Search for a grand tour of the Jupiter Galilean moons

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ABSTRACT

We make use of self-adaptation in a Differential Evolution algorithm and of the asynchronous island model to design a complex interplanetary trajectory touring the Galilean Jupiter moons (Io, Europa, Ganymede and Callisto) using the multiple gravity assist technique. Such a problem was recently the subject of an international competition organized by the Jet Propulsion Laboratory (NASA) and won by a trajectory designed by aerospace experts and reaching the final score of 311/324. We apply our method to the very same problem finding new surprising designs and orbital strategies and a score of up to 316/324.

Categories and Subject Descriptors
I.2.8 [Artificial Intelligence]: Problem Solving, Control Methods, and Search—Graph and tree search strategies, Heuristic methods

General Terms
Algorithms

Keywords
Self-adaptation, Differential Evolution, multi-criteria tree-search, interplanetary trajectory optimization

1. INTRODUCTION

The design of interplanetary trajectories can be profitably approached as a global optimization problem. In the last decade, a number of researchers [1, 9, 5, 14, 13, 10] have successfully applied different automated search techniques to these type of problems, helping to define a new approach to interplanetary mission design. In this context, the international Global Trajectory Optimization Competition (GTOC) series1, was born with the objective of fostering research in this area by letting different methods compete on one, difficult, well-defined, problem. During the competition, standard design methods based on a profound domain knowledge and on human-in-the-loop procedures, compete with more or less automated design techniques. Interestingly, evolutionary search techniques have been used by many of the competition participants, but have never obtained results that are competitive to those found by other types of optimization procedures. For an example of the complex and diverse solution strategies employed in the first edition of the competition one can refer to [7]. In this paper we describe an automated search procedure based on evolutionary techniques that is able to design multiple fly-by interplanetary trajectories and apply it to the latest problem released in the GTOC series, the sixth edition problem.

1.1 Brief problem description

The sixth edition of the GTOC series was organized by the Jet Propulsion Laboratory (JPL), and the problem was released on the 10th September 2012. The 33 international teams who registered to the event were given one month time to find a solution to arguably one of the most difficult problems ever formalized in the context of these competitions: mapping globally the four Galilean Jupiter moons (Io, Ganymede, Europa and Callisto) using multiple fly-bys and an electric propulsion spacecraft. The formal mathematical description (problem statement) can be found in [12]. Here we repeat some of the details to facilitate the reader. The inertial Jupiter centric frame \( F_i \) is indicated with \( i \), \( j \), \( k \). On each moon, a body frame \( F_b \) is defined and indicated with \( b_1 \), \( b_2 \), \( b_3 \). In such a body frame, the moon surface is considered to be divided into 32 faces \( F_c \), \( i = 1..32 \), similarly to the division used to stitch soccer balls (see Figure 1), defined by their cartesian coordinates in \( F_b \). We write \( F_i := \{ p_j, j = 1...n_i \} \) where \( n_i \) is the number of vertices of the \( i \)-th face (either 5 or 6), and \( p_j \) are the \( j \)-th vertex coordinates in \( F_b \). Each face, according to its scientific interest, is assigned a score ranging from 1 to 3 points. Each face on Europa is assigned double scores because of the higher scientific interest of this particular Jupiter moon.

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1See http://sophia.estec.esa.int/gtoc_portal/

2More formally, this happens when \( r_p \) lies within the pyramid having the face \( F_i \) as a basis and the moon center as a vertex.
2. PROBLEM $P_1$: THE JUPITER CAPTURE

In this section the first part of the trajectory, i.e. the Jupiter capture, is described as a global optimization problem and is indicated with $P_1(m)$. Formally, we define problem $P_1$ as a global optimization problem:

$$\text{find: } x \in \mathbb{R}^{16} \quad P_1(m): \begin{align*}
\text{to minimize:} & \quad \Delta V(x, m) = \sum_{i=0}^{3} \Delta V_i \\
\text{subject to:} & \quad lb \leq x \leq ub \\
& \quad \Delta V_i < c_T \alpha_{\text{max}}
\end{align*}$$

