Shifting the Link Weights in Networks

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***Abstract*— Transport in large networks follows near to shortest paths. A shortest path depends on the topology as well as on the link weight structure. While much effort has been devoted to understand the properties of the topology of large networks, the influence of link weights on the shortest path received considerably less attention. The scaling of all link weights in a graph by a positive number does not change the shortest path and most of the link weight distributions can be generated as a function of the uniform distribution. Hence, we compute analytically and by simulation the effect of shifting the uniform distribution for the link weights from $[0, 1]$ to $(a, 1]$ where $1 > a > 0$. The properties of the shortest path (hopcount and weight) vary for different a as well as the topology. Furthermore, when a is large, the traffic is more likely to follow the minimum hopcount shortest path, which leads to more balanced traffic traversing the network.**

I. Introduction

Routing in communication networks is based on shortest paths (or the best approximation) to obtain high efficiency of resources usage. We confine ourselves to additive and strict positive link weights (e.g. the delay, monetary cost, etc.) such that the shortest path is the minimizer of the sum of the link weights of any path between those two nodes. In this article, we concentrate on properties of shortest paths, in particular, the influence of the link weights on the shortest path.

Although link weights are obviously needed to compute a shortest path in a graph, in practice, little is known about the link weights. In fixed networks, link weights are usually chosen as part of an optimization process which is also termed as traffic engineering [1]. Here, we will not select the set of link weights to achieve the maximal traffic capacity and we will not infer link weights from the shortest path measurements [2]. Instead, we are interested in the combined modeling of the topology of the network and the link weights. We will first investigate how the link weight structure affects resulting routes.

Partial studies of effects of link weights on the shortest path in complex networks are found in [3] which characterize many biological, social and communication systems [4]. We investigated the influence of shifting the uniform distribution of link weights, because other link weight distributions can be generated as a function of the uniform distribution. The selection of the link weight structure is one of the key issues for network simulations, to which our work contributes.

In this paper, we will briefly review theory of the shortest path in Section II. The motivation to investigate the shifted uniform distribution in different classes of graphs is also explained. The shifted uniform distribution is defined in Section III. In the next Section IV and V, we show by simulation and by analytic computation how the characteristics of the shortest path change when the link weight distribution is shifted away from zero in random graph and square lattice. The results are summarized in Section VI.

II. The shortest path

In large networks, the link weights are hardly correlated and can be considered as independent to a good approximation. With uniformly distributed link weights, all links contribute to the sum, the weight of the shortest path and this case corresponds to weak disorder. Earlier in [12], it was shown that the Shortest Path Tree (SPT) in the complete graph with uniform (or exponential) link weights is precisely a Uniform Recursive Tree (URT). A URT is asymptotically the shortest path tree in the Erdös-Rényi random graph $G_p(N)$ (see e.g. [6]) with i.i.d. regular link weights and link density p above the disconnectivity threshold $p_c \sim \frac{\ln N}{N}$. The interest of the URT is that analytic modeling is possible (see e.g. [11, Part III]) and that it serves as a reasonable first order model to explain measurements in the Internet.

Since the shortest path is mainly sensitive to the smaller, non-negative link weights, the probability distribution of the link weights around zero will dominantly influence the properties of the resulting shortest path. Hence, if we add a constant to all link weights, the changes of the shortest path can be expected. Indeed, suppose that the shortest path contains many hops and the second shortest path only a few. In that case, there always exists a positive constant that, after added to all link weights, dethrones the initial shortest path.

Apart from being attractive in a theoretical analysis, the uniform distribution on $[0, 1]$ is the underlying distribution to generate an arbitrary other distribution and is especially interesting for computer simulations. Hence, this distribution appears most often in network simulations and deserves – for this reason alone perhaps – to be studied.

The understanding of the shortest path with independent, shifted uniformly distributed link weights will also give more insights into the stability of paths [18]. For instance, the changes in the shortest path due to the adding of constant noise to all link weights. The interest in understanding the stability of paths lies in the fact that it could direct efficient triggers for network updates.

