produces an increased electron injection from the source and a pronounced increase in the current drive as the drain voltage increases. On the other hand, the electric field intensity exhibits a sharp increase at the drain end of the gate for the HEMT with modified p-channel doping. Both electric field intensity and electron concentration at the source end for the HEMT with modified p-channel doping are much lower and less sensitive to the applied drain voltage compared with corresponding values for the HEMT's with both a modified n channel and an undoped channel. The low electron density and less spread in the electric field profile do not significantly increase the electron injection into the channel. As a result, drain current is less sensitive to applied drain voltage and output conductance is considerably reduced for the device with modified p-channel doping.

In summary, we have studied the effects of modified n- and p-channel doping on the device characteristics of 0.25 µm gate length In$_{0.53}$Al$_{0.47}$As/In$_{0.53}$Ga$_{0.47}$As/InP HEMT's. As gate voltage increases, the transconductance for the HEMT with modified p-channel doping is comparable to that of the undoped channel HEMT and is much higher than that for the HEMT with modified n-channel. Simultaneously, devices with modified p-channel doping exhibit reduced output conductance compared with those for the modified n-channel and undoped channel counterparts. Ensemble Monte Carlo simulations reveal that enhanced carrier confinement and reduced spread in the channel field profile are the primary physical mechanisms for the experimentally-observed improvement of the HEMT with modified p-channel doping. Results from this study indicate that employing proper modified p-channel doping is a promising approach for improved In$_{0.53}$Al$_{0.47}$As/In$_{0.53}$Ga$_{0.47}$As/InP HEMT operations.

REFERENCES


Origin of the Difference Between the Capacitance Intercept Voltage and the Built-In Potential

P. Van Vieghe

Abstract—For a diode with an abrupt, constant doping profile, the difference between the built-in potential $V_b$ and the capacitance intercept voltage $V_{bi}$ is shown to be at least $2(eT/q)$. The difference lies in the fact that $q/V_b$ reflects the potential energy of the equilibrium space charge in the diode whereas $q/V_{bi}$ is related to the work needed to create this space charge.

The built-in potential $V_b$ of a pn-junction is a basic parameter appearing in many fundamental formulas that describe electrical phenomena in semiconductor devices [1]. Despite its important role it is not straightforward how to determine $V_b$ experimentally. A common technique is to extract the capacitance intercept voltage $V_{bi}$ from a junction capacitance measurement $C(V)$. This is broadly considered a good measure for $V_b$, although the theory of the capacitance intercept voltage has a long history (see [2] for the references) a coherent physical understanding of both quantities has been lacking.

In this brief, we investigate the relation between $V_b$ and $V_{bi}$. The aim of the paper is basically of device physical interest although the determination of the doping concentration $N_a$ and the bandgap $E_g$ from capacitance measurements may benefit from the presented physical interpretation. For mathematical simplicity, we restrict ourselves to abrupt junctions with constant doping profiles in one dimension. For these diodes $V_{bi}$ is defined as the asymptotic voltage for which $C'(V_{bi}) = 0$. As there exists no voltage for which $C'(V) = 0$ there exists no capacitance values from the reverse and forward biased region.

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Let us start with the classical formula's in the literature [1] that assume Boltzmann statistics,
\[ V_{bi} = E_x + \frac{k_BT}{q} \ln \left( \frac{N_0}{N_i} + \frac{N_0}{N_i} \right) \]
and
\[ V_{in} = E_x + \frac{k_BT}{q} \ln \left( \frac{N_0}{N_i} + \ln \left( \frac{N_0}{N_i} \right) - 2 \right) \]
where the bandgap \( E_x \) is expressed in meV.

These expressions differ by an amount \( \Delta = V_{bi} - V_{in} = 2(k_BT/q) \). The difference \( \Delta \) has been attributed to the influence of free carriers in the Debye regions that compensate a fraction of the depleted ion charge. This results in a lowering of \( V_{in} \). But, the space charge in the reverse biased voltage regime from which \( V_{in} \) is extracted, is more depleted than the equilibrium space charge. Hence, we expect even more pronouncd free carrier effects for \( V_{in} \).