Consider the set $\mathcal{M} := \{i, e, g, c\}$ containing the four Galilean moons $i = Io, e = Ganymede, g = Callisto$. Given a sequence $m \in \mathcal{M}^4$ we want to find a vector $x$ encoding an interplanetary trajectory that visits, in sequence, the moons $m$ making use of maximum one deep space maneuver between two successive moons and one at the very beginning (at 1000 Jupiter Radii (JR) where the spacecraft is prescribed to start its journey [12]). The cumulative strength of the four allowed deep space maneuvers $\Delta V = \sum_{i=0}^{3} \Delta V_i$ is minimized and a maximum acceleration constraint on each $\Delta V_i$ is considered to account for the maximum acceleration $\alpha_{\text{max}}$ defined as $\tau_{\text{max}}/M$, where $M$ is the spacecraft mass. A detailed description of the decision vector $x$ (here also called trajectory encoding), of the objective function $\Delta V(x, m)$ and of the problem constraints follows.

2.1 Trajectory encoding

The capture trajectory is encoded into $x$ using a modification of the MGA-1DSM encoding described in [8]. The modification is necessary to account for the fact that the spacecraft starts its journey from a point at $R_s = 1000$ Jupiter Radii (JR) from Jupiter center and not from a planet as assumed in the original MGA-1DSM encoding. Using the concatenation operator "+" to highlight the four separate trajectory legs, a capture trajectory is encoded as follows: $x = [t_0, u, v, T_0] + [\beta_1, h_1, \eta_1, T_1] + [\beta_2, h_2, \eta_2, T_2] + [\beta_3, h_3, \eta_3, T_3]$, where all symbols are explained below.

2.1.1 1st leg

The launch date is encoded into $t_0$ as a continuous variable using the Modified Julian Date 2000 (MJD2000), defined as the number of days passed since 2000-01-01 at 00:00:00 hours. The starting position vector $r_0$ of the spacecraft is encoded by the next two variables $u, v$ as follow:

$$r_0 = 1000 R_s (\cos \theta \cos \phi + \sin \theta \cos \phi_3 + \sin \phi \mathbf{e}_k)$$

where $R_s$ is the Jupiter average radius and

$$\theta = 2 \pi u, \phi = \cos^{-1}(2v - 1) - \pi/2$$

The use of the variables, $u$ and $v$, is motivated by the need to have the position vector $r_0$ uniformly distributed over the starting sphere of radius $R_s$ when the encoding variables are sampled at random. The distribution would not be uniform if we were using directly $\theta$ and $\phi$. Finally the total duration of the first leg is encoded in $T_0$.

2.1.2 Following legs

For each of the following three legs, the starting velocity $v_{\text{out}}$ (outgoing spacecraft velocity at the moon $m$ in $F_m$), is encoded by the first two variables $\beta, h$ as follows:

$$v_{\text{out}} = v_{\text{in}} + |v_{\text{in}}| (\cos(\delta) \mathbf{e}_1 + \cos(\beta) \sin(\delta) \mathbf{e}_2 + \sin(\beta) \sin(\delta) \mathbf{e}_3) \quad (1)$$

A trajectory leg is a part of the whole trajectory between two consecutive moons.
Table 1: Lower and upper bounds, lb and ub, for the generic instance of $P_i$.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Lower</th>
<th>Upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_0$ [MJD 2000]</td>
<td>7305</td>
<td>11323</td>
</tr>
<tr>
<td>$u$</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$v$</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$T_0$ [days]</td>
<td>190</td>
<td>210</td>
</tr>
<tr>
<td>$\beta_i$</td>
<td>$-2\pi$</td>
<td>$2\pi$</td>
</tr>
<tr>
<td>$h_i$ [km]</td>
<td>50</td>
<td>2000</td>
</tr>
<tr>
<td>$\eta_i$</td>
<td>$-2\pi$</td>
<td>$2\pi$</td>
</tr>
<tr>
<td>$T_1$ [days]</td>
<td>0.1</td>
<td>5</td>
</tr>
<tr>
<td>$T_2$ [days]</td>
<td>5</td>
<td>100</td>
</tr>
<tr>
<td>$T_3$ [days]</td>
<td>20</td>
<td>55</td>
</tr>
</tbody>
</table>

