

[f]The set of L link weights is re-assigned randomly to the set of L links.

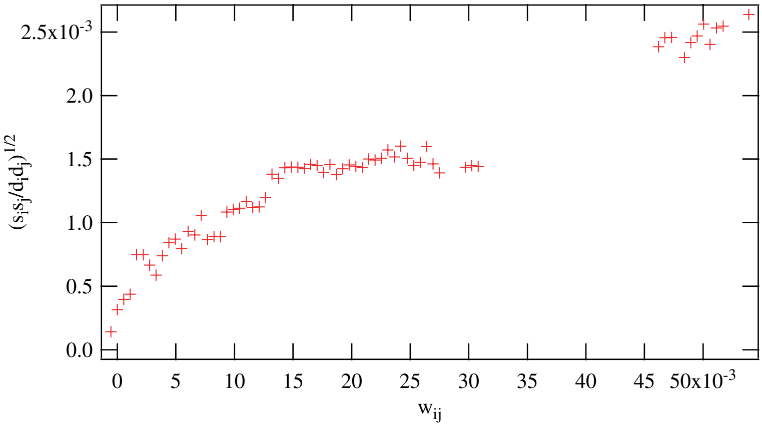


Fig. 9. The relation between $\sqrt{\frac{s_i s_j}{d_i d_j}}$ the geometric mean of the average link weight incident to node i and j and the link weight w_{ij}.

dense, because only one large link weight w_{ij} cannot lead to a high node strength s_i or s_j. Thus, a positive link weight correlation around a node can be expected. Specifically, the slope of $\frac{\sqrt{s_i s_j}}{N-1}$ as a function of w_{ij} reflects the relative strength of the link weight correlation surrounding a node.[g] Figure 9 illustrates the positive link weight correlation by[h] the positive slope of $\frac{\sqrt{s_i s_j}}{N-1}$ versus w_{ij}, which is consistent with the measure Δ_w. The $\frac{\sqrt{s_i s_j}}{N-1}$ and $\frac{s_i+s_j}{2(N-1)}$ in relation to w_{ij} similarly reveal the positive link weight correlation around a node as compared in [18].

3.3. *Node weight structure*

Both the node weight and the degree are a property of a node. Thus, measures related to the degree in the topology domain can be applied to the node weight domain. For example, the degree–degree correlation in the topology domain examines whether nodes with similar or opposite degrees tend to connect to each other. Correspondingly, we may study whether nodes with similar or dissimilar node weight tend to link to each other.[i]

Similar to the link weights, the node weights in the class of monetary transaction networks also well fit a power-law distribution $f_{w^n}(x) \sim x^{-1.3}$, as depicted in Fig. 10.

[g]The arithmetic mean $\frac{s_i+s_j}{2(N-1)}$ as a function of w_{ij} can also be used to measure link weight correlation and it illustrates the same result, as discussed in [18].

[h]Instead of making a scatter plot of all $(\frac{\sqrt{s_i s_j}}{N-1}, w_{ij})$ pairs, we divided the link weight range $[0, 1]$ into 100 bins and over each bin, we calculated the average $\frac{\sqrt{s_i s_j}}{N-1}$ corresponding to those w_{ij} that belong to the same bin.

[i]Note that such a node weight–node weight correlation involves in the information of topology although we focus on the node weight domain.

