Performance of Cell Loss Priority Management Schemes in Shared Buffers with Poisson Arrivals

PIET VAN MIEGHEM, JOHAN DAVID, GUIDO H. PETIT
Alcatel Corporate Research Center, Francis Wellesplein 1, B-2018 Antwerp - Belgium
mieghemp@rc.bel.alcatel.be

Abstract. The performances of two important cell loss priority mechanisms (CLPM) in shared buffers with N servers, Partial Buffer Sharing (N-PBS) and Push-Out Buffer (N-POB), are compared. Each arriving flow i is guided to a shared queue of size K where it is served by the corresponding i-th server. Further, these flows are Poissonean, independent of each other and consist of a uniformly distributed mix of high (H) and low (L) priorities. Subject to the required cell loss ratios for both priorities, clr_L^* and clr_H^*, we determine the maximum allowable traffic intensity λ_{max} for each incoming traffic flow as a function of the global priority mix in the buffer of size K. For an arbitrary CLPM, the maximum allowable load is achieved in the symmetrical traffic situation where all N links carry the same load. Only when $N > 1$, the N-POB scheme is found to be superior in performance to *N-PBS* over the whole priority mix range and for realistic cell loss ratio requirements. Moreover, the difference in performance between N-POB and N-PBS increases with increasing N, but decreases with increasing K. Finally, a methodology based on buffer size scaling has enabled the computation of the performance of the N-POB.

1. INTRODUCTION

Literature abounds in suggestions to tackle the connection admission control (CAC) problem in ATM switches. A smaller number of articles concentrates on priority management (1).

The majority of these discuss a particular priority scheme and then proceed to evaluate the performance of the priority algorithm in a single buffer [2, 5, 12, 13, 18, 19, 21, 22, 23, 28, 30, 33] or in a shared buffer [6, 16, 17, 20, 24, 31]. Generally one finds that the introduction of priorities enhances the number of customers that can be served adequately at the expense of increased complexity of the control algorithm. However, relatively few (e.g. [19, 22]) succeed in determining or proposing a concrete CAC-algorithm that is optimal given a certain priority scheme. Of course, there are different aims for optimization. Recently, Cidon *et al.* [7, 8] have presented a study on optimal buffer sharing trying to minimize the cell loss ratio of high priority traffic. The main purpose of this work is to optimize the overall traffic load in shared buffers. Most ATM switching fabrics [9, 32] possess *shared* buffers for different links because of efficiency reasons as demonstrated elsewhere [34]. As the performance of shared buffers can be further boosted by priority management schemes, it is worthwhile to investigate the combination of a shared buffer and a cell loss priority management scheme to achieve overall traffic load maximization.

Among buffer CLPM protocols [18, 19, 22], the Push-Out Buffer (POB) and the Partial Buffer Sharing (PBS) are the most well-known. In a POB, the push-out mechanism acts only if the buffer is completely filled and a high priority cell arrives. If there are low priority cells in the buffer, the arriving high priority cell pushes the low priority cell nearest (2) to the server out, all cells behind the pushed-out low priority cell ripple through over one position towards the server, and the arriving high priority cell takes its place at the tail of the queue in order to preserve cell sequence integrity. A PBS mechanism is somewhat simpler: if the buffer occupancy is below a threshold T, both low and high priority cells are allowed to enter, otherwise only high priority cells are accepted until complete buffer occupation.

In this article, we focus on a shared buffer with N incoming Poisson flows as drawn in Fig. 1. Each Poisson flow i has a priority mix α_i =Prob [cell of flow i has high priority H] and a traffic intensity λ_i independent of flow j. The second assumption is further coined as *inter-link* or *inter-flow* independence. All servers are deterministic

(1) For a more detailed literature overview, we refer to [25, 26].

(2) This push-out mode is coined FIFO push-out. Other types are LIFO and Random push-out where the first and an arbitrary low priority cell are pushed out respectively. Of these types, the FIFO POB has the best performance. The types are discussed in [25, 26]. In the sequel, the results computed are those of the Random push-out (R-POB) discipline because the performance of R-POB approximates the FIFO-POB very well but has for a single server ($N = 1$) a state space that is only quadratic in K, whereas that of the FIFO-POB is exponential in K [25, 26].

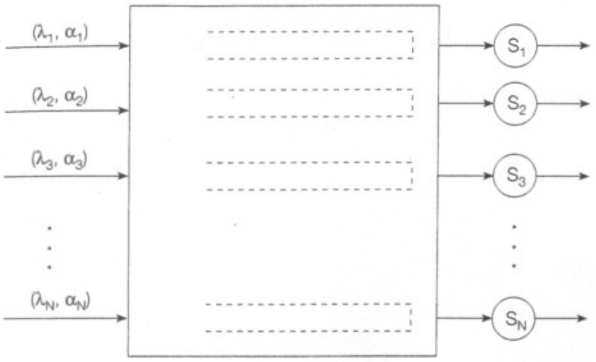

Fig. 1 - A sketch of the shared buffer of size K. The dashed open boxes represent the logical output queues for the different N links.

with service time equal to one time slot. The priority mechanism operates on the shared buffer as a whole and is unable to distinguish between cells of different flows. But, since the i-th server only processes cells of flow i, the present shared buffer system is not work conservative [22] because it violates the basic property that *no servers should be idle as long as the buffer is not empty*.

The performance of N-POB and N-PBS (N refers to the number of servers) is compared in a discrete-time analysis. The performance measure is the maximum allowable load *per link* $\lambda_{\max}$ in the shared buffer (sha) subject to the couple (3) ($clr^*_{\text{sha};L}$, $clr^*_{\text{sha};H}$) of cell loss ratio requirements for low and high priorities in the shared buffer. Hence, the analysis has been limited to the cell loss ratio as quality of service (QOS) measure and delay requirements are not considered.

In current standards, the QoS commitments on the Cell Loss Ratio for high and low priority cells is ATM Transfer Capability (ATC) dependent (cfr. ITU-T Rec. I.371, Geneva, May 1996). For the Deterministic Bit Rate ATC, a CLR commitment needs to be specified regardless of the CLP bit, whereas for the Statistical Bit Rate ATC (configurations 2 and 3), the QoS commitment on the cell loss ratio of the CLP = 1 cell flow (low priorities) is unspecified. Nevertheless, in the latter case, it could be a network operator policy to provide a QoS commitment for the CLP = 1 cell flow in order to differentiate his service from other network operators. This motivates the practical importance of the underlying study with two cell loss ratio requirements.

The restriction to Poisson flows entails that the arriving flows consist of many separate connections, none of them significantly bursty nor dominant in bandwidth. Although recent work [1, 3, 11, 14, 15] has shown that broadband traffic is self-similar in nature, the use of the Poisson process is motivated by several arguments. In order to handle arbitrary traffic profiles, many ATM switches incorporate spacers or shapers to smooth the traffic and to maximize link load efficiency. In these cases, Poisson's law is justified for engineering purposes. Further, the complexity and number of parameters of the queueing analysis with priority schemes in shared buffers virtually rule out arrival processes other than GI-processes if we are to end up with tractable and hence numerical and interpretable results. Finally, our work on priority mechanisms in single queues [25, 26] reveals that Markov Modulated arrival processes exhibit a performance curve for POB and PBS ($\lambda_{\max}$ versus α) similar in shape to that of a Poisson process.

The outline is as follows. First, general considerations about CLPM are discussed. Then, the maximum allowable link load in a shared buffer under the POB and PBS priority management schemes in a symmetrical traffic situation is compared as a function of N and K. The proofs of important results and the state equations for a shared buffer with PBS are given in the appendix.

(3) In the sequel, a^* denotes that a is given or fixed.

2. General results for CLPM

2.1. *Framework*

By virtue of the slotted nature of ATM, we concentrate on discrete-time systems where the servers work deterministically. The time unit, further called a time slot, equals the time needed to serve precisely one cell. If μ_i denotes the fraction of served i priorities per time slot, we have

$$\mu_A = \mu_H + \mu_L = 1 \tag{1}$$

where the subscripts refer to the aggregate (A), the low priority cells (L) and the high priority cells (H) respectively.

If α denotes the probability that an arriving cell has high priority, the mean number of arrivals per time slot equals

$$\lambda_A = \lambda_H + \lambda_L \tag{2}$$

where $\lambda_H = \alpha\, \lambda_A$ and $\lambda_L = (1-\alpha)\, \lambda_A$. Defining the traffic intensity as usual by $\rho = \lambda/\mu$, we observe that for a deterministic server holds that $\lambda_A = \rho_A$.

Since the system has a finite capacity of K queueing positions with an additional one for the server, in general cell loss will probably occur. We denote the cell loss ratio clr as the mean number of cells lost per time slot over the mean number of arrived cells of that type. Again the total number of lost cells consists of both priorities. From this fact we deduce a useful relation (4),

$$\begin{aligned} \lambda_A\, clr_A &= \lambda_L\, clr_L + \lambda_H\, clr_H \\ clr_A(\alpha) &= (1-\alpha)\, clr_L(\alpha) + \alpha\, clr_H(\alpha) \qquad \left(\alpha = \frac{\lambda_H}{\lambda_A}\right) \end{aligned} \tag{3}$$

(4) An alternative relation of the same nature is
$\lambda_A(1 - clr_A) = (1 - q[0])\, \mu_A$
where $q[0]$ is the probability that the buffer is empty.