where $v_m$ is the velocity of the moon $m$ at the beginning of the leg, $\dot{v}_{in} = v_{in} - v_m$ is the spacecraft velocity at the end of the previous leg (incoming velocity at the moon) and:

$$\hat{e}_1 = \dot{v}_{in}/|\dot{v}_{in}|$$
$$\hat{e}_2 = (\dot{e}_1 \times v_m)/(|\dot{e}_1| \times v_m)$$
$$\hat{e}_3 = \hat{e}_2$$

where $R_m$ is the moon radius and $\mu_m$, its non dimensional gravity parameter. Next, the variable $\eta_i$ encodes the position of a deep space maneuver which occurs at $t = t_0 + \sum_{j=0}^{i-1} T_j + \eta_i T_i$, and $T_i$ encodes the total leg duration.

2.2 Objective function

The objective function for problem $P_1$ is computed as follows. A single revolution Lambert’s problem [2] is solved between $r_0$ and $r_1$ with transfer time $T_0$. $r_1$ is the position of the first encountered moon at $t_0 + T_1$. The solution to the Lambert’s problem returns $v_0$ and $v_1$, i.e. the velocities at the beginning and at the end of the first trajectory leg. From these results, we compute $\Delta V_0 = abs(|v_0| - |v_1|)$, where $\Delta V_0$ is the magnitude of the prescribed starting velocity (i.e. 3.5 [km/s]). For each successive leg we then compute the velocity increments $\Delta V_i$ corresponding to the deep-space maneuver location encoded in $x$ using the standard procedure employed by the MGA-1DSM encoding [8]. The final objective function is the sum of all computed velocity increments, i.e., $\Delta V = \sum_{i=0}^{3} \Delta V_i$.

2.3 Constraints

The box bounds $lb$ and $ub$ used in problem $P_1$ are reported in Table 1. The most important bounds are those on the transfer times $T_i$. They, alone, select the type of capture we are looking for. In our case, we want to be captured in the Jupiter system in the shortest possible time and we aim to acquire a short orbital period to increase the frequency we will be able to score points with. We achieve this goal by forcing two moons to be visited on the very first revolution in a rapid succession and we thus select a very small upper bound on $T_i$. The possibility to have these types of “multiple-satellite-aided Jupiter capture trajectories” is discussed thoroughly in [11]. We then force the following orbit to have a relatively small period with an upper bound of 100 days on $T_2$, thus allowing, in the second leg, a $13:1$ resonance with the Ganymede orbit or a $5:1$ with Callisto. The upper bound on $T_3$, is then selected to be lower, i.e. 55 days, to complete the spacecraft insertion in the Galilean moon system (allowing only $3:1$ resonances with the outer moon Callisto$^5$).

The non linear constraint on the spacecraft $\Delta V_i$ accounts for the maximum acceleration $\tau_{max}$ allowed by the on-board spacecraft electric propulsion system. A constant $c$, here set to be 0.1, is also introduced to account for orbital effects while biasing the search towards purely ballistic captures (corresponding to $\Delta V = 0$)$^6$.

3. PROBLEM P$_2$: SINGLE TRANSFER BETWEEN MOONS

A fundamental building block to our overall strategy is a second global optimization problem we here indicate as $P_2$. This represents the problem of finding a minimum $\Delta V$ transfer between two moons $m_a$ and $m_{next}$ fixing the starting conditions. The transfer starts at $t_0$ from $m_a$ with a known absolute velocity $v_{in}$ and makes a fly-by over the $F_i$ face of $m_{next}$.

$$P_2(F_i, m_{next}) : \begin{array}{l}
\text{find: } x \in R^4 \\
\text{subject to: } lb_i(F_i) \leq x \leq ub_i(F_i)
\end{array}$$

where $x = [\beta, h, \eta, T]$ and, most importantly, the box bounds $lb$ and $ub$ depend on the face $F_i$ to be visited. While any particular instance of the problem $P_2$ depends on $F_i$, $m_{next}$, $m_a$, $v_{in}$ and $t_0$, we write $P_2(F_i, m_{next})$, neglecting those variables ($m_a, v_{in}, t_0$) that are fixed when a previous part of the trajectory exists (e.g. a solution to $P_1$ or to $P_2$).