We study the following complex network models: the Erdös-Rényi random graph $G_p(N)$, the square lattice and the scale-free graph. Traditionally, the complex networks have been modeled as Erdös-Rényi random graphs. Besides that, the

Erdös-Rényi random graphs are reasonably accurate models for peer-to-peer networks [13] and ad-hoc networks [7]. The square lattice, in which each node has four neighbors, is the basic model of a transport network as well as in percolation theory [10]. It is also frequently used to study the network traffic [15]. The scale-free graph [16] is proposed as model for complex networks that have a power-law degree distribution [5], such as the World Wide Web and the Internet.

III. Shifted uniformly distributed link weights

Any shifted uniformly distributed link weights w can be specified by

$$f_w(x) = \frac{1_{a_0 < x \le b}}{b - a_0} \tag{1}$$

The shifted link weight probability density function (1) can be considered as a result from adding a constant a_0 to a uniform link weight in $[0,1]$ when $b = 1 + a_0$.

The scaling of all link weights in the graph by a positive number does not change the shortest path. If (capital) W denotes the weight of the shortest path, the scaling of the link weights w by $\frac{1}{b}$, results in a weight $\frac{W}{b}$ of the shortest path with probability density function (*pdf*)

$$\begin{aligned} f_{\frac{W}{b}}(x) &= \frac{d}{dx}\Pr\left[\frac{W}{b} \le x\right] \\ &= \frac{d}{dy}\Pr[W \le y] \cdot \left.\frac{dy}{dx}\right|_{y=bx} = b f_W(bx) \end{aligned}$$

After scaling by $\frac{1}{b}$, the only specifier of the link weight is the parameter $a = a_0/b$ and (1) reduces to

$$f_w(x) = \frac{1_{a < x \le 1}}{1 - a}, \quad 0 \le a < 1 \tag{2}$$

IV. The shortest path in $G_p(N)$ with shifted uniformly distributed link weights

This Section is devoted to explain the curious behavior of the *pdf* of the weight and hopcount of the shortest path in the Erdös-Rényi random graphs $G_p(N)$ with shifted uniform link weights specified by (2). The main interest here lies in $a > 0$ because the case $a = 0$ is known in detail as mentioned in Section II.

A. The complete graph ($p = 1$)

Let us first confine to the complete graph K_N of which any other graph is a subgraph.

1) The case $\frac{1}{2} \le a < 1$: We use $w(P_{h=i})$ to denote the weight of a path with i hops. In the complete graph with link weights specified by (2) with $\frac{1}{2} \le a < 1$, the shortest path must be the direct link, because the weight $w(P_{h>1})$ of any path with $h > 1$ hops and the weight $w(P_{h=1})$ of the direct link between the source and destination nodes obey

$$\begin{aligned} w(P_{h>1}) &= \sum_{j=1}^{h} w(n_j \to n_{j+1}) \ge \sum_{j=1}^{2} w(n_j \to n_{j+1}) \\ &> 1 \ge w(P_{h=1}) \end{aligned}$$

Hence, the weight of the shortest path is uniformly distributed within $(a, 1]$.

The same idea can be applied to explain why all the pdfs of the weight of the shortest paths have certain uniformly distributed part when $0 < a < \frac{1}{2}$. The direct link w is always the shortest path with $w(P_{h=1}) = w$ provided $w \in (a, 2a]$. Thus, the probability density of the uniform part is $f_W(x) = \frac{1}{1-a}, x \in (a, 2a]$. There are two extreme cases. When $a = 0$, the uniformly distributed area becomes a point with value 1, which corresponds to the point $f_W(0) = 1$. When $a \ge \frac{1}{2}$, the *pdf* is uniformly distributed for $x \in (a, 1]$. Second, since the weight of the direct link $w(P_{h=1})$ is bounded by 1, the maximum possible number of hops in the shortest path P^* follows from $\min w(P_{h>1}) \le 1$ as $h < \left[\frac{1}{a}\right]$ where $[x]$ denotes the integer part of the real number x. Hence, if $\frac{1}{k+1} \le a < \frac{1}{k}$ for any integer $k \ge 1$, the shortest path has at most k hops.