The last relation, known as *the conservation law*, explicitly expresses the dependence on α. In addition, since we can write the aggregate cell loss ratio as a weighted mean, $clr_A = (\lambda_L\, clr_L + \lambda_H\, clr_H)/(\lambda_L + \lambda_H)$, we immediately find that $clr_H(\alpha) \le clr_A(\alpha) \le clr_L(\alpha)$ assuming that $clr_H(\alpha) \le clr_L(\alpha)$.

The cell loss ratio of the aggregate cell stream, $\widehat{clr}_A$, in the corresponding system without priority management is exactly described by the loss probability of that corresponding $G/D/1/K$ system. Formally, fixing all other traffic descriptors independent of the load ρ_A, we have

$$\widehat{clr}_A = f_K(\hat{\rho}_A) \tag{4}$$

where $f_K(x)$ is an increasing, continuous and positive function of x bounded by $0 \le f_K(x) \le 1$ and non-increasing in K. A priority mechanism can never lower the aggregate cell loss, hence we have

$$\widehat{clr}_A \le clr_A(\alpha) \tag{5}$$

and alternatively, for a same aggregate cell loss ratio requirement $\widehat{clr}_A = clr_A(\alpha) = clr_A^*$

$$\hat{\rho}_A \ge \rho(\alpha) \tag{6}$$

2.2. *Formal solution*

We are now in a position to treat the problem in more detail: *Given a priority management protocol, determine the maximal traffic intensity ρ_A subjected to the user's cell loss ratio requirements (clr_L^*, clr_H^*) such that $clr_L(\alpha) \le clr_L^*$ and $clr_H(\alpha) \le clr_H^* < clr_L^*$.* The latter inequality means that clr_H^* should be sufficiently smaller than clr_L^* in order for the priority scheme to have impact.

Since $f_K(x)$ is monotonously increasing, the inverse function exists which justifies to rewrite (4) as $\hat{\rho}_A = f_K^{-1}(\widehat{clr}_A)$. Further, the inverse function $g^{-1}(x)$ of an increasing function $g(x)$ is increasing. Using (6), we have $\rho(\alpha) \le f_K^{-1}(clr_A^*)$. Hence, the maximum allowable load $\rho_{\max}(\alpha)$ is found where $clr_A(\alpha)$ is maximal. Specifically, from (3) and the requirements on the cell loss ratios, we have

$$clr_A(\alpha) \le (1-\alpha)\, clr_L^* + \alpha\, clr_H^* \tag{7}$$

offering an upper bound for the maximal allowable load

$$\rho_{\max}(\alpha) \le f_K^{-1}\left[(1-\alpha)\, clr_L^* + \alpha\, clr_H^*\right] \tag{8}$$

Since the right hand side of (7) is decreasing in α due to the fact that $clr_H^* < clr_L^*$, so is (8). The upper bound (8) does not depend on the management protocol and indicates that for every value of $\alpha \in [0, 1]$ both requirements, $clr_L(\alpha) = clr_L^*$ and $clr_H(\alpha) = clr_H^* < clr_L^*$ are met.

2.3. *Shared buffers*

A flow (or link) dependent CLPM scheme leads to a multi-dimensional, non-linear optimization problem of an overwhelming complexity. In the present shared buffer ([5]) (Fig.1), we confine ourselves to a CLPM mechanism acting upon the shared buffer as a whole without distinguishing between the different input flows. This implies that the individual sets $\{clr_{Hi}^*, clr_{Li}^*\}$ are replaced by one collective cell loss ratio requirement ($clr_{\text{sha};H}^*$, $clr_{\text{sha};L}^*$) where we have chosen $clr_{\text{sha};H}^* = \min(clr_{Hi}^*)$ and $clr_{\text{sha};L}^* = \min(clr_{\text{sha};Li}^*)$. The aggregate priority mix in the shared buffer is $\alpha = 1/\Lambda \sum_{i=1}^{N} \alpha_i\, \lambda_i$ where $\Lambda = \sum_{i=1}^{N} \lambda_i$ is the total offered load. The purpose is to maximize the total throughput Λ over all N servers. For a single buffer, the *clr* requirement uniquely determines the maximum overall throughput, but in a shared buffer, it is perfectly possible that the clr requirement can be satisfied in a number of different ways. Among these, we choose the one that maximizes the total offered load $\Lambda = \sum_{i=1}^{N} \lambda_i$.

The basic symmetry property, *the equal balancing principle*, proved in Appendix A.2 reads.

Property 1 *In a shared buffer with inter-flow independent arrivals and a flow independent CLPM scheme, the aggregate load Λ, subject to the constraints that $clr_{\text{sha};H} \le clr_{\text{sha};H}^*$ and $clr_{\text{sha};L} \le clr_{\text{sha};L}^*$, is maximal in case of equal individual loads $\lambda_i = \lambda$ AND equal priority mixes $\alpha_i = \alpha$. This maximum is unique.*

Suppose that $N-1$ input flows are sending traffic each with priority mix $\alpha_i = \alpha^*$ and at the same maximum allowable load $\lambda_i = \lambda_{\max}(\alpha^*) = \lambda^*$ obtained when all N links carry symmetrical traffic subject to the same set of clr requirements ($clr_{\text{sha};L}^*$, $clr_{\text{sha};H}^*$). How the maximum allowable load λ_N on the N-th link varies as a function of α_N is estimated by the following property.

Property 2 *Since the symmetrical traffic situation corresponds to a global maximum (property 1), we have*

$$\alpha = \frac{\alpha_N\, \lambda_N + (N-1)\alpha^*\, \lambda^*}{\lambda_N + (N-1)\, \lambda^*} \tag{9}$$

$$N\, \lambda_{\max}(\alpha) \ge \lambda_N + (N-1)\, \lambda^* \tag{10}$$

from which an upperbound for λ_N is found as

$$\lambda_N \le N\, \lambda_{\max}\left[\frac{\alpha_N\, \lambda_N + (N-1)\alpha^*\, \lambda^*}{\lambda_N + (N-1)\lambda^*}\right] - (N-1)\lambda^* \tag{11}$$

This upper bound only requires knowledge of the symmetrical traffic situation through $\lambda_{\max}(\alpha)$. An investigation of this upper bound has led to the proposal of a safe CAC-rule covering the whole α-range: *the maximum allowable load λ_N should be the minimum of (11) and $\lambda_{\max}(\alpha)$.*

([5]) In absense of a priority mechanism, for each independent Poisson flow arriving at the shared buffer holds that $clr_i = clr_{\text{sha}}$, as demonstrated in Appendix A.1, property 3.

In view of its importance, we will further confine ourselves to the symmetrical traffic situation. Indeed, from the viewpoint of overall optimality or maximum aggregate throughput, a network operator should connect the links to switches to achieve a traffic loading as symmetrical as possible and even in the case of slight deviations from this symmetry, property 2 demonstrates how to handle this asymmetrical case. Highly asymmetrical cases occur due to inefficient network design or in inadequate load balancing strategies. Furthermore, since asymmetrical cases are characterized by a larger set of parameters than the symmetrical case, the development of a simple, efficient and real-time CAC is believed to be quite unfeasible.

3. Computation of the performances

3.1. *The single server POB*

For small α, the aggregate cell loss ratio will be mainly determined by $clr_L(\alpha)$ since there are hardly any high priority cells. Moreover, since generally $clr_H^* << clr_L^*$, we have from the conservation law (3) approximately that $clr_A(\alpha) \approx clr_L^*(1-\alpha)$. Invoking (8), we conclude that the maximal allowable load is dominated by the clr_L^* requirement. In this region, the cell loss ratio requirement for the low priority cell is precisely met ($clr_L(\alpha) = clr_L^*$), while for the high priority cells $clr_H(\alpha) < clr_H^*$.

Increasing α or the average number of high priority cells causes $clr_H(\alpha)$ to increase until $clr_H(\alpha) = clr_H^*$. At this point, denoted as α_k, both cell loss ratio requirements are precisely met (and this point is unique as follows by a continuity argument).

The situation is more complex for high values of α. From the definition of the priority mix α and the fact that the traffic intensity for the aggregate $\rho_A = \lambda_A$ since we have a deterministic server, the following inequality holds

$$\rho_A(\alpha) = \frac{\lambda_H(\alpha)}{\alpha} \le \frac{\lambda_H(1)}{\alpha} = \frac{\rho_A(1)}{\alpha} \tag{12}$$

because $\lambda_H(\alpha)$ is increasing in α. Invoking the characteristic property of a deterministic server (1), we can write

$$\rho_A(\alpha) = \frac{\rho_H(\alpha)\mu_H(\alpha)}{\alpha} = \frac{\rho_H(\alpha)}{\alpha}\left[1-\mu_L(\alpha)\right] \tag{13}$$

For sufficienly high α, $\lambda_{max}(\alpha)$ follows from (13). The problem is how to determine the service rate $\mu_L(\alpha)$ for the low priority cells. For values of α just exceeding α_k, the load will be limited by the high priority requirement such that $clr_H(\alpha) = clr_H^*$ while $clr_L(\alpha) < clr_L^*$. However, since $clr_H^* << clr_L^*$, we find that $clr_L(\alpha)$ still dominates the aggregate cell loss ratio $clr_A(\alpha)$. When $\alpha > \alpha_k$, the loss in low priority cells will be substantially due to the push-out mechanism leading to $clr_L(\alpha) \approx clr_{Lpo}(\alpha)$. The calculation of the push-out probability is exceedingly complicated and we believe it is only possible through solving the transition probability matrix.