The objective function $\Delta V(x)$ is computed by first computing $\Delta V_0$ from $v_{in}$ using Eq.(1-2), and then by propagating those initial conditions with Keplerian dynamics for the time $\eta T$. Finally, a single revolution Lambert’s problem [2] is solved to get to the target moon position in $t_0 + T$.

3.1 Face-targeting

When instantiating $P_2(F_i, m_{next})$ we need to specify the lower and upper bounds on the four variables $\beta, h, \eta, T$. The choice on the bounds on $\eta$, i.e. the position of the deep space maneuver, is rather simple: $\eta \in [0, 1]$ is set to allow small correction maneuvers at any points of the trajectory leg. For the transfer time, we use $T \in [0.1, \min(4T_m, 40)]$ (days), where $T_m$ is the largest period between the starting and final moon orbit. The maximum of 40 (days) on the upper bound avoids to consider long multiple revolution transfers when Callisto is either the starting or the targeted moon ($4T_m$ would then be roughly 64 days). The bounds on the remaining two variables, $\beta$ and $h$ are, instead, decided to have the resulting solution visit the chosen moon face $F_i$. The procedure to find these bounds is what we call face-targeting and it is based on the following results:

$^5$ The orbital period of Callisto is 16.7 days, while the orbital period of Ganymede is 7.15 days.

$^6$ The coefficient $c$ is often referred to gravity loss and, coarsely, accounts for the difference between having to accumulate a given velocity difference in a flat space or in a central gravity field.
Find visitable faces

Algorithm 1

\[ L := \emptyset \]

for all \( F_i \) in the moon \( m \) do
  for all \( p_j \in F_i \) do
    check if \( p_j \) is in the near side or above, below or in the visitable band.
  end for
  if (at least one vertex is visitable) or (at least one, but not all vertices are above the band) then set \( L = L \cup \{ F_i \} \)
end if
return \( L \)

Given a face \( F_i \in L \), we may now find the lower and upper bounds \( \beta_m, \beta_M, h_m, h_M \) that bracket such a face and bias the solution of the problem \( P_2 \) to fly over \( F_i \) and thus to score that face (if not scored already). We do so by using algorithm 2.

Algorithm 2

Face bracketing

Let \( F_i \in L \)

for all \( p_j \in F_i \) do
  If \( p_j \) is on the near side, project it to the far side.
  Compute \( h_j = \mu_m/(\hat{v}_{in} \cdot \hat{v}_{out})(1/\sin(\delta/2) - 1) - R_m \)
  Compute \( \beta_j = \arctan(\hat{v}_{in} \cdot \hat{v}_{out} \cdot \hat{e}_3) \)
end for

set \( \beta_m = \min_{j} \beta_j, \beta_M = \max_{j} \beta_j \)
set \( h_m = \max(\min_{j} h_j, h_{min}), h_M = \min(\max_{j} h_j, h_{max}) \)

4. SOLVING \( P_1 \) AND \( P_2 \)

All problems that derive from different instances of \( P_1(m) \) and \( P_2(F_i, m_{next}) \) are single objective, continuous global optimization problems that we solve by exploiting one powerful idea rooted in evolutionary computations: self-adaptation. By using self-adaptation we are able to not care about tuning algorithmic parameters, while still obtaining solutions for all different problems with an acceptable efficiency. This is a fundamental advantage as for each instance of \( P_1(m) \) or \( P_2(F_i, m_{next}) \) we are, essentially, creating a much different objective function landscape. Resonant transfers, multiple revolution transfers and very short transfers may or may not exist according to the particular instance, drastically changing the fitness landscape.