2) The case $\frac{1}{3} \le a < \frac{1}{2}$: When the direct link weight lies in $(a, 2a]$, the weight of shortest path is uniformly distributed as explained above. When the direct link weight lies in $(2a, 1]$, the one hop path and the $N - 2$ two hops paths compete to become the shortest path P^*. Hence,

$$\begin{aligned} f_W(x) &= f_{W|w(P_{h=1}) \le 2a}(x) \Pr[w(P_{h=1}) \le 2a] \\ &\quad + f_{W|w(P_{h=1}) > 2a}(x) \Pr[w(P_{h=1}) > 2a] \\ &= \frac{1_{a < x \le 2a}}{1-a} + \frac{1-2a}{1-a} \cdot f_{W|w(P_{h=1})>2a}(x) \cdot 1_{2a < x \le 1} \end{aligned}$$

Paths between a node pair with one or two hops are independent, because they do not have links in common and link weights are assumed to be independent. Then we arrive at

Theorem 1: In the complete graph K_N equipped with link weights uniformly distributed within $(a, 1]$ and $\frac{1}{3} \le a < \frac{1}{2}$, the *pdf* of the weight of the shortest path is

$$\begin{aligned} f_W(x) &= \frac{1_{a<x\le 2a}}{1-a} + \frac{1_{2a<x\le 1}}{1-a}\left(1 - \frac{1}{2}\left(\frac{x-2a}{1-a}\right)^2\right)^{N-2} \\ &\quad + \frac{(N-2)(1-x)(x-2a)}{(1-a)^3} \\ &\quad \times \left(1 - \frac{1}{2}\left(\frac{x-2a}{1-a}\right)^2\right)^{N-3} \cdot 1_{2a<x\le 1} \end{aligned} \tag{3}$$

Proof: See [14]. □

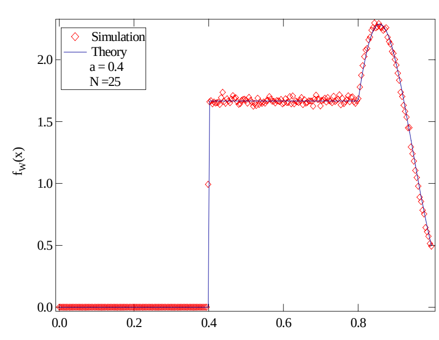

Fig. 1. The pdf of the weight of the shortest path in K_N with link weights specified by (2) with $a = 0.4$ both computed by (3) and simulated.

This analytic result (3) is verified by the simulation in Figure 1 for $a = 0.4$. We carry out 10^4 iterations, and find the shortest path between an arbitrary node pair in each generated $G_p(N)$ with the given link weight distribution.

3) The case $a < \frac{1}{3}$: When $a < \frac{1}{3}$, the same reasoning as above shows that the shortest path can have three or more hops. In general, paths with three or more hops can be overlapping, which prevent simple analytic derivations and necessitates a combinatorial approach as shown in [9].

B. The random graph ($p < 1$)

We will extend the previous analysis to the broader class of Erdös-Rényi random graphs $G_p(N)$.