We have investigated two types of POB: a conventional FIFO POB (as studied by Kröner *et al.* in continuous time [19]) and a R POB [25, 26]. The delimiter refers to the service discipline. Thus, R (random) means that all cells available have equal probability to be served as opposed to FIFO where invariably the cell in the position nearest to the server (or with longest waiting time) is removed from the queue. Clearly, the *R* POB does not obey the sequence integrity. However, as the cell loss ratio only weakly depends on the sequence order, the maximum allowable load of the R POB is expected to closely approach that of the FIFO POB, provided the cell loss ratio requirements are sufficiently stringent ($clr^* < 0.1$). Indeed, for both POB types the comparison in the maximum allowable load $\lambda_{max}(\alpha)$ versus α shows [25, 26] that both priority management systems exhibit very similar performances for λ_{max}.

3.2. *A fitting formula for the single server R POB*

Since $\lambda_{max}(\alpha)$ of a R POB in the $[0, \alpha_k]$ interval is sufficiently close approximated by (8), our objective is to find an estimate in $[\alpha_k, 1]$ accurate to within 1%.

Suppose for the moment that the value of α_k is known. We found that the data of the maximum allowable load determined via iterating on the matrix solution of the R POB to obey the cell loss ratio requirement, is well fitted by

$$\lambda_{max}(\alpha) = p_1 + \frac{p_2}{(\alpha+p)^2} \tag{14}$$

Introducing the additional information

$$\lambda_{max}(1) = f_K^{-1}(clr_H^*)$$

$$\lambda_{max}(\alpha_k) = f_K^{-1}\left[(1-\alpha_k)\, clr_L^* + \alpha_k\, clr_H^*\right]$$

the eq. (14) can be specified as

$$\lambda_{max}(\alpha) = \frac{1}{P}\left\{\lambda_{max}(1)\left[\frac{1}{(\alpha+p)^2} - \frac{1}{(\alpha_k+p)^2}\right] + \lambda_{max}(\alpha_k)\left[\frac{1}{(1+p)^2} - \frac{1}{(\alpha+p)^2}\right]\right\} \tag{15}$$

where $P = 1/(1+p)^2 - 1/(\alpha_k+p)^2$. An elegant approximation for $f_K^{-1}(x)$ in a discrete-time $M/D/1/K$ is given in [34].

The proposed fit (15) is a kind of weighted mean between $\alpha = \alpha_k$ and $\alpha = 1$ with weight function $(\alpha+p)^{-2}$. Apart from α_k, the only unknown is p for which we found $0.5 \le p \le 1$. The result is not very sensitive to variations in p (in contrast to α_k) when aiming at an accuracy of 1%. The remainder is therefore devoted to the study of α_k.

For a fixed ratio $\beta = clr_H^*/clr_L^*$ but variable K, we observed that $\log \alpha_k = A/K + B$. On the other hand, for a fixed buffer size K, we found that $\log \alpha_k$ is linear in $\log\beta$ for both high and low asymptotic values. In practical applications, β is often smaller than 10^{-3} and the low asymptotic regime is adequate for use. After rather extensive fitting this regime can properly (to about 1% accuracy) be modelled as

$$\alpha_k \approx 10^{-\frac{3}{2K}} \left(clr_L^*\right)^{\frac{1}{4K}} \beta^{\frac{1}{K}} \tag{16}$$

3.3. *The N-POB*

The performance of the N-POB is computed via simulation since the state space rapidly becomes intractably large, even for the random push-out discipline which has the smallest state space of the considered push-out strategies [25, 26]. The number of states in the N-POB system with random push-out equals $\binom{K+2N}{2N}$, e.g. if $N = 4$ and $K = 32$, the number of states is 61 523 748. Just as for the single POB, the N-POB curves consist of two regions with opposite curvature joining at the point α_k (Fig. 2). In the lower-region $[0, \alpha_k]$ the low priority clr requirement is dominating: all $\alpha \in [0, \alpha_k]$ have precisely $clr_L(\alpha) = clr_L^*$ while $clr_H(\alpha) < clr_H^*$. In the upper-region $[\alpha_k, 1]$ the converse holds: all $\alpha \in [\alpha_k, 1]$ have precisely $clr_H(\alpha) = clr_H^*$ while $clr_L(\alpha) < clr_L^*$.

Fortunately, the results (Fig. 2) illustrate that the maximum allowable load λ_{max} behaves similarly for various N. This similarity is not too unexpected. The global arriving process is a sum of Poisson arrivals and, hence, also Poissonean. Further, the priority mechanism operates on the buffer as a whole. These aspects lead to an operation equal to that in a single server. Thus, the only difference with a single server are the multiple servers. But, since the traffic situation is symmetrical so that each server behaves, on average, identically, we cannot expect large discrepancies in the priority mechanism either.

Since the single server case $N = 1$ can be computed precisely [19, 25, 26], the observation implies that the multi-server case can be derived from the single server case. Indeed, the two extreme values of α correspond to either completely low or high priority traffic for which the maximum allowable load can be calculated [10, 29, 34] and hence, the downwards shift $\Delta(\alpha)$ from the single server POB-curve to the N-POB for $\alpha = 0$ and $\alpha = 1$ is known. Assuming a linear interpolation for all $\alpha \in [0, 1]$ based on the two known extremes, $\Delta(\alpha) = \alpha\,\Delta(1) + (1-\alpha)\,\Delta(0)$, this yields the approximate performance of the N-POB. This technique is applied in section 4.3 for large K and more stringent cell loss ratio requirements than shown in Fig. 2.

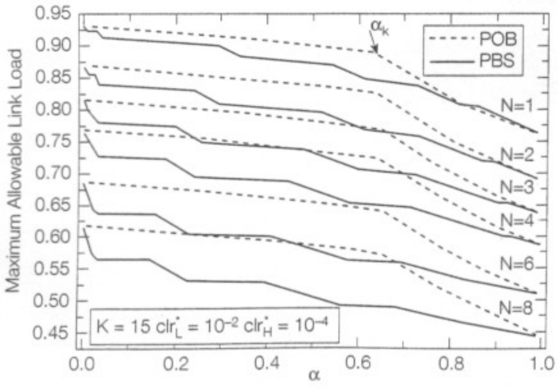

Fig. 2 - A comparison of the performance between the POB and PBS (with optimized threshold) for different N = 1,2,3,4,6,8. Each curve is computed for 100 values of α. The buffer size is $K = 15$ and the clr requirements are $clr_L^* = 10^{-2}$ and $clr_H^* = 10^{-4}$.

3.4. *The N-PBS*

3.4.1. Exact method

The steady state probabilities of the N-PBS are computed by solving the linear system $\pi = \pi M$, where π is the vector with the state probabilities and M the matrix with the transition probabilities (Appendix B.1). The number of states needed to completely describe the N-PBS system of buffer size K is $\binom{K+N}{N}$. Although this number is smaller than the number of states for the corresponding N-POB system $\binom{K+2N}{2N}$, it quickly grows too large for the system to be solved in a reasonable time, e.g. for $K = 15$ and $N = 4$, the number of states is 3876.

Fortunately, the number of states can be reduced considerably for a symmetrical traffic situation where $\lambda_i = \lambda$ and $\alpha_i = \alpha$ for all links i. States with the same probability can be grouped together into one state (6). The number of states to describe the system then can be reduced to

$$\sum_{r_1=0}^{\lfloor K/N \rfloor} \; \sum_{r_2=r_1}^{\lfloor K/(N-1) \rfloor - \sigma_2} \cdots \sum_{r_{N-1}=r_{N-2}}^{\lfloor K/2 \rfloor - \sigma_{N-1}} \; \sum_{r_N=r_{N-2}}^{K-\sigma_N} 1$$

where

$$\sigma_i = \sum_{k=1}^{i-1} r_k, \quad 2 \le i \le N$$

E.g. if $K = 15$ and $N = 4$ the number of states is only 295. This is a substantial saving compared to the general solution. The formulas for the elements of the transition matrix M can be found in Appendix B.2. A similar method was already used by Monterosso and Pattavina in [27] to study interconnection networks. They considered geometrically distributed cell arrivals and conditional serving of the cells in the buffer.

3.4.2. Approximate methods

As demonstrated above, the number of states in the exact system description increases so dramatically that it is useful only for small size (both K and N) systems. For more realistic values of K and N, several authors [4, 24, 27] have proposed approximate methods with a reduced number of states for problems similar to the N-PBS. All these methods assume symmetric traffic. We have used the bidimensional method proposed by

(6) E.g. for a buffer shared by two links with equal load and priority mix, the probability that there is one cell in the buffer for link 1 and two cells for link 2, is equal to the probability that there are two cells in the buffer for link 1 and one cell for link 2.

Bianchi and Turner in [4]. Each state corresponds to a number of cells in the buffer and a number of active servers. The formulas used to construct the transition matrix M for Poisson arrivals and deterministic servers have been derived in Appendix B.3.