In the case of \( P_1 \) (a more difficult problem with a higher dimensionality with respect to \( P_2 \) and a much larger search space mainly determined by the wide bounds on the \( t_0 \), the launch window), we also make use of a second tool: the asynchronous island model. This allows us to achieve a coarse-grained parallelization while introducing a migration operator that further accelerates evolution.

We tested three algorithms employing self-adaptation as a principle, a) differential evolution with self adaptation strategy (jDE) [3], the Corana’s Adaptive Neighbourhood Simulated Annealing (SA-AN) [4], and Covariance Matrix Adaptation Evolutionary Strategy (CMA-ES) [6]. We do not have space here to discuss the performances of these algorithms in connection with our problems, but we did perform extensive test sessions and our final decision converged on jDE as for its clear superiority in the context of \( P_1 \), where a good interaction with the asynchronous generalized island model is necessary to actually find any good solutions, and for its low run time in general.

It is worth anticipating here that, as explained in detail in a later section, to assemble one entire solution of the Jupiter’s moon tour, up to 1,000,000 different instances of \( P_2 \) need to be solved and one good solution to \( P_1 \) has to be determined. To give an idea of the performances of our chosen set up, we report, preliminarily, some results on the resulting computational efficiency. Figure 3 illustrates the results of 50 runs on three different instances of \( P_1(m) \), which consistently resulted in good final results: a) \( m = [g, c, c, c] \), b) \( m = [c, g, g, g] \), c) \( m = [g, i, i, i] \). Each run is made up to convergence, which we check every 40 generations and is defined by two criteria: fitness variability and chromosomic

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Figure 2: Visualization of the visitable band.

**Definition 1.** A point \( \hat{p} \) on the moon surface of radius \( R_m \) is visitable during a fly-by with incoming relative velocity \( \hat{v}_{in} \) if and only if \( \hat{v}_{in} \cdot \hat{p} \geq 0 \) and \( h \in [h_{min}, h_{max}] \), where:

\[
\begin{align*}
  h + R_m &= \mu_m/(\hat{v}_{out} \cdot \hat{v}_{out})(1/\sin(\delta/2) - 1) \\
  \hat{v}_{out} &= \hat{v}_{in} - 2(\hat{v}_{in} \cdot \hat{p})\hat{p} \\
  \delta &= \arccos\left(\frac{\hat{v}_{in} \cdot \hat{v}_{out}}{\hat{v}_{in} \cdot \hat{v}_{out}}\right)
\end{align*}
\]

and \( h_{min}, h_{max} \) were prescribed by the JPL problem statement [12] as the minimum and maximum fly-by altitude on each moon. If \( \hat{v}_{in} \cdot \hat{p} < 0 \) the vertex is said to be in the near side of the moon. For the cases in which \( \hat{v}_{in} \cdot \hat{p} \geq 0 \), if \( h < h_{min} \) the vertex is said to be above the visitable band, if \( h > h_{max} \) it is said to be below.

If a point is visitable according to the above definition, then a moon-centric hyperbola exists and has its closest approach radius \( r_p \) aligned with \( \hat{p} \). Essentially, at each fly-by the moon surface can be represented as in Figure 2, where the area made by all visitable points is shown to form a band (the visitable band) that is symmetric with respect to the incoming relative velocity and that is placed in the far side of the moon.

**Definition 2.** A face \( F := \{ \hat{p}_j, j = 1..n \} \) defined by its \( n \) vertices is visitable if and only if \( a_j \geq 0, j = 1..n \) exist such that the point \( \sum_{j=1}^{n} a_j \hat{p}_j \) is visitable.

In other words, a face is visitable if at least one of its points is visitable. In order to compute the set \( L \) containing all visitable faces, one can use the following algorithm:

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Our experiments are done on a Intel(R) Xeon(R) CPU - X5355 @ 2.66GHz.
variability \( f_{\text{tot}} \leq 1 \times 10^{-2} \) and \( x_{\text{tot}} \leq 1 \times 10^{-3} \). Eight islands are used and one individual is migrated along a ring topology each 100 generations. Each island evolves a population of 20 individuals. The asynchronous migration model implementation and the jDE implementation are made available via the open source PaGMO/PyGMO project\(^8\). In the case of \( P_2 \), where one single island is used (i.e. no migration), we report that our set-up is able, on average, to solve 10 problem instances each second.