1) The Case $\frac{1}{2} \leq a < 1$: The *pdf* of the weight of the shortest path consists of two parts: the uniform part when the direct link exists and the more complicated part when the direct link does not exist. For the second part, the *pdf* starts from $2a$, since $ah < w(P_h) \leq h$. By the law of total probability, we have

$$\begin{aligned} f_W(x) &= f_{W|P_{h=1}}(x) \Pr[P_{h=1} \text{ exists}] \\ &\quad + f_{W|P_{h>1}}(x) \Pr[P_{h=1} \text{ does not exist}] \\ &= \frac{p}{1-a} \cdot 1_{a<x\leq 1} + (1-p) f_{W|P_{h>1}}(x) \end{aligned} \tag{4}$$

The probability $\Pr[w(P^*_{h=2}) \leq 3a]$ that the shortest path with two hops is smaller than $3a$, the lower bound of the weight of a 3 hops path, can be derived as

$$\Pr[w(P^*_{h=2}) \leq 3a] = \begin{cases} 1 - (\frac{p^2}{2}(3 - \frac{1}{1-a})^2 + 1 - p^2)^{N-2} \\ \quad \text{for } \frac{1}{2} < a \leq \frac{2}{3} \\ 1 - (1-p^2)^{N-2} \\ \quad \text{for } \frac{2}{3} < a \leq 1 \end{cases}$$

$$\geq 1 - (1 - \frac{1}{2}p^2)^{N-2}$$

which increases as a and tends to 1 for N sufficiently large and $p \leq 1$. This justifies the approximation

$$f_{W|P_{h>1}}(x) \approx f_{w(P^*_{h=2})}(x) \tag{5}$$

where the possibility that the shortest path has more than 2 hops is neglected. The *pdf* of the weight of the shortest path in this case can be calculated [14] as

$$\begin{aligned} f_W(x) &\approx \frac{p}{1-a} \cdot 1_{a<x\leq 1} + p^2(1-p)(N-2) \cdot 1_{2a<x\leq 1+a} \\ &\quad \times \frac{x-2a}{(1-a)^2}\left(1 - 0.5p^2\left(\frac{x-2a}{1-a}\right)^2\right)^{N-3} \\ &\quad + p^2(1-p)(N-2)\frac{2-x}{(1-a)^2} \cdot 1_{1+a<x\leq 2} \\ &\quad \times \left(0.5p^2\left(\frac{2-x}{1-a}\right)^2 + 1 - p^2\right)^{N-3} \end{aligned} \tag{6}$$

The third part is very small when p and N are large enough and can be approximated by 0. The approximation in (5) is more precise for larger a. Therefore, we examine the worst case $a = 0.5$. When $a = 0.5$, (6) becomes

$$\begin{aligned} f_W(x) &\simeq 2p \cdot 1_{0.5<x\leq 1} + 4p^2(1-p)(N-2)(x-1) \\ &\quad \times (1 - 2p^2(x-1)^2)^{N-3} \cdot 1_{1<x\leq 1.5} \end{aligned} \tag{7}$$

Simulation in Figure 2 confirms the precision of (6).

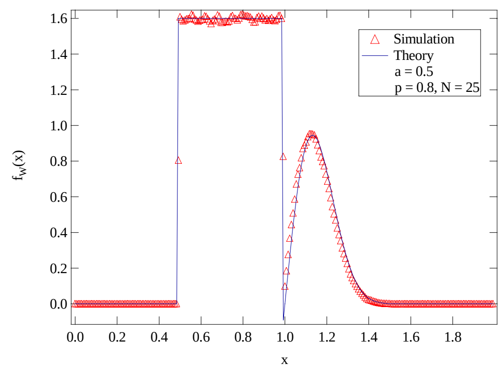

Fig. 2. The pdf of the weight of the shortest path with $a = 0.5$, $p = 0.8$ and $N = 25$ computed by (7) and by simulations.

2) The case $a < \frac{1}{2}$: Similar to the corresponding case $a < \frac{1}{3}$ for the complete graph, no simple analysis is expected for this case due to the dependence of paths that compete to be the shortest. Simulation results are shown in Figure 3.