The number of states for this approximate method is

$$1+\frac{(N-1)N}{2}+(K-N)N$$

For $K = 15$ and $N = 4$ we merely have 51 states. This is a considerable reduction compared to the general method. But this approximate method, too, possesses a non-structured matrix preventing the use of dedicated, fast algebraic solution algorithms. For $N = 1$ the approximate method is exact and for $N > 1$ it provides quite a tight upper bound for the cell loss ratio. In Fig. 3 the exact maximum allowable loads are compared with those computed with the approximate method. The agreement is good and, moreover, the approximate method leads to safe-side values for the maximum allowable load due to the overestimation of the cell loss ratios.

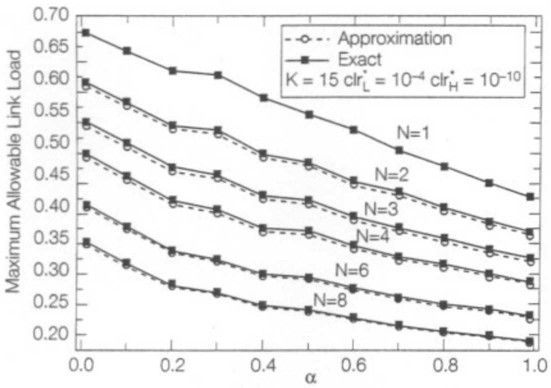

Fig. 3 - A comparison of the exact performance and the bidimensional approximation for N-PBS with different links N with optimized threshold. The buffer size is $K = 15$ and the clr requirements are $clr_L^* = 10^{-4}$ and $clr_H^* = 10^{-10}$.

3.4.3. The optimized threshold

Compared to POB, a PBS-scheme has an additional parameter, the threshold T, which needs to be optimized to render the maximum allowable load.

The maximum allowable load is plotted in Fig. 4 for $N = 1$ link, a buffer size $K = 15$ and fixed positions of the threshold. Similarly to the POB, two different regimes are observed: the flat decreasing, concave curves represent regions where the low priority cell loss ratio requirement clr_L^* is determinant, whereas the hyperbole-like curves correspond to a region dominated by the high priority cell loss ratio requirement clr_H^*. The desired maximum allowable load is the maximum envelope of all these curves and is clearly a concatenation of regions alternatingly dominated by the high and low priority cell loss ratio requirement. Due to the discrete settings of T, the maximum allowable load under PBS

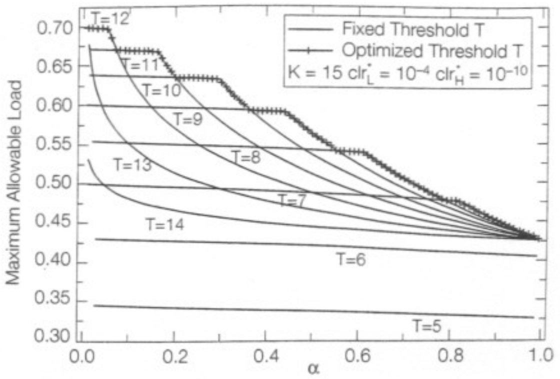

Fig. 4 - The influence of the threshold T on the maximum allowable load in a 1-PBS for $K = 15$, $N = 1$ and the clr requirements (10^{-4}, 10^{-10}).

features a stepwise shape. The longer the buffer size K, the more integer values of T are available, resulting in a smoother maximum allowable curve.

4. Partial buffer sharing versus push-out buffer

In this section, we compare the maximum allowable load *per link* λ versus priority mix α for both N-PBS and N-POB systems.

4.1. *Varying the number of links N*

Fig. 2 illustrates some noteworthy aspects. First, for high values of α and low N, the performance of N-PBS is slightly better than that of a N-POB. This is more pronounced in larger buffers [25, 26]. Second, for increasing N, the performance of N-PBS tends to deteriorate with respect to the N-POB. Finally, we observe that the shape of the N-PBS changes with N in that the "wobbles" shift to lower α_s when N increases. This effect is averse since it excludes that the multi-server PBS can be derived from the (much simpler) single server one as was the case for the N-POB. Hence, the computation of the maximum allowable load for large K and N can only be found sufficiently accurate via the proposed approximate method.

4.2. *Varying the cell loss ratio requirements*

When comparing the performance of a N-PBS and N-POB for an identical low priority cell loss ratio requirement clr_L^* as a function of a varying high priority cell loss ratio requirement clr_H^* (Fig. 5), the N-POB scheme clearly excels the N-PBS, especially in the low priority range $\alpha < \alpha_k$. We point out that in the high priority range $\alpha > \alpha_k$, the PBS-scheme with threshold optimization can outweigh the performance of the POB-scheme (especially in single queues [25, 26]).

In Fig. 6, the optimized threshold versus α as a function of the high priority cell loss ratio requirement clr_H^* is shown. As expected, the optimized threshold T decreases to lower values with more stringent clr_H^*.

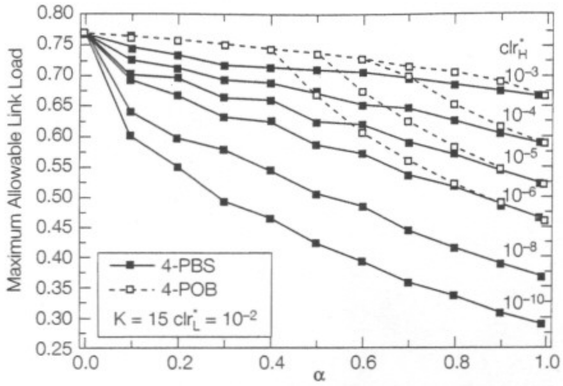

Fig. 5 - A comparison in performance between the 4-POB and 4-PBS (with optimized threshold) for different high priority cell loss ratio requirements, {10–3, 10–4, 10–5, 10–6, 10–8, 10–10}. The buffer size is $K = 15$ and the low priority cell loss ratio requirements is $clr^*_L = 10^{-2}$.

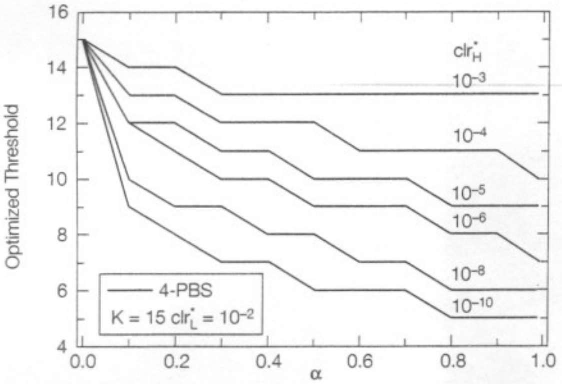

Fig. 6 - The corresponding optimized threshold in the 4-PBS for the setting of previous Fig. 5.

4.3. *Varying the buffer size K*

The performance of the *N*-POB has been approximated via scaling as explained in section 3.3, while for the *N*-PBS the bidimensional method is used. As illustrated in Fig. 7, for small buffer sizes, the *N*-POB obviously outweighs the *N*-PBS in performance. For larger buffer sizes, the differences become smaller. However, since the maximal gain $\lambda_{max}(0) - \lambda_{max}(1)$ due to priority management tends to zero for $K \to \infty$, the efficiency of a priority mechanism diminishes. Thus, for large buffers, a CLPM cannot be justified.

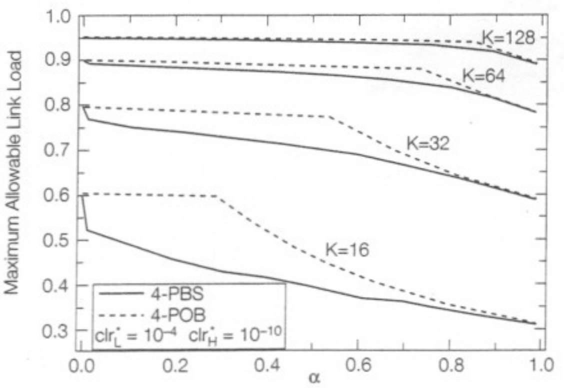

Fig. 7 - The performance of the 4-POB (dotted line) and 4-PBS (full line) for different buffer sizes K but with the same cell loss ratio requirements (10^{-4}, 10^{-10}).

Fig. 8 shows the normalized optimal threshold *T/K*. As only integer values of *T* are possible, the curves for small *K* clearly exhibit plateaus. This confinement is responsible for the poorer performance of *N*-PBS compared to *N*-POB in small buffers. For large buffers, the discreteness of *T* ceases to be the limiting factor. Another feature in Fig. 8 is the discontinuity at $\alpha = 0$, where the optimized threshold *T* precisely equals *K* because there are no high priorities. But, for an arbitrarily small portion of high priority cells ($\alpha > 0$), the optimized threshold *T* can never equal *K* because the stringent high priority cell loss ratio requirement (with $clr^*_H < clr^*_L$) forces the PBS to reserve at least one buffer position for only high priority cells. Finally, we mention that the behaviour of the normalized optimal threshold for $N = 4$ links is very similar to that for a single $N = 1$ link for which we refer to [25, 26].