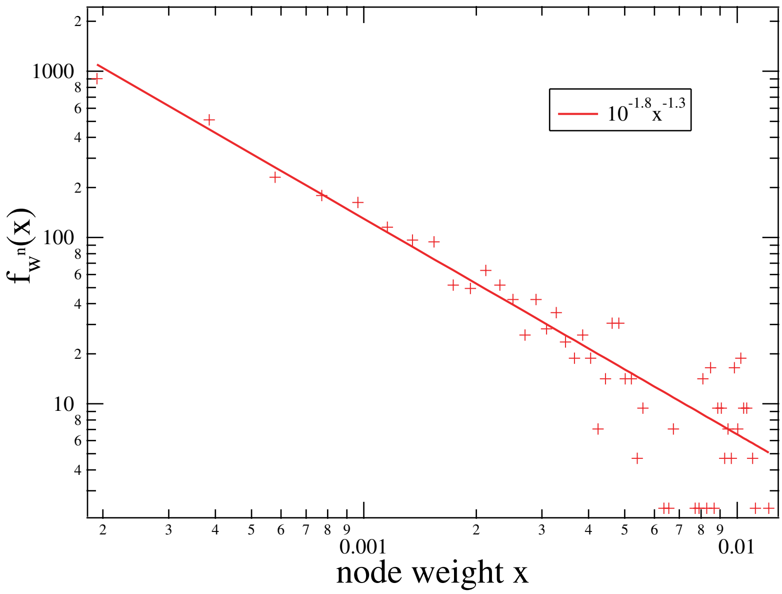
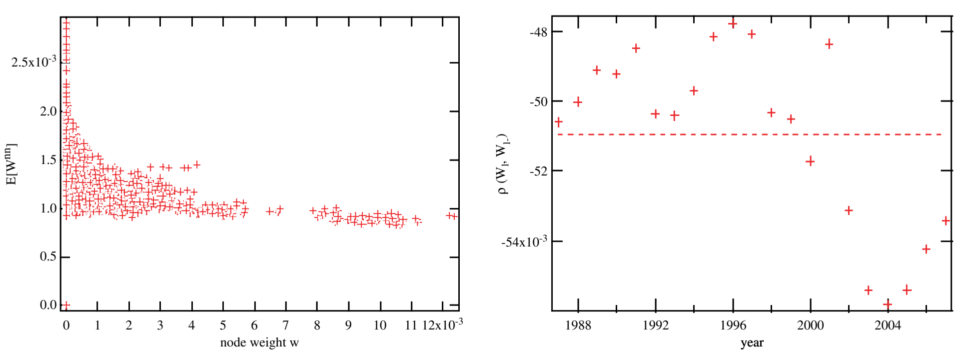


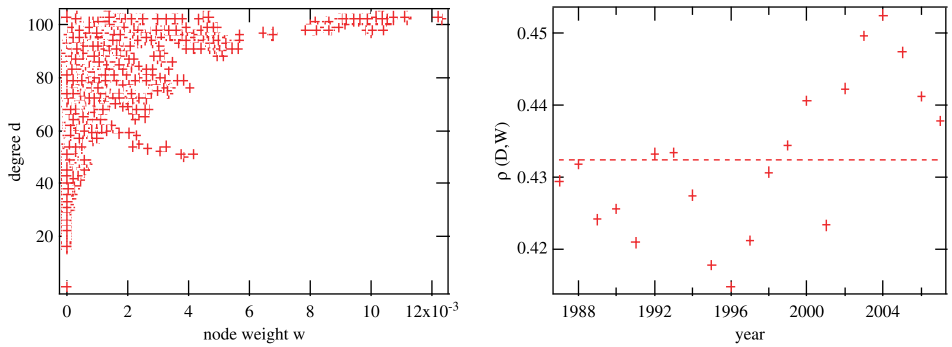
Fig. 10. The probability density function $f_{w^n}(x)$ of node weights.

With respect to the node weight–node weight correlation, we examine the relation between the weight w of a node and the average weight of its direct neighbors $E[W^{nn}]$, similar to the degree–degree correlation. The scatter plot in Fig. 11(a) shows that if a node has a large node weight, the average weight of its neighbors $E[W^{nn}]$ is small. When a node has a small weight, $E[W^{nn}]$ varies dramatically but is large on average. The class of monetary transaction networks shows disassortativity in the weight of nodes that are connected by a link. A large number of nodes



(a) (b)

Fig. 11. (a) The scatter plot of the weight w of a node and the average node weight of its direct neighbors $E[W^{nn}]$. (b) Node weight correlation $\rho(W_{l+}, W_{l-})$ of connected node pairs in each monetary transaction network measured from 1987 and 2007, where the dotted line is the average over the 21 years.



