Topological analysis of public transport networks’ recoverability

NAS group seminar

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Margarita-Salas call for postdoctoral research abroad by Universidad de Cádiz (funded by NextGenerationEU).

2 years @ TU Delft + 1 year @ Universidad de Cantabria.

Joined the Smart Public Transport Lab (T&P department @ CiTG) on Feb 2022, supervised by Oded Cats.
**Motivation**
Reducing the impact of disruptions is key for providing attractive PT services.

Many works assessing vulnerability and robustness of PTN.

Little is known on the topological aspects of the recovery process once disruptions occur.
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Inspiration
We study the notion of recoverability, inspired by the work of Sun et al. (2021) for optical networks.
Problem formulation

- $G_0(N, L_0)$ be the $L$-space representation of a PTN
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- Rebound ratio:
  $$\eta = \frac{S_r}{S_f}$$
Performance metrics and failure/recovery strategies

Performance metrics

Connectivity

\[ C_G = \frac{\sum_{i \neq j \in G} 1_{\text{exists a path } (i,j)}}{N \times (N - 1)} \]

Efficiency

\[ E_G = \frac{\sum_{i \neq j \in G} 1/\text{travel time}(i,j)}{N \times (N - 1)} \]
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Failure/recovery strategies

- *Random*: links are iteratively removed/addicted to the graph randomly and uniformly.
- *Greedy (C_G)*: selects the link to remove/add that would render the largest decrease/increase in \( C_G \).
- *Greedy (E_G)*: same but with the \( E_G \) performance metric.
Preliminary results

Studied scenarios

(a) Berlin (172 nodes, 183 links)  
(b) Paris (302 nodes, 359 links)

**Figure:** Metro networks used in the experiments

**Experiments**
We simulated a random failure process of $K = 30$ steps on each city and report the average results for 10 independent executions.
 Preliminary results

Figure: \( C_G \) metric

\[
\begin{array}{|c|c|c|c|}
\hline
\text{city} & \text{metric} & \text{recovery strategy} & \\
& & \text{random} & \text{greedy} C_G & \text{greedy} E_G \\
\hline
\text{Berlin} & C_G & 0.833 (12.748;10.619) & 1.141 (12.748;14.545) & 1.060 (12.748;13.500) \\
& E_G & 0.904 (9.897;8.943) & 1.098 (9.897;10.869) & 1.102 (9.897;10.910) \\
\hline
\text{Paris} & C_G & 0.725 (3.121;2.263) & 1.591 (3.121;4.968) & 1.321 (3.121;4.122) \\
& E_G & 0.938 (3.474;3.260) & 1.240 (3.474;4.307) & 1.293 (3.474;4.491) \\
\hline
\end{array}
\]

Table: Average rebound ratio (\( \eta \)), performance loss (\( S_f \)), and gain (\( S_r \)) with random failure and varying recovery strategies. Notation: \( \eta \ (S_f;S_r) \).
Preliminary results

**Figure: $E_G$ metric**

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**Table:** Average rebound ratio ($\eta$), performance loss ($S_f$), and gain ($S_r$) with random failure and varying recovery strategies. Notation: $\eta (S_f;S_r)$. 
**Outlook**

**Conclusions**
We are starting research on a topological approach to measure recoverability in PTNs inspired by previous works in the field of optical networks.

Preliminary results showed that greedy recovery heuristics are able to rebound quickly from the loss of performance during the failure process.

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**On-going and future work**
- devise other failure and recoverability strategies
- incorporate new performance metrics
- test on a large number of PTNs worldwide
- incorporate equity in the recovery strategy
Thanks for your time :)  

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