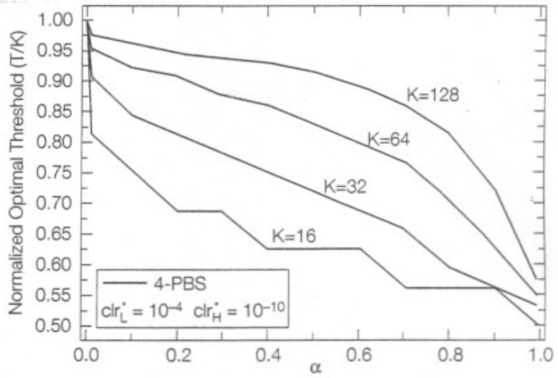

Fig. 8 - The normalized optimal threshold *T/K* in the 4-PBS versus α with cell loss ratio requirements (10^{-4}, 10^{-10}).

5. Conclusions

For Poisson arrivals, the symmetrical traffic situation is proved to result in optimal loading irrespective of the CLPM. The performances of two cell loss priority mechanisms in a shared buffer, *N*-POB and the *N*-PBS, are compared. We found that the *N*-POB is superior to the *N*-PBS for small *K* and sufficiently large *N*. For large buffers, the difference in performance between *N*-POB and *N*-PBS diminishes, as expected because the efficiency of a CLPM mechanism vanishes with increasing buffer sizes *K*. In addition, for large buffer sizes *K* and large *N*, we have shown that the maximum allowable load for the *N*-POB is easier to compute than for the corresponding *N*-PBS due to scaling in *K* and *N*. The lack of similarity in the performance curves of the *N*-PBS prevents simple scaling rules and, unfortunately, requires the solution of a large linear set in high precision.

Lastly, when the number of links *N* increases, the *N*-POB always performs better than the *N*-PBS over the whole priority mix region α and for realistic cell loss ratio requirements.

APPENDIX A

Properties of a shared buffer

A.1. *No priorities*

Property 3 *If each link carries Poissonean traffic that is inter-link independent then it holds for each link i leaving the shared buffer that* $clr_i = clr_{\text{sha}}$.

Proof:

- First we note that $clr = \langle$number of cells lost/number of arriving cells$\rangle_{av}$ where the average is over all time slots.
- Since the arrivals are Poissonean and inter-link independent, the arrivals are independent of the buffer occupation. Only if the shared buffer is entirely occupied, we have cell losses. In that case, any cell that arrives is lost. On average, we expect a cell arriving on link i with probability λ_i. Hence, the cell losses are proportional to λ_i implying that the cell loss *ratio* clr_i is independent of i. Thus, all Poisson flows experience the same cell loss ratio.

In fact, this property 3 is a corollary of the PASTA property [35].

Property 4 *In a shared buffer with inter-link independent arrivals, the aggregate load* Λ, *subject to the constraint that* $clr_{\text{sha}} \le clr^*_{\text{sha}}$, *is maximal if the individual link loads are equal whence* $\lambda_i = \lambda$.

Proof:

- The demonstration relies on symmetry. Indeed (7), we have

$$clr_{\text{sha}} = f(\lambda_1, \lambda_2, \ldots, \lambda_N; K) \equiv f(\{\lambda_i\}; K) = f(\{\lambda_i\}_{\text{all permutations}}; K) \tag{A.1}$$

- The maximization of Λ limited by (A.1) leads to a Lagrange function,

$$L = \Lambda + \xi\,[f(\{\lambda_i\}; K) - clr^*_{\text{sha}}]$$

with Lagrange multiplier ξ. The optimal solution obeys the set

$$\frac{\partial L}{\partial \lambda_i} = 1 + \xi \frac{\partial f}{\partial \lambda_i} = 0 \qquad (1 \le i \le N)$$

$$\frac{\partial L}{\partial \xi} = f(\{\lambda_i\}; K) - clr^*_{\text{sha}} \le 0$$

The first N equations indicate that $\partial f/\partial \lambda_i = -1/\xi$ while symmetry (A.1) implies that $\partial f/\partial \lambda_i = \partial f/\partial \lambda_j$ for all i and j. Hence, $\lambda_i = \lambda$ obeys these equations. The last equation or (A.1) will specify λ. It is readily verified that this solution is indeed a maximum.

A.2. *With priorities*

Proof of the equal balancing principle (property 1):

- Again the demonstation relies on symmetry properties. Since the priority scheme operates on the shared buffer as a whole, the link address i cannot be distinguished. Hence, we can write

$$clr_{\text{sha};H} = f_H(\Lambda, \alpha; K) \tag{A.2}$$

$$clr_{\text{sha};L} = f_L(\Lambda, \alpha; K)) \tag{A.3}$$

with $\Lambda = \sum_{i=1}^N \lambda_i$ and $\alpha = 1/\Lambda \sum_{i=1}^N \alpha_i \lambda_i$.

- The maximization of Λ with respect to λ_i limited by (A.2, A.3) leads to a Lagrange function,

$$L = \Lambda + \xi\,[f_H(\Lambda, \alpha; K) - clr^*_{\text{sha};H}] + \zeta\,[f_L(\Lambda, \alpha; K) - clr^*_{\text{sha};L}] \tag{A.4}$$

where ξ and ζ are Lagrange multipliers (8). The optimal solution obeys the set

$$\frac{\partial L}{\partial \lambda_i} = 1 + \xi \frac{\partial f_H}{\partial \Lambda}\frac{\partial \Lambda}{\partial \lambda_i} + \zeta \frac{\partial f_L}{\partial \Lambda}\frac{\partial \Lambda}{\partial \lambda_i} + \xi \frac{\partial f_H}{\partial \alpha}\frac{\partial \alpha}{\partial \lambda_i} + \zeta \frac{\partial f_L}{\partial \alpha}\frac{\partial \alpha}{\partial \lambda_i} = 0 \qquad (1 \le i \le N)$$

$$\frac{\partial L}{\partial \xi} = f_H(\Lambda, \alpha; K) - clr^*_{\text{sha};H} \le 0$$

$$\frac{\partial L}{\partial \zeta} = f_L(\Lambda, \alpha; K) - clr^*_{\text{sha};L} \le 0$$

The first N equations can be rewritten as

$$1 + \xi \frac{\partial f_H}{\partial \Lambda} + \zeta \frac{\partial f_L}{\partial \Lambda} + \left[\xi \frac{\partial f_H}{\partial \alpha} + \zeta \frac{\partial f_L}{\partial \alpha}\right]\frac{\alpha_i - \alpha}{\Lambda} = 0 \qquad (1 \le i \le N) \tag{A.5}$$

Multiplying the i-th eq. (A.5) by λ_i and summing over all i yields

$$\Lambda\left[1 + \xi \frac{\partial f_H}{\partial \Lambda} + \zeta \frac{\partial f_L}{\partial \Lambda}\right] = 0 \tag{A.6}$$

Combining (A.6) with (A.5) gives

(7) It is assumed that the cell loss ratio is completely determined by the average arrival rates or first moments λ_i. In this sense, the equal balancing principle is more generally applicable than just to Poisson arrivals.

(8) Notice that we have chosen in L the equality sign for the constraints for simplicity. Since $f_H(\Lambda, \alpha; K)$ and $f_L(\Lambda, \alpha; K)$ are increasing in Λ, the optimal solution will be the lowest upper bound.

$$\left[\xi\frac{\partial f_H}{\partial\alpha}+\zeta\ \frac{\partial f_L}{\partial\alpha}\right]\frac{\alpha_i-\alpha}{\Lambda}=0 \tag{A.7}$$

Two cases can be distinguished. In case all $\alpha_i = \alpha$, the N eq. (A.5) reduce to

$$\xi\frac{\partial f_H}{\partial\Lambda}+\zeta\ \frac{\partial f_L}{\partial\Lambda}=-1 \tag{A.8}$$

$$\xi\frac{\partial f_H}{\partial\alpha}+\zeta\ \frac{\partial f_L}{\partial\alpha}=\kappa \tag{A.9}$$

where κ is a real number. In case not all α_i are equal, the N eq. (A.5) reduce to

$$\xi\frac{\partial f_H}{\partial\Lambda}+\zeta\ \frac{\partial f_L}{\partial\Lambda}=-1 \tag{A.10}$$

$$\xi\frac{\partial f_H}{\partial\alpha}+\zeta\ \frac{\partial f_L}{\partial\alpha}=0 \tag{A.11}$$

In any case, the N equations reduce to a set that does not depend on the link address i. Hence, all λ_i should be equal. Again, the nature of the problem indicates that this solution $\lambda_i = \lambda$ AND $\alpha_i = \alpha$ is indeed a maximum.

Before proving the uniqueness of the symmetrical solution, we need additional results based on the conservation law (3). The average cell stream consists of high and low priorities,

$$\Lambda = \Lambda_H + \Lambda_L$$

with $\Lambda_H = \sum_{i=1}^{N} \alpha_i \lambda_i$ and $\Lambda_L = \sum_{i=1}^{N} (1-\alpha_i)\lambda_i$. The cell loss ratio may be written analogously to that for a single buffer,

$$clr_{\text{sha}}(\alpha) = \alpha\, clr_{\text{sha;}H}(\alpha) + (1-\alpha)\, clr_{\text{sha;}L}(\alpha) \tag{A.12}$$

where $\alpha = \Lambda_H/\Lambda$.