Why then do we not see their influence on \( V_{bi} \)? This observation poses the question as what \( \Delta \) means or what phenomenon allows for this scission. Intuitively, one is eager to conclude that both quantities should be the same and that \( V_{in} \) is the only accurate (and simple) way to measure \( V_{bi} \) (as is usually done as already mentioned above). Equation (2) for \( V_{in} \) is exact under the assumptions of constant quasi-fermi levels over the diode and Boltzmann statistics.

In order to gain a deeper understanding, it is instructive to briefly focus on \( V_{bi} \), to derive a more accurate expression than (1).

Invoking the neutrality condition, equation 30 on p. 27 in [1] gives
\[ V_{bi} = E_x + \frac{k_BT}{q} \ln \left( \frac{N_0}{N_i} \right) + \ln \left( \frac{N_0}{N_i} \right) \]
\[ + \ln \left( \frac{N_0}{N_i} \right) \left( \frac{1}{1 + \frac{4n_i^2}{N_0^2}} \right) \]
(7)

The last terms are at most of order \( n_i^2/N_0^2 + n_i^2/N_0^2 << 1 \) and more than sufficient to justify (1). In addition their sign is such that \( \Delta \) becomes even larger than \( 2(k_BT/q) \). In conclusion so far, within the assumption of Boltzmann statistics, we find that the classical expression of \( V_{bi} \) is accurate within a fraction much smaller than \( 2(k_BT/q) \). However, this derivation does not give a hint to unravel a deeper physical reason for the difference \( \Delta \).

In order to investigate the origin of the discrepancy, we now turn our attention to \( V_{in} \). Only for abrupt junctions with a constant doping profile, a general description assuming the Fermi-Dirac statistics and an arbitrary density of state \( g(E) \) can be presented. From previous work [2], [3] on the capacitance of an abrupt diode (written for a symmetrical junction \( N_0 = N_c = N \) for reasons explained in [2]), we find
\[ V_{in} = E_x + \frac{k_BT}{q} \left( \frac{1}{N} \sum_{n=-L}^{n=L} \frac{dn}{\partial E(x)} \left( \frac{1}{N} \sum_{m=-L}^{m=L} \frac{dn}{\partial E(x)} \right) \right) \]
(8)

where the notation is clarified in the appendix. The built-in potential, written in the same general formalism, is
\[ V_{in} = E_x + \frac{k_BT}{q} \left( \frac{Z(N)}{N} \right) \]
(9)

As a result, we observe that the difference \( \Delta = (k_BT/q)Z(N) \) is now doping concentration dependent and is plotted for GaAs in Fig. 1. For low doping concentrations, \( Z(N) \) tends to 2 (the Boltzmann non-degenerate regime), while it increases significantly (roughly \( Z(N) \sim N^{3/2} \)) for heavy doping. Now, expression (9) can be transformed into an integral as demonstrated in appendix B, yielding
\[ V_{in} = E_x + \frac{k_BT}{q} \left[ \frac{1}{N} \int_{-L}^{L} \left( \frac{1}{N} \sum_{m=-L}^{m=L} \frac{dn}{\partial E(x)} \left( \frac{1}{N} \sum_{n=-L}^{n=L} \frac{dn}{\partial E(x)} \right) \right) \right] \]
(10)
or in the assumption that the bandgap $E_d$ does not depend on doping concentration (and thus disregarding heavy doping effects),

$$V_{in} = \frac{1}{N} \int_0^N \left[ E_d + \frac{k_BT}{q} \left( R^{-1}\left( \frac{n}{N} \right) + R^{-1}\left( \frac{p}{N} \right) \right) \right] \, dN. \tag{14}$$