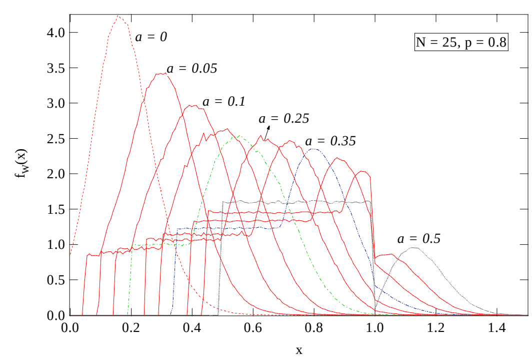

Fig. 3. The pdf of the weight of the shortest path in $G_{0.8}(25)$ with $a \leq 0.5$ (in steps of $\Delta a = 0.05$).

C. Summary

In the complete graph, we have shown that the case $a \geq \frac{1}{3}$ is analytically tractable. Earlier [8], the case for $a = 0$ has been computed analytically, which leaves the case $a \in \left(0, \frac{1}{3}\right)$ open as a problem that still requires an analytic solution.

In random graphs, if a is not too small, almost all shortest paths are shown to consist of a few hops which seems to agree with practice in multi-hop wireless networks. In these networks where the link weight represents the delay, the value of a is indeed bounded from below by (a) the propagation delay and (b) the minimum processor time to transmit an IP packet. On the other hand, interpretations of simulations that target e.g. to compare routing algorithms or protocols should take the quite small hopcount into account when a shifted uniform link weight distribution as (1) is used in small world networks.

V. The shortest path in a square lattice with shifted uniformly distributed link weights

The Erdös-Rényi random graphs $G_p(N)$ belong to the class of "small-world" graphs [17], where the average hopcount of the shortest path is usually small, with average on the order $O(\log N)$. In a lattice with N nodes, the hopcount of the shortest path is much larger, on average of the order $O(\sqrt{N})$. In this Section, we investigate the weight and the hopcount of the shortest path in a two-dimensional square lattice with shifted uniformly distributed link weight specified by (2). Two cases are studied: (a) the source and destination are positioned at the diagonal points and (b) they are randomly chosen among the N nodes in the lattice.

A. The source and destination are fixed at diagonal points

For the class of square lattices with N nodes, the minimum hopcount between the diagonal points is $h_{\min} = 2\sqrt{N} - 2$ and the number of paths with such minimum hopcount is $\binom{2x}{x}$, where $2x = h_{\min}$. Figure 4 shows the *pdf* $f_W(x)$ of the

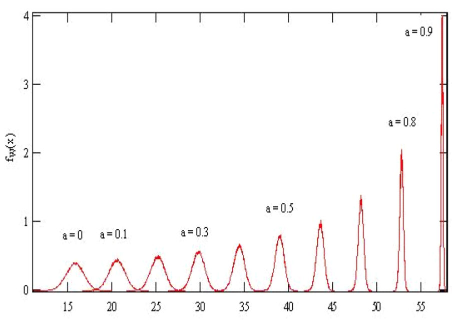

Fig. 4. The pdf of the weight of the shortest path in square lattice with $0 \leq a < 1$ (in steps of $\Delta a = 0.1$) and $N = 1024$.

weight of the shortest path for different values of $0 \leq a < 1$ in a square lattice with $N = 1024$ nodes. Each *pdf* with a specified a resembles a Gaussian which is characterized by its mean and standard deviation. This is in contrast to the random graph, where the *pdf* changes dramatically as a increases as shown previously in Section IV.As shown in Figure 5, both the

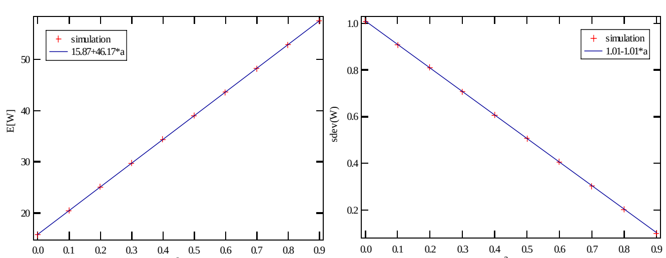

Fig. 5. The average and standard deviation of the weight of the shortest path in a square lattice with 1024 nodes and $0 \leq a < 1$ (in steps of $\Delta a = 0.1$).

average and standard deviation of the weight of the shortest path seem to be linear with a. When a is large, and exactly $a = 1$, the shortest path must have $h_{\min} = 2\sqrt{N} - 2 = 62$ hops. In this case, the average weight of the shortest path must be linear with a.