Since the cell loss ratio increases for increasing loads, we have

$$\left.\frac{\partial clr_{\text{sha;}H}}{\partial\Lambda}\right|_{\alpha=\text{const}} > 0 \tag{A.13}$$

$$\left.\frac{\partial clr_{\text{sha;}L}}{\partial\Lambda}\right|_{\alpha=\text{const}} > 0 \tag{A.14}$$

The variation of $clr_{\text{sha;}H}(\alpha)$ with respect to α at constant load Λ is

$$\left.\frac{\partial clr_{\text{sha;}H}}{\partial\alpha}\right|_{\Lambda=\text{const}} > 0 \tag{A.15}$$

$$\left.\frac{\partial clr_{\text{sha;}L}}{\partial\alpha}\right|_{\Lambda=\text{const}} > 0 \tag{A.16}$$

because $(\partial\, clr_{\text{sha;}H})/(\partial\alpha) = (\partial\, clr_{\text{sha;}H})/(\partial\Lambda_H)\ \partial\Lambda/\partial\alpha = (\partial\, clr_{\text{sha;}H})/(\partial\Lambda_H)\ \Lambda > 0$ and similar arguments lead to (A.16). The latter can also be obtained by differentiating (A.12) with respect to α at constant Λ. Indeed, because $(\partial\, clr_{\text{sha}})/(\partial\,\alpha)\,|_{\Lambda=\text{const}} = 0$, we find

$$\alpha\left.\frac{\partial clr_{\text{sha;}H}}{\partial\alpha}\right|_{\Lambda=\text{const}} + (1-\alpha)\left.\frac{\partial clr_{\text{sha;}L}}{\partial\alpha}\right|_{\Lambda=\text{const}} = clr_{\text{sha;}L}(\alpha) - clr_{\text{sha;}H}(\alpha) > 0 \tag{A.17}$$

Property 5 *The maximum obtained in property 1 (the equal balancing principle) with individual link loads $\lambda_i = \lambda$ AND equal priority mixes $\alpha_i = \alpha$ is unique.*

Proof:

- The Lagrange function L (A.4) can be interpreted as a constrained total throughput. This interpretation requires that both ξ and ζ are negative. For, if the incoming loads λ_i are such that the cell loss ratio is exceeded, the constrained total load must be smaller than the incoming total load Λ. A similar argument holds for the opposite case.
- The variation of L (A.4) with respect to α_i at constant Λ is

$$\frac{\partial L}{\partial\alpha_i}=\left[\xi\frac{\partial f_H}{\partial\alpha}+\zeta\ \frac{\partial f_L}{\partial\alpha}\right]\frac{\lambda_i}{\Lambda} \tag{A.18}$$

Now, if $clr^*_{\text{sha;}L} > clr^*_{\text{sha;}H}$, the constrained total load L is strictly decreasing with α_i, because $(\partial L)/(\partial\alpha_i) = (\partial L)/(\partial\Lambda_H)\ (\partial\Lambda_H)/(\partial\alpha)\ (\partial\alpha)/(\partial\alpha_i) = \alpha_i\ (\partial L)/(\partial\Lambda_H)$ and $(\partial L)/(\partial\Lambda_H) < 0$ since $clr^*_{\text{sha;}L} > clr^*_{\text{sha;}H}$. But, in case not all α_i are equal, an optimized solution leads to $(\partial L)/(\partial\alpha_i) = 0$ on (A.11) for all α_i. This contradicts the previous argumentation and shows that a maximum solution cannot have all α_i different.
- Finally, we will show that it is possible for the only maximum (all $\lambda_i = \lambda$ AND $\alpha_i = \alpha$) to obey that ζ and ξ are negative. First, the condition $(\partial L)/(\partial\alpha_i) < 0$ in (A.18) implies that $\kappa < 0$ on (A.9). Further, let us denote $det = (\partial f_H)/(\partial\Lambda)\ (\partial f_L)/(\partial\alpha) - (\partial f_L)/(\partial\Lambda)\ (\partial f_H)/(\partial\alpha)$ and on (A.13, A.14, A.15, A.16) we find that $det < 0$. Then, solving (A.8) and (A.9) for ξ and ζ leads to

$$\zeta=\frac{\kappa\dfrac{\partial f_H}{\partial\Lambda}+\dfrac{\partial f_H}{\partial\alpha}}{det}$$

$$\xi=-\frac{\dfrac{\partial f_L}{\partial\alpha}+\kappa\dfrac{\partial f_L}{\partial\Lambda}}{det}$$

Demanding that $\zeta < 0$ and $\xi < 0$ leads to the inequalities

$$-\frac{\dfrac{\partial f_H}{\partial\alpha}}{\dfrac{\partial f_H}{\partial\Lambda}}<\kappa<-\frac{\dfrac{\partial f_L}{\partial\alpha}}{\dfrac{\partial f_L}{\partial\Lambda}}$$

Again, using (A.13, A.14, A.15. A.16), we readily verify that κ is bounded by a negative and a positive number. These bounds are compatible with the condition that $\kappa < 0$, hence, we can always find such a κ.

APPENDIX B
State equations for N-PBS

B.1. *Exact solution - general method*

In this method a vector v^j of length N corresponds to each state π_j:

$$v^j = (v_1^j, v_2^j, \ldots, v_N^j)$$

Each entry v_i^j denotes the number of cells in the buffer for link i at the end of the service time. The number of cells in the buffer at the beginning of the next service cycle is stored in the vector v^{j1+} where $v_i^{j1+} = \max(v^{j1} - 1, 0)$, $i = 1 \ldots N$. The following notation will be used:

$$\Lambda = \sum_{i=1}^{N} \lambda_i,\ \Lambda_H = \sum_{i=1}^{N} \alpha_i \lambda_i,\ \Lambda_L = \sum_{i=1}^{N} (1-\alpha_i)\lambda_i,$$

$$N_{c1+} = \sum_{i=1}^{N} v_i^{j1+},\ N_{c2} = \sum_{i=1}^{N} v_i^{j2}$$

and the Poisson arrival law is written as $P(\lambda, i) = \lambda^i / i!\, e^{-\lambda}$.

The transition probability to go from a state π_{j1} to a state π_{j2} can now be computed. The number of cells in a logical queue cannot decrease going from v^{j1+} to v^{j2}, therefore

$$M_{j1,j2} = 0, \text{ if } \exists\, i, i \in \{1, 2, \ldots, N\} : v_i^{j2} < v_i^{j1+}$$

1) when $N_{c2} < K$:
   a) when $N_{c1+} < T$ and $N_{c2} < T$.
   All arriving cells are admitted in the buffer. No cells are discarded. Using the vectors v^{j1+} and v^{j2}, we know exactly how many cells have to arrive on every link. Invoking the independence of arrivals between different links yields the transition probability

$$M_{j1,j2} = \prod_{i=1}^{N} P\left(\lambda_i, v_i^{j2} - v_i^{j1+}\right)$$

   b) when $N_{c1+} \geq T$ and $N_{c2} \geq T$.
   Only the arriving high priority cells are stored in the buffer. All low priority cells are discarded. As before, the transition probability is

$$M_{j1,j2} = \prod_{i=1}^{N} P\left(\alpha_i \lambda_i, v_i^{j2} - v_i^{j1+}\right)$$

   c) when $N_{c1+} < T$ and $N_{c2} \geq T$.
   This is the hardest case. $N_{c2} - N_{c1+}$ cells have to be stored in the buffer. All the arriving cells are accepted until the threshold T is reached and above T only high priority cells are admitted. The number of cells of link i to be stored is $m_i = v_i^{j2} - v_i^{j1+}$. Of these m_i cells, k_i cells are stored in buffer positions lower than T. The remaining number of high priority cells will be denoted by $n_i = m_i - k_i$. Further, the set $\{k_i\}$ is confined by $\sum_{i=1}^{N} k_i = T - N_{c1+}$.

To compute the transition probability two functions are introduced. The function $f_1(.)$ computes the probability that among the first $T - N_{c1+}$ cells there are k_1 cells of link 1, k_2 cells of link 2, ..., k_N cells of link N. Defining $N_T = T - N_{c1+}$, the function f_1 (.) is

$$f_1(k_1, k_2, \ldots, k_{N-1}) = \begin{cases} 0 & \text{if } k_N < 0 \text{ or } k_N > m_N \\ \binom{N_T}{k_1}\binom{N_T - k_1}{k_2}\cdots\binom{N_T - \sum_{l=1}^{N-2} k_l}{k_{N-1}} & \\ \times\left(\frac{\lambda_1}{\Lambda}\right)^{k_1}\left(\frac{\lambda_2}{\Lambda}\right)^{k_2}\cdots\left(\frac{\lambda_{N-1}}{\Lambda}\right)^{k_{N-1}}\left(\frac{\lambda_N}{\Lambda}\right)^{k_N} & \text{if } 0 \leq k_N \leq m_N \end{cases} \tag{B.1}$$

with $k_N = T - N_{c1+} - \sum_{i=1}^{N-1} k_i$.