(a) (b)

Fig. 12. (a) Scatter plot of the degree d and the node weight w of each node. (b) The linear correlation coefficient $\rho(D, W)$ of the degree and the node weight of a node in each monetary transaction network measured from 1987 and 2007, where the dotted line is the average over the 21 years.

have a degree 103 (see Fig. 5) and thus have almost the same $E[W^{nn}]$, which results in the floor in the scatter plot 11(a). As shown in Fig. 12(a), activity clusters with high node weight $w > 6 \times 10^{-3}$, are almost connected to all the other clusters. This explains why the average weight of neighbors $E[W^{nn}]$ remains the same for nodes with a large weight.

The node weight–node weight correlation $\rho(W_{l^+}, W_{l^-})$ can be as well computed via (1) by replacing the degree with the node weight. Weak disassortativity[j] $\rho(W_{l^+}, W_{l^-}) < 0$ is observed in each monetary transaction network measured from 1987 and 2007, as shown in Fig. 11(b). In other words, activity clusters, with a small amount of internal money flow tend to interact with clusters that have a huge amount of internal money flow.

3.4. *Correlation among topology, link weight and node weight structure*

We are going to investigate the correlation between any two of the three dimensions: topology, node weight and link weight structure. In order to exemplify the methodology in a simple way, we mainly explore the correlations among the corresponding elementary features in these three dimensions: degree, node weight and

[j]The scatter plot of $E[W^{nn}]$ versus node weight w reveals the node weight correlation in a straightforward way via detailed views at each node. The quantitative metric assortativity $\rho(W_{l+}, W_{l-})$ depends on the node weight distribution, which is a power-law distribution in the studied networks. The assortativity in the rank of node weight will be different from that of the node weight $\rho(W_{l+}, W_{l-})$, because (a) the transformation from the node weight (power-law distributed) to the rank of the node weight (uniformly distributed) is a nonlinear strictly increasing transformation. (b) The linear correlation $\rho(X, Y)$ is not invariant under nonlinear strictly increasing transformation T such that $\rho(T(X), T(Y) \neq \rho(X, Y)$ [19]. The weak node weight disassortativity $\rho(W_{l+}, W_{l-}) < 0$, thus, may result from the power-law distribution of node weight.

average link weight incident to a node, which equals the strength of a node divided by its degree s/d.

3.4.1. *Topology and node weight structure correlation*

The relation between the degree d and the node weight w of each node is depicted in Fig. 12(a). Activity clusters with a large node weight tend to have monetary transactions with many other activity clusters. Activity clusters with extremely high node weight $w > 6 \times 10^{-3}$, are almost connected to all the other clusters. The positive correlation between the degree and weight of a node $\rho(D, W) > 0$ is identified in each monetary transaction network instance measured from 1987 and 2007 in Fig. 12(b). The positive correlation between the weight and the degree of a node manifests that a cluster with more money circulating inside tends to be capable to cooperate with more activity clusters.

3.4.2. *Topology and link weight structure correlation*

We examine the correlation between the degree and the average link weight around a node. Node strength s is the product of the degree d and the average link weight incident to that node s/d. Therefore, the correlation between the node strength s and the degree d is expected to be more positive than the correlation between the degree d and the average link weight around a node s/d. In social networks where individuals (nodes) are connected via collaborations (links) and the link weights quantify the number of collaborations between two individuals, the node strength s and the average link weight incident to a node s/d have been thoroughly studied [20] since they stand for the total number of collaborations a person has been involved and the likelihood that an individual collaborate with the same partner respectively. Figure 13(a) shows the scatter plot of the degree of a node versus the average link