A quite interesting observation is that we can rewrite (14) as

$$V_{in}(N) = \frac{1}{N} \int_0^N V_n(n) \, dN. \tag{15}$$

or, alternatively,

$$qN \cdot V_{in}(N) = \int_0^N V_n(n) \, q \cdot dN. \tag{16}$$

The interpretation of (16) is as follows. For a certain doping concentration $n$, a well-defined space charge is built up in the diode that generates a potential $V_n(n)$ over the diode. This state corresponds to a potential energy $q \cdot V_n(n)$. The energy needed to increase the space charge at $q \cdot n$ by an amount $q \cdot dN$ equals $q \cdot dN \cdot V_n(n)$. Thus, from (16), we see that $V_{in}$ is related to the total energy $qN \cdot V_{in}(N)$ needed to build up a space charge from 0 to the final concentration $N$. Equation (16) shows the essential difference between the built-in potential $V_{in}$ and the capacitance intercept voltage $V_{in}$ for a doping concentration $N$. $V_{in}$ represents the potential energy needed to add (or remove) an arbitrary small charge to (or from) the established space charge at doping concentration $N$ while $V_{in}$ is related to the work involved to create this space charge.

Clearly, these two energies are in general not the same [4], hence $V_{in}$ does not equal $V_{in}$. In fact from (15), $V_{in}$ equals the built-in potential averaged over all carrier concentration up to $N$, $\langle V_n(n) \rangle_N$, which is always smaller than $V_{in}(N) = \max(V_n(n))$.

The way in which $V_{in}$ is obtained, is consistent with the interpretation above. The capacitance intercept voltage is found from extrapolation of $C^{-1}(V)$ in reverse (or low forward) biased region. For increasing negative applied voltages, an increasing space charge is built up. This information of the space charge built-up energy must be enclosed in $V_{in}$. More specific as suggested by its definition, $V_{in}$ reflects the creation energy of the equilibrium space charge.

Finally, notice that (15) is general and also applies to Boltzmann statistics as readily may be verified. Equations (15) and (16) are proposed based on a particular doping profile. A proof of (15) and (16) for an arbitrary profile seems hardly possible due to the lack of a general solution—even assuming constant quasi-Fermi levels—of the nonlinear Poisson equation (of the form $d^2\psi/dx^2 = g(\psi(x)) + f(x)$, where $g(\psi(x))$ and $f(x)$ are related to the free carriers and doping profile, respectively) preventing accurate analytical calculations. Relying on the space charge layer approximation for this problem may introduce errors as large as the discrepancy between $V_{in}$ and $V_{in}$ and is thus here inadequate.

**Appendix A**

**The $\Phi$ Transform**

In order to account for an arbitrary density of state (DOS) function, the $\phi$ transform is introduced which is defined as

$$\phi(g(\xi), y) = \int_{-\infty}^{\infty} \frac{g(\xi)}{1 + \exp(\xi - y)} \, d\xi = \mathcal{R}(y) \tag{A1}$$

The number of electrons (holes) $n(p)$ in a semiconductor with a DOS $g_{eff}(E)$ and quasi-Fermi level $\xi_{eff}$ then reads

$$\begin{cases}
\mathcal{N} = N_c \cdot \phi\left( \frac{2}{\sqrt{\pi}} \sqrt{E} \xi_{eff}, \frac{E}{k_B T} \right) \\
\mathcal{P} = N_v \cdot \phi\left( \frac{2}{\sqrt{\pi}} \sqrt{E} \xi_{eff}, \frac{E}{k_B T} \right)
\end{cases} \tag{A2}$$

Particularly, if the density of states is parabolic, i.e., $g(E) = \sqrt{E}$ for $E > 0$ and $g(E) = 0$ for $E < 0$, the number of electrons written in the formalism of (A1) with Heavyside’s step function $\theta(x)$, yields

$$\mathcal{N} = N_c \cdot \phi\left( \frac{2}{\sqrt{\pi}} \sqrt{E} \xi_{eff}, \frac{E}{k_B T} \right) \tag{A3}$$

where $F_s(x)$ is the Fermi–Dirac integral of order $p$.