Suppose now that after the first N_T cells, still $l - N_T$ cells arrive at the buffer. The function $f_2(.)$ computes the probability that among these $l - N_T$ cells there are exactly n_1 high priority cells for link 1, n_2 for link 2, ... The other $l - N_{c2} + N_{c1+}$ cells are low priority cells and are discarded.

$$f_2(n_1, \ldots, n_N, l) = \binom{1 - N_T}{n_1}\binom{1 - N_T - n_1}{n_2}\cdots\binom{1 - N_T - \sum_{p=1}^{N-1} n_p}{n_N} \times \left(\frac{\alpha_1\lambda_1}{\Lambda}\right)^{n_1}\left(\frac{\alpha_2\lambda_2}{\Lambda}\right)^{n_2}\cdots\left(\frac{\alpha_N\lambda_N}{\Lambda}\right)^{n_N} \times \left(\frac{\sum_{p=1}^{N-1}(1-\alpha_p)\lambda_p}{\Lambda}\right)^{1 - N_{c2} + N_{c1+}} \tag{B.2}$$

With these definitions the transition probability from state j_1 to state j_2 equals

$$M_{j1,j2} = \sum_{l = N_{c2} - N_{c1+}}^{\infty} P(\Lambda, l) \times \left[\sum_{k_1=0}^{m_1}\sum_{k_2=0}^{m_2}\cdots\sum_{k_{N-1}=0}^{m_{N-1}} f_1(k_1, k_2, \ldots, k_{N-1}) \cdot f_2(n_1, n_2, \ldots, n_{N-1}, n_N, l)\right] \tag{B.3}$$

2) when $N_{c2} = K$

a) when $N_{c1+} < T$.

Before computing this transition probability, two new functions are introduced. The function $f_3(.)$ is the probability that having l arriving cells, at least $K - T$ high priority cells arrive, after the first $T - N_{c1+}$ cells. One may verify that

$$f_3\left(1, N_{c1+}, K, T\right) = 1 - \sum_{p=0}^{(K-T)-1} \binom{1-N_T}{p} \left(\frac{\Lambda_H}{\Lambda}\right)^p \left(\frac{\Lambda_L}{\Lambda}\right)^{1-N_T-p}$$

where $N_T = T - N_{c1+}$.

The function $f_4(.)$ computes the probability that among the stored $K - T$ high priority cells, there are exactly n_1 cells for link 1, n_2 for link 2,…:

$$f_4\left(n_1, n_2, \ldots, n_N\right) = \binom{K-T}{n_1}\binom{K-T-n_1}{n_2}\cdots\binom{K-T-\sum_{i=1}^{N-2} n_i}{n_{N-1}} \times \left(\frac{\alpha_1 \lambda_1}{\Lambda_H}\right)^{n_1} \cdots \left(\frac{\alpha_N \lambda_N}{\Lambda_H}\right)^{n_N}$$

The transition probability from state j_1 to state j_2 reads

$$M_{j1,j2} = \sum_{l=K-N_{c1+}}^{\infty} P\left(\Lambda, l\right) \times \left[\sum_{k_1=0}^{m_1} \cdots \sum_{k_{N-1}=0}^{m_{N-1}} f_1\left(k_1, k_2, \ldots, k_{N-1}\right) \cdot f_3\left(1, N_{c1+}, K, T\right) f_4\left(n_1, n_2, \ldots, n_N\right)\right]$$

b) when $N_{c1+} \geq T$.

Here, we only have to consider the high priority cells. Of these high priority cells only the first $K - N_{c1+}$ cells will enter the buffer. The other cells are discarded when the buffer is full. In those first $K - N_{c1+}$ cells we have to find exactly $m_i = v_i^{j2} - v_i^{j1+}$ cells for each link (i = 1,…, N). The transition probability is thus:

$$M_{j1,j2} = \sum_{l=K-N_{c1+}}^{\infty} P\left(\Lambda_H, 1\right) \cdot \left[\binom{K-N_{c1+}}{m_1} \cdot \binom{K-N_{c1+}-m_1}{m_2} \cdots \binom{K-N_{c1+}-\sum_{i=1}^{N-1} m_1}{m_N} \times \left(\frac{\alpha_1 \lambda_1}{\Lambda_H}\right)^{m_1} \left(\frac{\alpha_2 \lambda_2}{\Lambda_H}\right)^{m_2} \cdots \left(\frac{\alpha_N \lambda_N}{\Lambda_H}\right)^{m_N}\right] \quad \text{(B.4)}$$

B.2. *Exact solution - symmetrical traffic*

In this appendix, the formulas used to construct the transition matrix for the N-PBS are derived employing the vector method proposed by Monterosso and Pattavina [27]. The traffic is assumed to be symmetrical: $\lambda = \lambda_i$ and $\alpha = \alpha_i$, $i = 1, \ldots, N$. The following definitions are used:

$$\lambda_H = \alpha\, \lambda,$$

$$\lambda_L = (1 - \alpha)\, \lambda$$

For this method each state π_j again corresponds with a vector

$$v^j = (v_1^j, v_2^j, \ldots, v_N^j)$$

but with the additional constraint that $v_1 \leq v_2 \leq \ldots \leq v_N$.

Before computing the transition probabilities, the number of ways to move from the state $j1$ to a state $j2$, denoted by $f\,(j1, j2)$, has to be known. Recalling the definitions of the previous section, to compute the number of possible transitions, all the possible distributions of $N_{c2} - N_{c1+}$ cells (N_{c1+} is the number of cells *after* serving) over N links are examined. Define $N_c = N_{c2} - N_{c1+}$, the number of cells that have to arrive to induce a transition from state j_1 to state j_2. The number of possible transitions is then:

$$f\left(j1, j2\right) = \sum_{r_1=0}^{N_c} \sum_{r_2=0}^{N_c-\sigma_2} \cdots \sum_{r_{N-1}=0}^{N_c-\sigma_{N-1}} F\left(r, j1, j2\right)$$

where

$$\sigma_i = \sum_{j=1}^{i-1} r_j, \quad i = 2, \ldots N,$$

$$r = \left(r_1\, r_2 \ldots r_{N-1}\, r_N\right), \quad \text{with } r_N = N_c - \sigma_N$$

and

$$F\left(r, j1, j2\right) = \begin{cases} 1 & \text{if } v^{j2} = \text{ord}\left(v^{j1+} + r\right) \text{ and } r_N \geq 0 \\ 0 & \text{if } v^{j2} \neq \text{ord}\left(v^{j1+} + r\right) \text{ and } r_N < 0 \end{cases}$$

The function ord(.) sorts the elements of a vector in non-decreasing order.

Using this definition of $f\,(j1, j2)$, we are able to compute the transition probabilities. Obviously, the transition probability is zero when $N_{c2} < N_{c1+}$. The other transition probabilities are:

1) when $N_{c2} < K$.

a) when $N_{c1}+ < T$ and $N_{c2} < T$.

All arriving cells are stored in the buffer.

$$M_{j1,j2} = f\,(j1, j2) \cdot P\,(N\,\lambda, N_c)$$

b) when $N_{c1+} \geq T$ and $N_{c2} \geq T$.
Only arriving high priority cells are stored in the buffer.

$$M_{j1,j2} = f(j1, j2) \cdot P(N\lambda_H, N_c)$$

c) when $N_{c1+} < T$ and $N_{c2} \geq T$.
All arriving cells are stored until the buffer is filled with T cells. Subsequently only high priority cells are stored. However, exactly $N_c - N_T$ high priority cells still have to arrive to induce the transition. A formula proposed by Kröner *et al.* [19] is used. The resulting transition probability is,

$$M_{j1,j2} = f(j1,j2) \sum_{l=N_c}^{\infty} P(N\lambda, l) \cdot \binom{l-N_T}{N_c-N_T} \alpha^{N_c-N_T} (1-\alpha)^{l-N_c}$$

with $N_T = T - N_{c1+}$.

2) when $N_{c2} = K$.

a) when $N_{c1+} < T$

$$M_{j1,j2} = f(j1,j2) \cdot \left\{ \sum_{l=N_c}^{\infty} P(N\lambda, l) \left[1 - \sum_{k=1}^{K-T-1} \binom{l-N_T}{k} \alpha^k (1-\alpha)^{l-N_T-k} \right] \right\}$$

An infinite number of cells might arrive. However, the state $j2$ can only be reached if at least $K - T$ high priority cells arrive, after the first N_T cells that are certainly stored in the buffer.

b) when $N_{c1+} \geq T$.
The state $j2$ can only be reached when at least $N_c = K - N_{c1+}$ high priority cells arrive. The transition probability is thus:

$$M_{j1,j2} = f(j1,j2) \sum_{l=N_c}^{\infty} P(N\lambda_H, l) = f(j1,j2) \left[1 - \sum_{l=0}^{N_c-1} P(N\lambda_H, l) \right]$$

B.3. *Approximate bidimensional method*

The state equations for the N-PBS are approximated, based on the bidimensional principle of Bianchi and Turner [4] where each state corresponds with a vector $v = (s, c)$, with s denoting the number of cells in the buffer and c the number of active servers.

Before calculating the transition probabilities, preliminary results are needed. Let $R(s_1, c_1, a_1, c_2)$ denote the probability that when there are initially s_1 cells in the buffer for c_1 servers, c_1 cells are served and a_1 cells are stored in the buffer, then the resulting number of active servers is c_2. Recall that our state is measured just before the cells in the buffer are served. Due to the deterministic server discipline, c_1 cells are always served, if the number of active servers (being the number of non-empty queues) is c_1.

To compute $R(s_1, c_1, a_1, c_2)$, let c denote the number of active servers, when after c_1 cells are served. Introducing the functions

$S(s_1, c_1, c)$: The probability that having s_1 cells in the buffer for c_1 servers before serving, there are still c servers active *after* the serving of c_1 cells. Of course, $S(s_1, c_1, c) = 0$ when $c > c_1$.