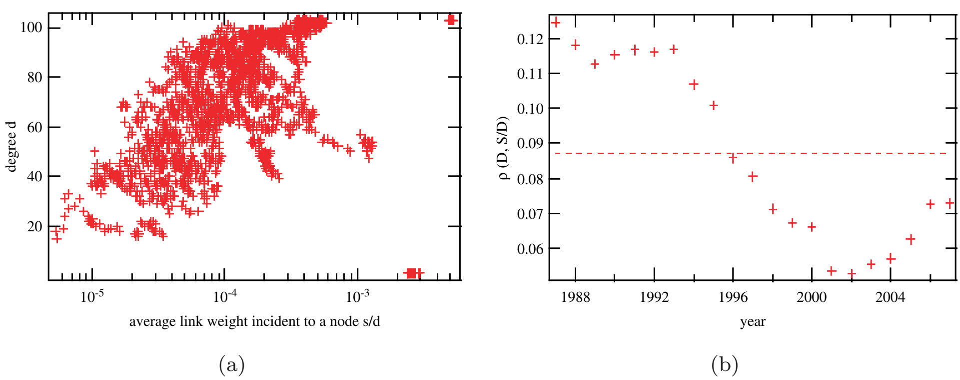


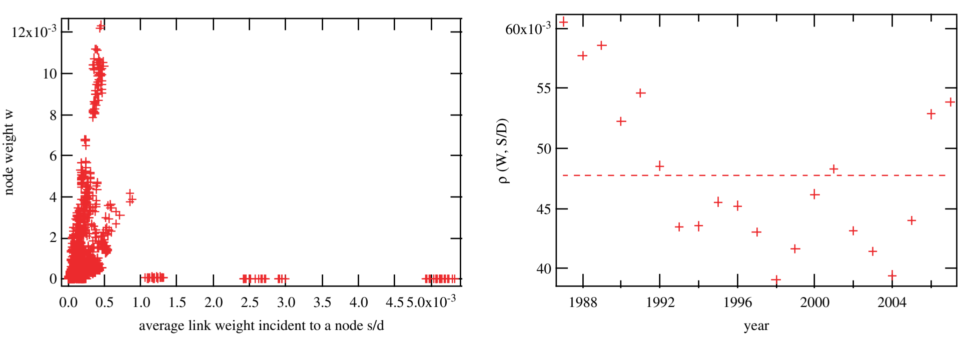
Fig. 13. (a) Scatter plot of the degree d and the average link weight incident to the node s/d in linear-log scale. (b) The linear correlation coefficient $\rho(D, S/D)$ of the degree and the average link weight incident to a node in each monetary transaction network measured from 1987 and 2007, where the dotted line is the average over the 21 years.

weight incident to the node. It reveals that if a node has a high degree, the links connected to the node possess, on average, a high link weight.[k] The degree and the average link weight incident to a node are positively correlated in each network instance measured from 1987 and 2007 as depicted in Fig. 13(b). The positive correlation between the degree of a node and the average link weight incident to the node implies that a cluster cooperating with many other clusters exchanges a large amount of money via each of these links. The correlation $\rho(D, S/D)$ continuously decreases during the period 1987–2001, as shown in Fig. 13(b) and subsequently recovers from 2002 on, which correspond in time the turning-point in the number of links, the degree correlation, and the link weight correlation around a node.

Other correlations between the topology and the link weight dimension examine, for instance, the correlations between the link weight w_{ij} and the degrees of the end-point nodes d_i and d_j [9].

3.4.3. *Link weight and node weight correlation*

Figure 14 shows the weak positive correlation between the node weight and the average link weight around a node via both the scatter plot and the correlation coefficient. This is consistent with earlier observations: (a) the positive correlation between degree and weight of a node and (b) the positive correlation between degree and average link weight around the node. The average link weight incident to a node larger than 1.0×10^{-3} appears for node 78, 103 and 105. Node 105 (the households activity cluster) is a hub. It delivers the expenses and salaries of all Dutch households from and to each of all the other activity clusters except for activity cluster 104. The positive correlation between node weight and s/d does not hold for these three clusters. In general, a cluster with a large amount of value



(a) (b)

Fig. 14. Scatter plot of the node weight w and the average link weight incident to a node s/d.

