Indirect influence in social networks as an induced percolation phenomenon

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Motivation- behaviors can be contagious in social networks
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Publishing behavior in collaboration networks

- Indirect influence in behavior contagion
  - Obesity experiment lasting 32 years
    - person-to-person spread of obesity
    - extended to three degrees of separation
  - Repeatedly found in behaviors of happiness, smoking, drug, alcohol, loneliness, among others.

Motivation- Indirect influence in behavior contagion

Motivation- Indirect influence in behavior contagion

• Indirect influence observed in ecological trait evolution.

Scientists behaviors:

<table>
<thead>
<tr>
<th>State 1</th>
<th>State 0</th>
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Prob. of being in state 1

Motivation- Indirect influence in behavior contagion

• Indirect influence observed in scientific collaboration
Research question

What are the potential underpinning mechanisms for indirect influence in behavior contagion,

so that we can propose models to mimic the contagion process?

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Indirect influence as an induced percolation

- **Model of Induced percolation:**

$$m_i = \text{Max indirect (second) neighbors in state } 1$$

$$k_i = \text{direct neighbors in state } 1$$

$$d_i = \text{second neighbors in state } 1$$

Induced index $$m_i$$: Max indirect (second) neighbors in state 1 of those Direct neighbors in state 1

Indirect influence as an induced percolation

- **Model of Induced percolation:**

Induced index $$m = 2$$

Order parameter GOUT: corresponds to the largest spreading coverage.
Indirect influence as an induced percolation

- **Model of Induced percolation:**

  Induced index $m = 2$

  Maintain

  $x$: Prob. of starting node in state 1
  $y$: Prob. of an end node in state 1
  $P_{\infty}$: Order parameter GOUT

  $$x = \sum_{k_{\text{in}}, k_{\text{out}}} P \left( k_{\text{in}}, k_{\text{out}} \right) \frac{1}{k} \left[ 1 - (1 - y)^{k_{\text{out}}} \right]$$

  $$y = \sum_{k_{\text{in}}, k_{\text{out}}} k_{\text{out}} P \left( k_{\text{in}}, k_{\text{out}} \right) \left( \frac{k_{\text{in}}}{k} \right) \left( 1 - x \right)^{k_{\text{in}} - y} \left[ 1 - \left( 1 - y^{k_{\text{out}}} \right) \right]$$

  $$P_{\infty} = \sum_{k_{\text{in}}, k_{\text{out}}} P \left( k_{\text{in}}, k_{\text{out}} \right) \left[ 1 - (1 - y)^{k_{\text{in}}} \right]$$
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Rich critical behaviors induced by indirect influence

• Order parameter GOUT on directed nets
  – $\langle k \rangle$: average degree
  – $m$: induced index

• GOUT on undirected nets:

J Xie, X Wang, L Fang, JH Zhao, Y Moreno, Y Hu, Induced Percolation on Networked Systems, PNAS, 119(9), 2022.
Rich critical behaviors induced by indirect influence

- GOUT on mixed nets:
  - $\langle k \rangle$: average degree
  - $m$: induced index
  - $p$: proportion of directed links

Rich critical behaviors induced by indirect influence

- Size distribution $P(s)$ of small clusters at the critical point of induced percolation ($m = 4$) on undirected networks.
Rich critical behaviors induced by indirect influence

- Size distribution $P(s)$ of small clusters at the critical point of induced percolation ($m = 4$) on undirected networks.

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Take-away message

- Indirect influence in social networks as an induced percolation phenomenon
- Induced percolation leads to rich critical behaviors depending on a single network parameter.

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