5. ASSEMBLING ONE MOON TOUR

Our overall strategy to solve the GTOC6 problem consists of assembling in cascade multiple solutions to problems \( P_1 \) and \( P_2 \). We start with a trajectory that captures our spacecraft in the Jupiter system (i.e. one solution to \( P_1(\mathbf{m}) \)), and we append to it, one by one, solutions to \( P_2(F_i, m_{\text{next}}) \). As there are four moons and, on average, 15 visitable faces, the average branching factor is 60. This means that at the \( n^{16} \) step a full search tree contains \( 60^n \) trajectories. This creates a tree \( T \) of possibilities that soon becomes intractable.

In our notation, the generic tree node is defined by \( N := \{m, t, v_m, \Delta V, f\} \) and represents an interplanetary trajectory (solution to the GTOC6 problem), that at the epoch \( t \) visits the moon \( m \) with an incoming velocity \( v_m \), having cumulated a velocity change of \( \Delta V \) and having previously visited all moon faces in \( f \). Each node can thus be assigned a score that is computed as a function of \( \text{Algorithm 3} \) find a capture trajectory \( N_{\text{root}} \)

\[
\text{while } \Delta V \geq 100 \text{ (m/s)} \text{ and } \Delta T = t - t_0 \geq 0.9 \text{ (years)} \text{ do}
\]

\[
\text{pick a random } \mathbf{m} \in M^4
\]

\[
\text{solve } P_1(\mathbf{m})
\]

\[
\text{end while}
\]

\[
\text{let } m, t \text{ be the last moon in } \mathbf{m},
\]

\[
\text{let } t \text{ be the epoch at the last moon,}
\]

\[
\text{let } v_m \text{ be the spacecraft velocity at } t,
\]

\[
\text{let } \Delta V \text{ be the cumulative velocity increment,}
\]

\[
\text{let } f \text{ the set of all moon faces visited,}
\]

\[
\text{let } N_{\text{root}} := \{m, t, v_m, \Delta V, f\}
\]

5.1 The lazy race tree search

As explained above, starting from a root node \( N_{\text{root}} \) that is an outcome of Algorithm 3, we define a tree \( T \) where each node \( N \) represents a solution (i.e. a trajectory) to our problem. The tree depth, defined as \( d \), represents the number of fly-bys performed (after capture). The tree dimension make a complete search intractable. We thus need to deploy a different strategy. A first attempt could be to search the tree implementing a depth first search by adding pruning criteria to make sure to trim uninteresting branches and to complete the search in a reasonable time. The advantage would be to prioritize an early exploration of the tree leaves. The problem with this approach is that it is impossible to estimate how much of the tree would be pruned out by a certain criterion, and thus each search could easily be too short or too long. A breadth first search strategy, carrying over to the next depth a constant number of nodes that is selected by a ranking criteria, solves this problem having a time complexity that grows linearly with the tree depth, which is a characteristic that is highly desirable in our case. In such an algorithm, nodes that have equal depth (i.e. number of fly-bys) need to be ranked and we would then be forced to compare trajectories that possess different times of flight, cumulative \( \Delta V \) and score using an aggregated objective function, which introduces a lot of greediness in the search\(^5\). We thus introduce a novel algorithm that we call Lazy Race Tree Search (LRTS) able to overcome the problems mentioned above. The basic idea, outlined in Algorithm 4, is to only rank (and eventually select and prune) nodes that have similar time of flight, not equal tree depth. Introducing the set \( U \) containing all unbranched nodes, initialized to a set containing only the root node \( N_0 \), at each iteration we look in \( U \) for nodes having time of flight smaller than the minimum time of flight \( T_0 \) among them plus a small \( \epsilon \). We then select \( b f \) (branching factor) nodes to branch, using the ranking criteria, and we remove all selected nodes from \( U \). The algorithm stops when \( T_0 \) is larger than the 4 years allowed by the problem definition.