[k]The points with degree 1 correspond to activity cluster 103 measured in each of the 21 years. This super node 105 delivers the expenses and salaries of all Dutch households from and to each of all the other 103 activity clusters except for activity cluster 103.

inside the cluster, is likely to exchange a large amount of monetary transactions with other cooperative clusters.

The correlation between node weight and link weight can be further explored by examining the relation of a link weight w_{ij} versus the node weights w_i and w_j of the two end nodes of the link. Since node weight and the average link weight incident to a node is positively correlated, we expect that two nodes with large node weights w_i and w_j are likely connected by a link with high link weight w_{ij}.

3.4.4. *Summary*

The class of monetary transaction networks is featured by: (1) a high link density and a large number of hubs. They determine, to a large extent, other topology related properties such as a short average distance and a small diameter. The large number of hubs also explains the floor and ceiling observed in the scatter plots of Figs. 11(a) and 12(a). The high link density results from the fact that a network at a higher hierarchical level, like the activity cluster level, is denser than that at a lower hierarchical level, as discussed later in Sec. 4. (2) A power-law like link weight and node weight distribution that illustrates the diversity of the amount of monetary transactions in between activity clusters and within activity clusters, respectively. (3) A positive correlation among node weight, degree and the average link weight incident to a node. It implies that an activity cluster with a large amount of money flowing inside exchanges a large amount of money with each of many other activity clusters. (4) A disassortativity in degree and node weight, which reflects that activity clusters with lower internal transaction volume collaborate with fewer clusters, but preferably with those containing higher internal flows. This properties will be explained and compared with other complex networks in the next section. (5) a positive link weight correlation of links incident to a node. In other words, activity clusters tend to transact with their neighbors an equal amount.

When we look at the evolution of the network year by year from 1987 to 2007, we deem the follow findings important: (1) the number of links and link weight correlation around a node continuously increase until the year 2001 and 2003 respectively. The correlation between degree and average link weight incident to a node decreases until 2001 and 2002. The period 2001–2003 coincides in time with the crash of the Internet bubble in the year 2001, which suggests that the evolution of the monetary transaction network may reflect the performance of our economic system (see Sec. 5).

4. Similarity with Topological Community Overlays

Nodes of complex networks can be clustered into topological communities such that the link densities within communities are higher than that between communities. A number of algorithms has been proposed to find communities in complex networks based on their network topologies. A topological community overlay can be constructed upon a network, called the underlying network, by (a) aggregating each topological community into a node and (b) connecting two nodes in the overlay if in the underlying network at least one link connects nodes residing in the two corresponding communities respectively. Node weight in the community overlay is usually used to represent a certain property of an underlying community such as its number of nodes/links. The link weight in the overlay may e.g. represent the number of links between two communities in the underlying network.

In the monetary transaction network as shown in Fig. 1, activity clusters sharing a similar functioning form a functional community, which is further aggregated into a sector, a node at a higher layer. The monetary transaction network at sector level is, thus, a functional community overlay upon the transaction network at activity cluster level. In a similar way, the network at activity cluster level is again a functional community overlay upon a lower level network, where those lower level elements sharing a similar functioning form a community, aggregated as an activity cluster. Generally, classification systems that describe all economic activities discriminate five different aggregation layers. Correspondingly, the monetary transaction network could be decomposed into five hierarchical aggregation layers as well. The network at a higher hierarchical level is denser. For example, the network at the highest level, the sector level is almost a complete graph. In this work, we focus on the monetary transaction network at the activity cluster level considered as a functional community overlay.

In the construction of topological community overlays and functional community overlays, communities are derived from the connections of the nodes (topology) and from the functioning of the nodes respectively. Despite the evident different nature of topological and functional community overlays, they surprisingly share similar properties in degree–degree correlation (as well as node weight–node weight correlation) and node weight distribution.