Finally, $y = R\xi_{eff}(x)$ denotes the inverse function or solution of $z = R_y(x)$ with respect to $y$ and the subscript $\lambda$ refers either to the conduction band ($c$) or valence band ($v$), while the integrated $\phi$ transform corresponds to

$$\int_{-\infty}^{\infty} \phi(g(\xi), y) \, du = I_{\phi}(y) = \int_{-\infty}^{\infty} g(x) \, dx \tag{A4}$$

with the following, easily verified property.

$$\int_{-\infty}^{\infty} \phi(g(\xi), y) \, du = I_{\phi}(y) = \int_{-\infty}^{\infty} g(x) \, dx \tag{A5}$$

**Appendix B**

**The Equivalence Between (9) and (13)**

The demonstration that (13) is equivalent to (9) uses the following identity of inverse $f^{-1}(y)$.

$$\int_a^b f^{-1}(y) \, dy = bf^{-1}(b) - af^{-1}(a) - \int_{f^{-1}(a)}^{f^{-1}(b)} f(y) \, dy \tag{B.1}$$

The proof of (B.1):

- Partial integration of the left hand side gives
  $$\int_a^b f^{-1}(y) \, dy = y \cdot f^{-1}(y) \bigg|_a^b - \int_a^b y \cdot \frac{df^{-1}(y)}{dy} \, dy$$

- From the definition of an inverse function, we have $f(f^{-1}(y)) = y$, which yields after differentiation,
  $$\frac{df^{-1}(y)}{dy} = \frac{1}{f'(f^{-1}(y))}$$
Substitution into the integral and making a change in the independent variable \( y = f(u) \) or \( u = f^{-1}(y) \) finally leads to (B.1)

Using (B.1) with \( f^{-1}(y) = R_{c}^{-1}(y/N_c) \) gives

\[
\int_{0}^{b} R_{c}^{-1}(y/N_c) \, dy = b \left( R_{c}^{-1} \left( \frac{b}{N_c} \right) \right) - a \left( R_{c}^{-1} \left( \frac{a}{N_c} \right) \right) + N_c \int_{0}^{b} R_{c}^{-1}(y/N_c) \, R_{c}(y) \, dy.
\]

Since \( R_{c}^{-1}(y) = \mathcal{O}(\ln(y)) \), by choosing \( a = 0 \), \( R_{c}^{-1}(a) \) vanishes and \( R_{c}^{-1}(a) \) tends to minus infinity. Applying (A5) with \( B(R_{c}^{-1}(a)) \) as \( a = 0 \), we obtain with \( b = N_c \)

\[
\int_{0}^{N_c} R_{c}^{-1} \left( \frac{y}{N_c} \right) \, dy = N_c \left( R_{c}^{-1} \left( \frac{N_c}{N_c} \right) \right) - N_c \mathcal{O} \left( R_{c}^{-1} \left( \frac{N_c}{N_c} \right) \right).
\]

A similar relation can be written for the valence band. By adding these together we establish the equivalence between (9) and (13).

As an example, we consider a parabolic DOS and Fermi-Dirac statistics (A3) to obtain

\[
V_{\text{sat}} = E_f + \frac{k_b T}{q} \left( F_{1/2} \left( \frac{N_c}{N_v} \right) + F_{1/2} \left( \frac{N_v}{N_c} \right) \right) - Z_{\text{val}}(N_c)
\]

with

\[
Z_{\text{val}}(N_c) = \frac{N_v}{N_c} \cdot F_{1/2} \left( \frac{N_c}{N_v} \right) + \frac{N_c}{N_v} \cdot F_{1/2} \left( \frac{N_v}{N_c} \right)
\]

that is equivalent to

\[
V_{\text{sat}} = E_f + \frac{k_b T}{q} \int_{0}^{1} \left[ F_{1/2} \left( \frac{n}{N_c} \right) + F_{1/2} \left( \frac{n}{N_v} \right) \right] dn.
\]

The Boltzmann counterpart follows from

\[
1 \int_{0}^{N_c} \left( \ln \left( \frac{n}{N_c} \right) + \ln \left( \frac{n}{N_v} \right) \right) dn = \ln \left( \frac{N_c}{N_v} \right) + \ln \left( \frac{N_v}{N_c} \right) - 2
\]

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REFERENCES