6. TREE SEARCH RESULTS

In order to assess the performance of LRTS on our problem, while also trying to beat the best solution known for

\(^8\)http://pagmo.sourceforge.net/pygmo/index.html

\(^5\)In a preliminary implementation of this algorithm we could not reach the score threshold of 300
Algorithm 4 Lazy Race Tree Search

\[ \mathcal{U} := \{ \mathcal{N}_{\text{root}} \} \text{ (the set of unbranched nodes)} \]

\[ \text{let } t_0 \text{ be the starting epoch of the trajectory} \]

\[ \text{while } T_0 \leq 4 \text{ years do} \]

\[ \text{let } T_0 := \min \{ t - t_0 \} \]

\[ \text{let } \mathcal{S} := \{ \mathcal{N} \in \mathcal{U} \mid (t - t_0) \in [T_0, T_0 + 1] \} \]

\[ \text{select } b_f \text{ nodes in } \mathcal{S} \text{ and branch them} \]

\[ \text{remove from } \mathcal{U} \text{ all nodes in } \mathcal{S} \]

end while

the GTOC6 problem (i.e., the trajectory scoring 311 found by the group at Polytechnic of Turin and La Sapienza, University of Rome), we launch a large number (360) of tree searches. The LRTS uses a branching factor \( b_f = 50 \), a time of flight bin of \( \epsilon = 5 \) days and a ranking method defined by the trajectory score augmented by the maximum potential number of points scored on the current moon \( m \) (with ties broken by a secondary ranking method defined as the number of non scoring fly-bys). A gravity loss coefficient of \( c = 0.1 \) is used when instantiating the various \( P_2 \). Each search starts from a different root node \( \mathcal{N}_{\text{root}} \) found by running once Algorithm 3. The results are summarized in Figure 4. Most trajectories found are above the threshold of 300 points, some (34 to be exact) have a higher score than the currently known best of 311. We here note that the overall trajectory \( \Delta V \) is consistently very small thanks to the pruning at 100 m/s in Algorithm 3 and to the very small gravity loss coefficient \( c \) selected. As a consequence, all solutions found by LRTS can be transformed into low-thrust trajectories (i.e. trajectories that can be flown by an electric propulsion system such as that prescribed by the Jet Propulsion Laboratory problem statement). On average, one tree search is completed in 16.8 hours. The average depth of the searched tree is 132. Using 32 CPUs, the whole run lasts 7.87 days. During the entire run an estimated number of half a billion (500,000,000) different global optimization problems (of type Multiple Gravity Assist with one Deep Space Maneuver MGA-1DSM) are solved by the self-adaptive jDE deployed in our asynchronous island model. It is interesting to note how only the sequences \( m = [c, g, g, g] \), \( m = [g, i, i, c] \), \( m = [g, i, i, g] \), \( m = [g, i, i, i] \), \( m = [c, g, c, c] \), \( m = [c, i, c, c] \), \( m = [g, c, c, c] \), \( m = [g, c, c, c] \) were found to be able to define a suitable capture trajectory and, in any case, all best trajectories used the Callisto, Ganymede, Ganymede, Ganymede sequence, indicating that such an entry strategy results in particularly good root nodes for LRTS.

7. Our Solution

In this section, we describe in more details the solution with the highest score found by our tree search (i.e. the left most point in Figure 4). It has a total score of 316 out of the maximum score of 324, visiting 120 moon faces against the 115 faces visited by the previously best known solution (winner of the 6th edition of the GTOC which scores 311). From Table 2, we note that our solution almost maps all faces on three of the four moons: full scores on Europa and Ganymede and one point less on Io. The solution was sent according to the required competition format to the Jet Propulsion Laboratory experts (the GTOC6 organizers) who kindly validated the new solution using their trajectory analysis tools.