$T(c, a_1, c_2)$: The probability that having c active servers *after* serving and a_1 arriving cells are stored in the buffer, the number of active servers is c_2. From the definition it follows that $T(c, a_1, c_2) = 0$ when $c_2 < c$ as the number of active servers cannot decrease on arrival of a_1 cells.

We obtain

$$R(s_1, c_1, a_1, c_2) = \sum_{c=0}^{c_1} S(s_1, c_1, c) \cdot T(c, a_1, c_2)$$

Bianchi and Turner [4] have demonstated that

$$S(s_1, c_1, c) = \binom{c_1}{c_1 - c} \frac{\binom{s_1 - c_1 - 1}{c - 1}}{\binom{s_1 - 1}{c_1 - 1}}$$

and

$$T(c, a_1, c_2) = \frac{c_2}{N} T(c, a_1 - 1, c_2) + \left(1 - \frac{c_2 - 1}{N}\right) T(c, a_1 - 1, c_2 - 1)$$

when $c < c_2 \leq c + a_1$ while for $c_2 = c$

$$T(c, a_1, c) = \left(\frac{c}{N}\right)^{a_1}$$

With these definitions, the construction of the transition matrix is as follows. In the state j_1 (j_2) there are s_1 (s_2) cells in the buffer for c_1 (c_2) servers. In order to simplify the formulas below, we write $s_1^+ = s_1 - c_1$ and $N_c = s_2 - s_1^+$.

The transition probability is zero when $s_2 < s_1^+$. The other transition probabilities are:

1) when $s_2 < K$

a) when $s_1^+ < T$ and $s_2 < T$.
All arriving cells are stored in the buffer.

$$M_{j1,j2} = R(s_1, c_1, N_c, c_2) \cdot P(N\lambda, N_c)$$

b) when $s_1^+ \geq T$ and $s_2 \geq T$.
Only arriving high priority cells are stored in the buffer.

$$M_{j1,j2} = R\,(s_1, c_1, N_c, c_2) \cdot P\,(N\,\lambda_H, N_c)$$

c) when $s_1^+ < T$ and $s_2 \geq T$.

All arriving cells are stored until the buffer is filled with T cells. Subsequently only high priority cells are stored.

$$M_{j1,j2} = R(s_1, c_1, N_c, c_2) \sum_{l=N_c}^{\infty} P(N\lambda, l) \cdot \binom{1-N_T}{N_c - N_T} \alpha^{N_c - N_T} (1-\alpha)^{l-N_c}$$

with $N_T = T - s_1^+$.

2) when $s_2 = K$.

a) when $s_1^+ < T$

$$M_{j1,j2} = R(s_1, c_1, N_c, c_2) \cdot \left\{ \sum_{l=N_c}^{\infty} P(N\lambda, l) \left[1 - \sum_{k=1}^{K-T-1} \binom{l-N_T}{k} \alpha^k (1-\alpha)^{l-N_T-k} \right] \right\}$$

An infinite number of cells might arrive. However, the state $j2$ can only be reached if at least $K - T$ high priority cells arrive, after the first N_T cells that are certainly stored in the buffer. The following definitions were used: $N_T = T - s_1^+$ and $N_c = K - s_1^+$.

b) when $s_1^+ \geq T$.

The state $j2$ can only be reached when at least $K - s_1^+$ high priority cells arrive. The transition probability is thus:

$$M_{j1,j2} = R(s_1, c_1, N_c, c_2) \sum_{l=N_c}^{\infty} P(N\lambda_H, l) = R(s_1, c_1, N_c, c_2) \left[1 - \sum_{l=0}^{N_c - 1} P(N\lambda_H, l) \right]$$

with $N_c = K - s_1^+$.

Acknowledgement

This work has been sponsored by the Flemish Institute for the Promotion of Scientific and Technological Research in Industry.

Manuscript received on May 27, 1996.

REFERENCES

[1] O. Narayan, A. Erramilli, W. Willinger: *Experimental queueing analysis with long-range dependent packet traffic.* "IEEE/ACM Trans. Networking", Vol. 4, No. 2, April 1996, p. 209-223.

[2] I. F. Akyildiz, X. Cheng: *Analysis of a finite buffer queue with different scheduling and push-out schemes.* "Performance Evaluation", Vol. 19, 1994, p. 317-340.

[3] K. R. Krishnan, B. V. Rao, D. P. Heyman: *Performance of finite-buffer queues under traffic with long-range dependence.* "IEEE Globecom'96 (London)", 18th-22nd November 1996, p. 607-611.

[4] G. Bianchi, J. S. Turner: *Improved queueing analysis of shared buffer switching networks.* "IEEE ACM Trans. on Networking", Vol. 1, No. 4, Aug. 1993, p. 482-489.

[5] Y. Z. Cho, C. K. Un: *Analysis of the M/G/1 queue under a combined preemptive/nonpreemptive priority discipline.* "IEEE Transactions on Commun.", Vol. 41, No. 1, January 1993, p. 132-141.

[6] A. K. Choudhury, E. L. Hahne: *Space priority management in a shared memory ATM switch.* "Proc. IEEE Globecom'93", December 1993, p. 1375-1383.

[7] I. Cidon, L. Georgiadis, R. Guerin, A. Khamisy: *Optimal buffer sharing.* "IEEE JSAC", Vol. 13, No. 7, Sept. 1995, p. 1229-1240.

[8] I. Cidon, A. Khamisy, L. Georgiadis, R. Guerin: *Optimal buffer sharing.* "IEEE INFOCOM'95", Boston, Vol. 1, No. 1a.4.1, April 2-6 1995, p. 24-31.

[9] M. de Prycker: *Asynchronous Transfer Mode: Solution for Broadband ISDN.* Ellis Horwood, New York, third edition, 1995.

[10] E. Desmet, B. Steyaert, H. Bruneel, G. H. Petit: *Tail distributions of queue length and delay in discrete-time multiserver queueing models, applicable in ATM networks.* "ITC-13", Copenhagen, Denmark, June 19-26 1991, p. 1-6.

[11] P. Droz, J. Le Boudec: *A high-speed self-similar ATM VBR traffic generator.* "IEEE Globecom'96", London, 18th-22nd November 1996, p. 586-590.

[12] Z. Dziong, K.-Q. Liao, L. Mason: *Effective bandwidth allocation and buffer dimensioning in ATM based networks with priorities.* "Computer Networks and ISDN Systems", Vol. 25, 1993, p. 1065-1078.

[13] G. Gallassi, G. Rigolio, L. Fratta: *Bandwidth assignment in prioritized ATM networks.* "Proc. IEEE Globecom'90", Vol. 505, No. 2, 1990, p. 852-856.

[14] M. W. Garrett, W. Willinger: *Analysis, modeling and generation of self-similar VBR video traffic.* "ACM SIGCOMM'94", London, Sept. 1994.

[15] S. Giordano, M. Pagano, R. Pannocchia, F. Russo: *A new call admission control scheme based on the self similar nature of multimedia traffic.* "IEEE Globecom'96", 1996, p. 1612-1618.

[16] H. E. J. Hernandez-Valencia, F. G. Bonomi: *Simulation of a simple loss/delay priority scheme for shared memory ATM fabrics.* "Proc. IEEE Globecom'93", December 1993, p. 1389-1394.

[17] T.-Y. Huang, J.-L.C. Wu: *Performance analysis of ATM switches using priority schemes.* "IEE Proc.-Commun.", Vol. 141, No. 4, Aug. 1994, p. 248-254.

[18] T.-Y. Huang, J.-L. Chen Wu, J. Wu: *Priority management to improve the QOS in ATM networks.* "IEICE Trans. Commun.", Vol. E 76-B, No. 3, March 1993, p. 249-257.

[19] H. Kröner, G. Hebuterne, P. Boyer, A. Gravey: *Priority management in ATM switching nodes.* "IEEE J. Select. Areas Commun.", Vol. 9, No. 3, April 1991, p. 418-427.

[20] M. Krunz, H. Hughes, Y. Parviz: *Congestion control in ATM networks using multiple buffers and priority mechanisms.* Proc. of Sum. Computer Simulation Conf. (SCSC '93), Boston, USA, July 1993, p. 566-571.

[21] M. Libby, H. Hughes: *Priority management for selective cell scheduling and discarding in ATM networks.* Proc. of Sum. Computer Simulation Conf. (SCSC '93), Boston, USA, July 1993, p. 572-577.

[22] A.Y.-M. Lin, J.A. Silvester: *Priority queueing strategies and buffer allocation protocols for traffic control at an ATM integrated broadband switching system.* "IEEE J. Select. Areas Commun.", Vol. 9, No. 9, Dec. 1991, p. 1524-1536.

[23] A. Y.-M. Lin, J. A. Silvester: *Priority queueing strategies for traffic control at a multichannel ATM switching system.* "Proc. IEEE Globecom '91", Vol. 8B, No. 2, 1991, p. 234-238.

[24] J. F. Meyer, S. Montagna, R. Paglio: *Dimensioning of an ATM switch with shared buffer and threshold priority.* "Computer Networks and ISDN Systems", Vol. 26, 1993, p. 95-108.