Assume that we have three graphs G_1, G_2 and G_3, which have the same topology, a square lattice. The links in G_1 are uniformly distributed within $(0, 1]$. The graph G_2 with uniform links distributed within $(b, 1+b]$ is obtained by adding a constant b to all links of G_1. After scaling all links in G_2 by $\frac{1}{1+b}$, we obtain G_3 which has the shifted uniformly distributed link weights specified by (2) with $a = \frac{b}{1+b}$. The shortest paths in G_2 and G_3 are the same, because the shortest path will not change when all the links are scaled. When a is large, the shortest path in G_2 and G_3 has hopcount $h_{\min}$. Moreover, it is equal to $P^*_{h_{\min}}$, the shortest path among paths with $h_{\min}$ hops in G_1. Hence, in G_3, which corresponds to the graph we simulated, the average weight of the shortest path obeys

$$\begin{aligned} E[W_3(P^*)] &= \frac{E[W_1(P^*_{h_{\min}})] + b * h_{\min}}{1+b} \\ &= (h_{\min} - E[W_1(P^*_{h_{\min}})]) * a + E[W_1(P^*_{h_{\min}})] \end{aligned} \quad (8)$$

where $E[W_1(P^*_{h_{\min}})]$ is the average weight of $P^*_{h_{\min}}$ in G_1. Our simulation results show that when $a \geq 0.5$, the shortest path always has $h_{\min}$ hops, which indicate that (8) only holds for $a \geq 0.5$. For any $0 \leq a < 1$,

$$E[W_3(P^*)] \geq \frac{E[W_1(P^*)] + b * h_{\min}}{1+b}$$

where the shortest path in G_3 with weight $W_3(P^*)$ may be different from the shortest path in the corresponding G_1 with weight $W_1(P^*)$. The reasons, why in Figure 5, the average weight seems always linear with a, are:

- In G_1 where $a = 0$, the average weight of the shortest path $E[W_1(P^*)]$ is very close to $E[W_1(P^*_{h_{\min}})]$. Curve fitting of the $E[W_3(P^*)]$ with $a \geq 0.5$ indicates that $E[W_1(P^*_{h_{\min}})] = 15.93$ while simulation results show that $E[W_1(P^*)] = 15.77$.
- The hopcount of the shortest path in G_1 $E[H_1(P^*)] = 64.2$ is very close to $h_{\min} = 62$.

Similarly, when a is large, the variance of $W_1(P^*_{h_{\min}})$ in G_1 is equal to the variance of $W_2(P^*)$ in G_2. However, the variance in G_3 is $Var[W_3(P^*)] = (\frac{1}{1+b})^2 \cdot Var[W_1(P^*_{h_{\min}})] = (1-a)^2 \cdot Var[W_1(P^*_{h_{\min}})]$. Hence, the standard deviation is

$$\sigma[W_3(P^*)] = -a * \sigma[W_1(P^*_{h_{\min}})] + \sigma[W_1(P^*_{h_{\min}})]$$

Since the $W_1(P^*_{h_{\min}})$ is close to $W_1(P^*)$ in G_1, the standard deviation $\sigma[W_3(P^*)]$ of the weight of the shortest path in G_3 is almost linear with a.

B. The source and destination are chosen randomly

The analysis can be extended to a more general case, where the source and destination nodes are randomly chosen. We show by simulation again the two points: in G_1 or when $a = 0$, the hopcount of the shortest path is very close to $h_{\min}$ and the weight of the shortest path $W(P^*)$ is very close to $W(P^*_{h_{\min}})$.