The scores on each flyby of the moons are plotted in Fig. 5, in which different colors indicates the points received: 3, 2, 1, or 0 for flyby on faces visited before, while Europa receives double points. From the plot, we notice that the flyby sequence and scores obtained are quite well structured instead of scattered evenly. The flybys on the same moon and scores are clustered together. They are also mostly ordered from first visiting the high score faces then the low score faces. From the dynamical point of view, visiting the same moon repeatedly implies the use of a resonance transfer (i.e. integer ratio of the orbital periods of the spacecraft and the moon) and is rather easy to obtain, while switching between moons may result in a time gain, but it is subject to stricter planetary phasing consideration and may thus cost more \( \Delta V \) or just be not possible. Our solution clearly makes use of both possibilities trying to take advantage of quick moon

### Table 2: Summary of our Jovian tour.

<table>
<thead>
<tr>
<th>Moon</th>
<th>No. of Flybys</th>
<th>Scores Obtained/Max. Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Io</td>
<td>37</td>
<td>67/68</td>
</tr>
<tr>
<td>Europa</td>
<td>39</td>
<td>136/136</td>
</tr>
<tr>
<td>Ganymede</td>
<td>40</td>
<td>60/60</td>
</tr>
<tr>
<td>Callisto</td>
<td>25</td>
<td>53/60</td>
</tr>
<tr>
<td>Total</td>
<td>141</td>
<td>316/324</td>
</tr>
</tbody>
</table>
switches as much as possible and otherwise mapping the same moon on resonances. This overall strategy, which we call moon-hopping, is very different from that employed by the GTOC6 winning trajectory that, instead, mapped one moon at a time.

From Fig. 5 we observe how the whole trajectory can be divided into four phases. In Phase 1, starting from a fast Callisto-Ganymede flyby using a 2.3 days long transfer, the spacecraft is captured in the Jovian system. A few more flybys at Ganymede are then able to further reduce the orbital period, allowing a visit to the high-score faces of the inner moons, Io and Europa. Phase 2 consists of mostly high-score flybys at Ganymede, mixed with flybys at the mid-score faces of Io. In the 3rd phase, the spacecraft visits Callisto and Io on faces with high and low scores, respectively. The remaining low-scores faces in Europa, Ganymede, and Callisto are visited in the final phase. The trajectories of the 4 phases are plotted in Figure 6.

Solutions from the tree searches are under an impulsive MGA-1DSM model, which is a good approximation of the low-thrust propulsion model. In the MGA-1DSM model, our solution has a total ∆V of 83 m/s, which is mostly used in the first leg for the capture and that implies the rest of the trajectory is basically ballistic. We optimized this solution for final mass in a continuous low-thrust model [15], as required by the problem. As the trajectories coming out from the search are mainly ballistic (i.e. very little ∆V is used) we did not encounter any problem in such a low-thrust conversion. The final mass of the trajectory is 1036 kg and the total time of flight is 3.9 years. The final mass is mainly accounting for the mass penalty that needs to be assigned at each successive Jupiter close encounter as described in the JPL problem statement [12]. We did not check such a penalty during the tree search, only at the end. As a consequence some of the trajectories (roughly one out of ten) in Figure 4 are actually infeasible for the GTOC6 problem (luckily not the highest ranked one).

8. CONCLUSIONS

The design of extremely complex interplanetary trajectories including dozens of multiple gravity assists may be performed autonomously by computers with the aid of carefully planned searches powered by evolutionary algorithms. We consider the case of the GTOC6 challenge proposed by the Jet Propulsion Laboratory and improve significantly over the best known solution (winner of the competition) using a new completely automated scheme that makes extensive use of self-adaptation and of the asynchronous island model. Compared to the previous best solution the trajectory found by our computers appears to be using a quite different orbital strategy, frequently hopping between moons, which is of interest to interplanetary trajectory designers.

9. ACKNOWLEDGMENTS

We thank Dr. Anastassios Petropoulos from Jet Propulsion Laboratory (the organizer of GTOC6) for having numerically validated our trajectory to be a solution to the GTOC6 problem.

10. REFERENCES

Figure 6: Visualization of the four phases of the moon tour. Phase 1: leg 1-51 (top-left); Phase 2: leg 52-78 (top-right); Phase 3: leg 79-115 (bottom-left); Phase 4: leg 116-141 (bottom-right). Colors on the trajectory represent scores obtained on the upcoming flyby (same color scheme as Fig. 5).