Newman [14] observed a disassortative degree–degree correlation in technological and biological networks and the opposite (assortativity) concerning social networks. Our recent work [21] shows that topological community overlays constructed upon 82 real-world complex networks are disassortative in degree–degree and node weight–node weight correlation, where the weight of a node can be the number of nodes or links of the lower level community corresponding to this node. Specifically, the overlay most likely has a smaller degree–degree correlation than its underlying network $\rho_{\text{overlay}}(D_{l^+}, D_{l^-}) < \rho(D_{l^+}, D_{l^-})$ and is mostly disassortative $\rho_{\text{overlay}}(D_{l^+}, D_{l^-}) < 0$ [21]. The monetary transaction network at activity cluster level, a functional community overlay upon a lower level, also shows disassortativity in degree–degree correlation (see Fig. 6) and in node weight–node weight correlation (Fig. 11), where the node weight is the amount of money circulating inside an activity cluster. The disassortativity in the topological and functional community overlays can be understood by the following two aspects: (a) Newman has suggested assortative degree correlation can be explained by the presence of evident communities in the network, assuming that almost all nodes within each community will be homogeneously connected and will therefore have approximately the same degree.

Otherwise, a network is disassortative. (b) After aggregating each community into one node, the community overlay does not seem to possess communities any more. One supportive example is the Internet, a network of Autonomous Systems (AS) which are collections of routers under the control of one or more network operators. The Internet at AS level, a functional overlay, is more disassortative than that at the router level: $\rho_{AS}(D_{l^+}, D_{l^-}) = -0.189 < \rho_{\text{router}}(D_{l^+}, D_{l^-}) = -0.024$. The node weight (number of nodes/links or amount of money inside a community) is mostly positively correlated with the degree of that node. Therefore, the disassortativity in node weight–node weight correlation can be expected.

The node weights in the monetary transaction network seem likely to follow a power-law distribution. When the community size or the number of elements in a community is considered as the node weight, surprisingly, such a power-law node weight distribution is widely observed in the topological community overlays upon a large number of real-world networks [22].

5. Conclusion

Having combined the disciplines of economic data research and complex network research resulted in observations and additional insights about the developments and changing features of the Dutch national economy. In this work, we construct a transaction network from the recorded monetary data for each of the 21 years over the period 1987–2007. This network describes the monetary transactions among 105 activity clusters. Our work contributes to the following two aspects.

Firstly, we propose a systematic network representation of a multi-weighted network, which includes both its node weights and its link weights. In the monetary transaction network, for example, the monetary transactions between activity clusters are described by link weights. The monetary transactions within activity clusters are described by node weights.

Secondly, by applying methodologies/metrics of complex network theory, we observe important features of the monetary transaction network. (a) The network differs from most real-world complex networks in its high link density and the large number of hubs. (b) The power-law like node weight (community size) distribution and the disassortativity in degree–degree correlation (and node weight–node weight correlation) of nodes connected by a link turn out to be the generic features of topological community overlays upon various complex networks. Surprisingly, these properties are also exhibited by the monetary transaction network at the activity cluster level, which can be regarded as a functional community overlay. (c) An activity cluster with a large internal flow is likely to cooperate with many other clusters via high volume monetary transactions. Activity clusters with lower internal transaction volume collaborate with fewer clusters, and preferably with those containing higher internal flows (disassortative node weight correlation). (d) Activity clusters are spreading out transaction amounts more equally with their neighbors rather than transacting only high values with a preferred, small group of partners (assortative link weight correlation around a node). This correlation becomes stronger over time. The number of links increases as well. Organizations are increasingly able to outsource the development and production of their product components and retain only access to (not ownership of) resources. These resources are now globally available, which is a prime source of innovation [23]. The link weight correlation and the number of links continuously increase until a trend change in 2003 and 2001 respectively. Furthermore, the correlation between degree and average link weight incident to a node continuously decreases until 2002. These three observations remind us of the massive rise of Digital Information Networks, Mobile Communications, the introduction of the World Wide Web and the crash of the Internet bubble around 2001. The evolution of network properties may provide more insights about the performance of our economical system.