As shown in Figure 6, with 10^6 iterations simulation, the hopcount of the shortest path is very close to the minimum

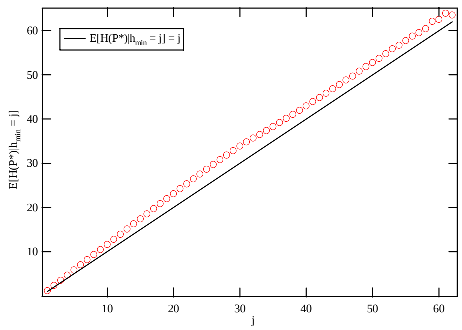

Fig. 6. The average hopcount of the shortest path in a square lattice with $N = 1024$ and $a = 0$, given the minimum hopcount is j.

hopcount $H(P^*) \approx h_{\min}$. The shortest path subject to a given hopcount is more complex to calculate than the unconstrained shortest path problem. In fact, that problem is NP-complete. We observe that, when with a large enough, all shortest paths follow the shortest minimum hopcount path. Hence, we have $W_1(P^*_{h_{\min}}) = W_3(P^*) \cdot (1 + b) - h_{\min} \cdot b$,where $a = \frac{b}{1+b}$. The problem of calculating the shortest minimal hopcount path can then be reduced to calculating the shortest path in the corresponding graph G_3.

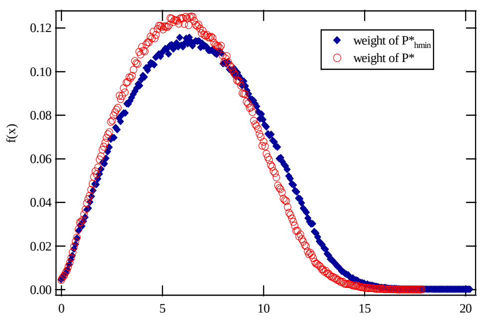

Fig. 7. The weight of the shortest path $W(P^*)$ and of the shortest minimum hopcount path $W(P^*_{h\,\min})$ in a square lattice with $N = 1024$ and $a = 0$.

The *pdf* of weight of the shortest minimum hopcount path $W_1(P^*_{h_{\min}})$ and the weight of the shortest path $W_1(P^*)$ are shown in Figure 7 to be close with average $E[W_1(P^*_{h_{\min}})] = 6.77$ and $E[W_1(P^*_h)] = 6.31$.

In summary, after adding a small constant to all links in a square lattice, as the definition of G_1 and G_2 in Section V-A, the routing in the lattice is more stable than that in the random graph. The constant link weight b added may be caused by e.g. reserving certain resources of the network or by the delay due to a traffic jam. The difference between the weight of the updated shortest path and weight of the original shortest path can be upper bounded by $(H(P^*) - h_{\min}) * b$ while $H(P^*) \approx h_{\min}$. Similarly, the traffic can be routed along the shortest minimal hopcount path. Then the difference between its weight and the weight of the updated shortest path is at most $W_1(P^*_h) - W_1(P^*_{h_{\min}})$, which is small and decreases to zero when b is large.

VI. Conclusion

We have analyzed the effect of shifting the uniform distribution for the link weights from $[0, 1]$ to $(a, 1]$ where $a > 0$. By choosing a larger value of the link weight parameter a, the shortest path is more probable to have a smaller hopcount and the network resources are used more efficiently with balanced traffic traversing the network. In the Erdös-Rényi random graph, the case that $a > 0$ causes the properties of the shortest path (hopcount and weight) to be dramatically different than for a small $(a \to 0)$. However, the shortest paths in the square lattice are more stable in contrast to the small-world graphs. The intuition is that, in respective of the link weights, if $h_{\min}$ is large, the $i.i.d.$ link weights only seem a small perturbation of the $w = 1$ case. As a final remark, the scale-free networks are tree-like sparse graphs. There are few paths between the source and destination nodes [18]. Hence, the scale-free networks are expected to be stable when link weights are shifted.

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