A GEOMETRICAL STRATEGY FOR THE IDENTIFICATION OF STATE SPACE MODELS OF LINEAR MULTIVARIABLE SYSTEMS WITH SINGULAR VALUE DECOMPOSITION

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CONCLUDING REMARKS

REFERENCES


4.2 Identification algorithm: version 2

While the identification approaches derived in section 4.1, essentially rely on the singular value decomposition of the observability space, the following theorem shows that this space only depends upon the observable poles of the system.

This theorem has been generalized (section 4.2) to obtain the projection of the observable part of the state space into a dynamical imbedded subspace of lower dimensionality, where the observable part of the state space are lost in the projection. The geometrical situation can rigorously be described via the concept of principal angles between subspaces, which is a generalization of the orthogonal complement of the row space of the input Hankel matrix UH.

Theorem 2 (in section 4.2) guarantees that the observable part of the state space is lost in the projection of the observable part of the state space into a dynamical imbedded subspace of lower dimensionality. The orthogonality of the observable poles guarantees that the observable part of the state space are not lost in the projection. The geometrical situation can rigorously be described via the concept of principal angles between subspaces, which is a generalization of the orthogonal complement of the row space of the input Hankel matrix UH.

Proof: Consider a linear system described by the state space model:

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) \\
y(t) &= Cx(t) + Du(t)
\end{align*}
\]

where \(x(t)\) is the state vector, \(u(t)\) is the input, and \(y(t)\) is the output. The observability index of the system is denoted by \(n\), which is the number of rows of the observability matrix.

The observability index is defined as the number of rows of the observability matrix that are linearly independent. The observability index is often used to determine the number of states of a system. However, it is important to note that the observability index is not necessarily equal to the number of independent states of a system. For example, a system may be observable in one coordinate system but not in another. Therefore, it is important to consider the observability index in conjunction with other system properties, such as controllability.

The observability index is a useful tool in the design and analysis of control systems. It can be used to determine the minimum number of sensors required to fully observe a system and to design observers that can estimate the state of a system. However, it is important to note that the observability index is not a direct measure of the system's performance. For example, a system may be observable but not controllable, or vice versa. Therefore, it is important to consider both the observability and controllability indices when analyzing a system. 

4.3 Identification algorithm: version 3

While the identification approaches derived in section 4.1, essentially rely on the singular value decomposition of the observability space, the following theorem shows that this space only depends upon the observable poles of the system.

Theorem 3 (in section 4.2) guarantees that the observable part of the state space is lost in the projection of the observable part of the state space into a dynamical imbedded subspace of lower dimensionality. The orthogonality of the observable poles guarantees that the observable part of the state space are not lost in the projection. The geometrical situation can rigorously be described via the concept of principal angles between subspaces, which is a generalization of the orthogonal complement of the row space of the input Hankel matrix UH.

Proof: Consider a linear system described by the state space model:

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\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) \\
y(t) &= Cx(t) + Du(t)
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where \(x(t)\) is the state vector, \(u(t)\) is the input, and \(y(t)\) is the output. The observability index of the system is denoted by \(n\), which is the number of rows of the observability matrix.

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5. Properties of the new identification approach

As will be explained in this section, the optimal choice for the matrix \(k\) is simply to be used in the construction of the observability matrix. The following theorem states that the observability matrix constructed in this way is an optimal identification excitation signal.

For the identification version 1 (section 4.1) the following inequalities allow enough 'space' to estimate the observable dynamical order \(n\). Once this is done, it follows from a similar observation as was used in section 4.2.

\[
\begin{align*}
\text{Rank}(\tilde{X}_1) &\geq \text{Rank}(X_1) \\
\text{Rank}(\tilde{X}_2) &\geq \text{Rank}(X_2)
\end{align*}
\]

This result is important.

In this section, we discuss the three most important without mentioning too many free data.

This section is important.

For the identification version 2 (section 4.3) the following inequalities should be satisfied:

\[
\begin{align*}
\text{Rank}(\tilde{X}_1) &\geq \text{Rank}(X_1) \\
\text{Rank}(\tilde{X}_2) &\geq \text{Rank}(X_2)
\end{align*}
\]

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5.3 The computational requirements.

Although computational details will be reported elsewhere, we briefly summarise in this paper some important observations.

Identification approach version 1 (section 4.2)

The configuration of the row space of the block Hankel matrix \( H \) with respect to that of the block Hankel matrix \( \tilde{H} \) requires a Grassmannian orthogonality procedure. Proceeding, the Hankel structure could be exploited.

- The resulting matrix \( \tilde{H} = [H, \tilde{H}] \) is a matrix to be discussed in section 9.2. A possibly overdetermined matrix, with much more columns than rows. Hence, its singular value decomposition can be computed in finite time by fast performing a RB decomposition (R for row orthonormal) and then computing the SVD of the matrix \( \tilde{H} \).

Identification approach version 2 (section 4.3)

- The singular value decomposition of a largely overdetermined concatenated matrix \( \hat{H} \) is computed. This can again be achieved by first computing the RB factorisation, followed by the SVD of \( \hat{H} \).

As adaptive versions of the presented identification algorithms that may be used for the identification of time-varying linear systems, an adaptive versions of the presented algorithms have been derived: One which allows for a linear least squares projection interpretation of the singular value decomposition for structured matrices.

7 Conclusions

In this paper, a survey was given of geometrical concepts for a new identification scheme. The properties of the singular value decompositions are exploited to compute a state space model from noisy input-output observations. Two variants have been derived: One which allows for a linear least squares projection interpretation of the singular value decomposition for structured matrices.

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References


Figure 4.