Acknowledgment

Herewith we cordially thank John Hoffmans (KPN Royal) for sharing his unifying vision on networks and providing his constructive feedback.

References

[1] Schweitzer, F., Fagiolo, G., Sornette, D., Vega-Redondo, F. and White, D. R., Economic networks: What do we know and what do we need to know? *Adv. Complex Syst.* **12**(4/5) (2009) 407–422.

[2] Ten Raa, T., *The Economics of Input-Output Analysis* (Cambridge University Press, 2005).

[3] Newman, M. E. J., The structure and function of complex networks, *SIAM Rev.* **45** (2003) 167–256.

[4] Da Costa, L., Rodrigues, F. F. A., Travieso, G. and Villas Boas, P. R., Characterization of complex networks: A survey of measurements, *Adv. Phys.* **56**(1) (2007) 167–242.

[5] Watts, D. J., Small Worlds, *The Dynamics of Networks Between Order and Randomness* (Princeton University Press, Princeton, New Jersey, 1999).

[6] Albert, R. and Barabasi, A., Emergence of scaling in random networks, *Science* **286**(5439) (1999) 509–512.

[7] Serrano, M. A., Boguñá M. and Vespignani, A., Patterns of dominant flows in the world trade web, *Journal of Economic Interaction and Coordination* **2** (2007) 111.

[8] Fagiolo, G., Reyes, J. and Schiavo, S., World-trade web: Topological properties, dynamics, and evolution, *Phys. Rev. E* **79** (2009) 036115.

[9] Barrat, A., Barthélemy, M., Pastor-Satorras, R. and Vespignani, A., The architecture of complex weighted networks, *PNAS* **101**(11) (2004) 3747–3752.

[10] Onnela, J.-P., Saramäki, J., Hyvonen, J., Szabó, G., Argollo de Menezes, M., Kaski, K., Barabási, A.-L. and Kertész, J., Analysis of a large-scale weighted network of one-to-one human communication, *New J. Phys.* **9** (2007) 179.

[11] Ángeles Serrano, M., Boguñá, M. and Pastor-Satorras, R., Correlations in weighted networks, *Phys. Rev. E* **74** (2006) 055101R.

[12] Baken, N., van Belleghem, N., van Boven E. and de Korte, A., Unravelling 21st century riddles — Universal network visions from a human perspective, *J. Commun. Netw.* **5**(4) (2006).

[13] United Nations Statistics Department, *ISIC Revision* **4** (New York, 2008).
[14] Newman, M. E. J., Assortative Mixing in Networks, *Phys. Rev. Lett.* **89** (2002) 208701.
[15] Newman, M. E. J., Mixing patterns in networks, *Phys. Rev. E* **67** (2003) 026126.
[16] Van Mieghem, P., Wang, H., Ge, X., Tang, S. and Kuipers, F. A., Influence of Assortativity and degree-preserving rewiring on the spectra of networks, *Eur. Phys. J. B* **76**(4) (2010) 643–652.
[17] Ramasco, J. J. and Gonçalves, B., Transport on weighted networks: When the correlations are independent of the degree, *Phys. Rev. E* **76** (2007) 066106.
[18] Wang, H., Douw, L., Hernández, J. M., Reijneveld, J. C., Stam, C. J. and Van Mieghem, P., Effect of tumor resection on the characteristics of functional brain networks, *Phys. Rev. E* **82** (2010) 021924.
[19] Van Mieghem, P., *Performance Analysis of Communications Systems and Networks* (Cambridge University Press, Cambridge, U.K., 2006).
[20] Ramasco, J. J. and Morris, S. A., *Phys. Rev. E* **73** (2006) 016122.
[21] Ge, X. and Wang, H., Community structures of real-world complex networks, to appear in *Eur. Phys. J. B*.
[22] Clauset, A., Newman, M. E. J. and Moore, C., Finding community structure in very large networks, *Phys. Rev. E* **70** (2004) 066111.
[23] Prahalad, C. K. and Krishnan, M. S., *The New Age of Innovation* (Tata McGraw Hill, 2008).