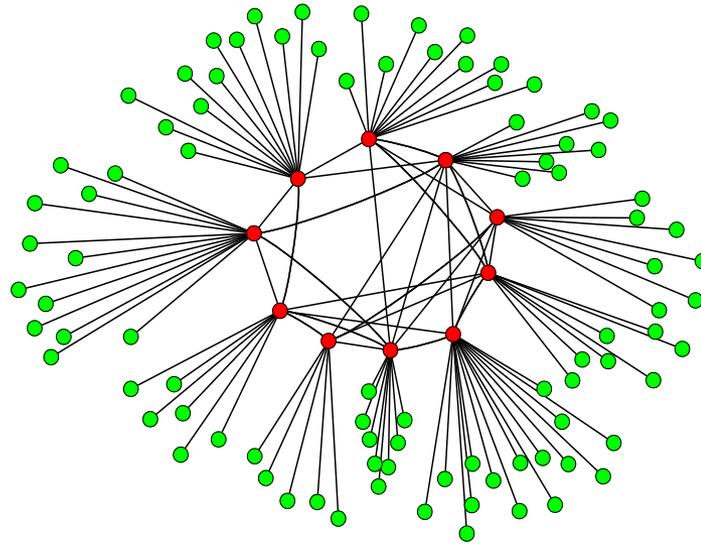


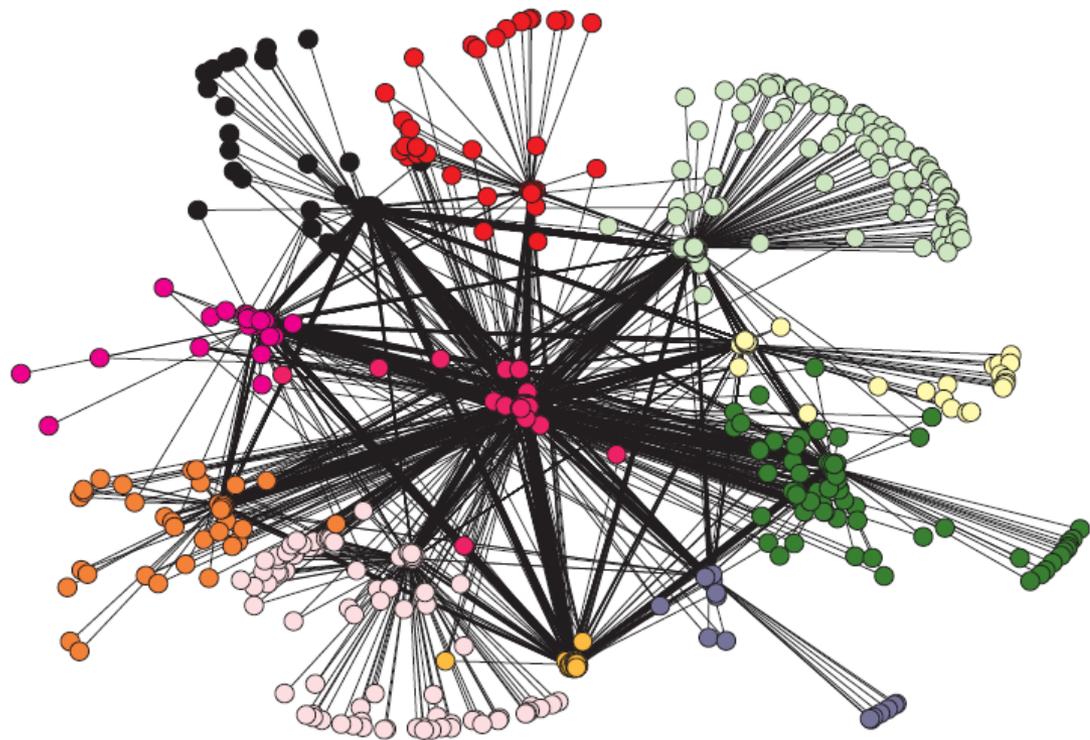
# Network Reconstruction for the prediction of spreading processes



Diego Garlaschelli

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University of Leiden  
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# The challenge: reconstructing (interbank) networks from partial information



## Systemic risk

risk of collapse of entire (financial) system crucially depends on

## network topology

which is however unknown

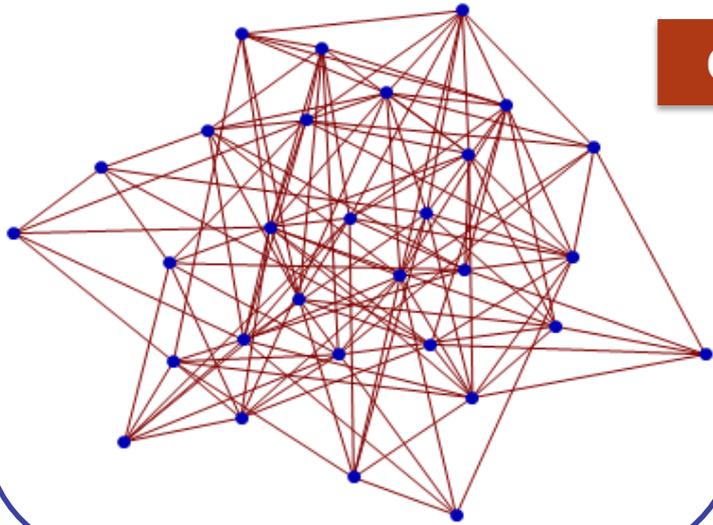
## Public information:

each bank's total exposure towards the **aggregate** of all other banks

## Hidden information:

each bank's individual exposure towards each **single** bank

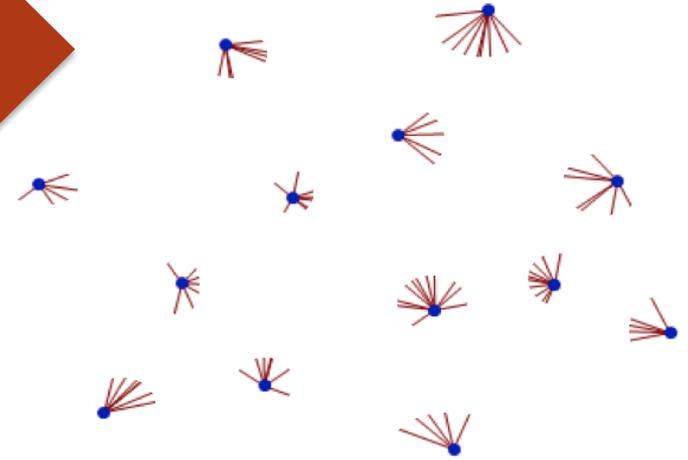
**Original network**  
(unknown/hidden)



$G^*$

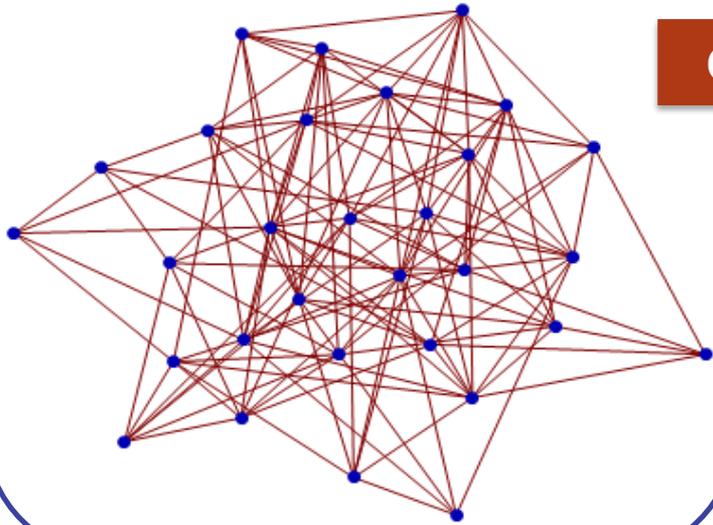
**CONFIDENTIALITY**

**Local properties**  
(known/public)



$\vec{C}(G^*)$

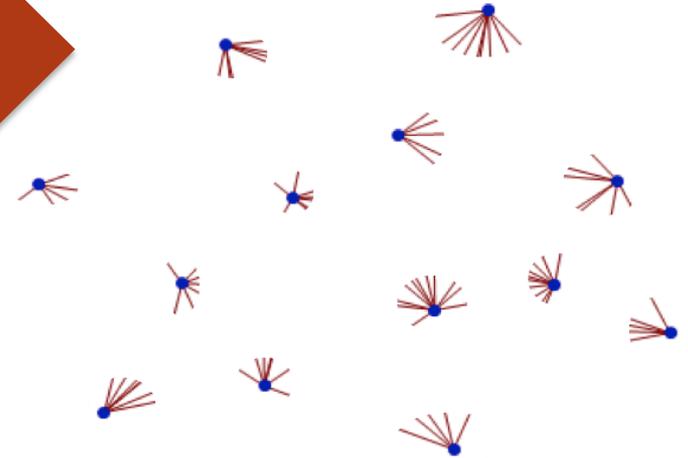
**Original network**  
(unknown/hidden)



$G^*$

**CONFIDENTIALITY**

**Local properties**  
(known/public)



$\vec{C}(G^*)$

**NETWORK RECONSTRUCTION**

**OUR GOAL** : Can we (statistically) reconstruct the original network in such a way that...

- 1) *Privacy is protected*
- 2) *Higher-order effects are correctly predicted*



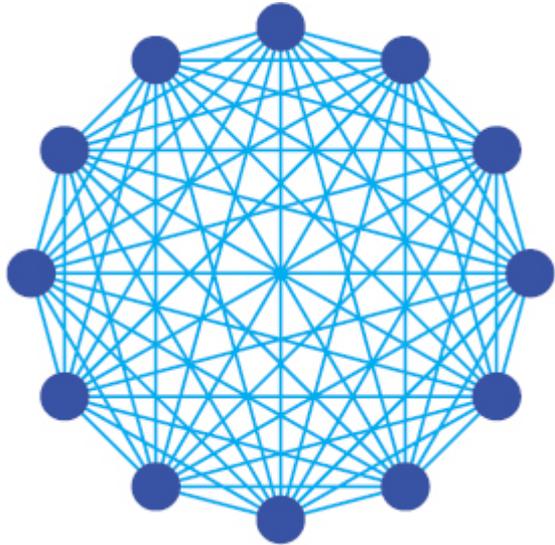
# Traditional deterministic (dense) solution

$w_{i \rightarrow j}$  : true (unknown) link weights of  $G^*$

$\tilde{w}_{i \rightarrow j}$  : reconstructed from margins  $\vec{C}(G^*) = \begin{cases} s_i^{in} = \sum_{j \in V} w_{j \rightarrow i} \\ s_i^{out} = \sum_{j \in V} w_{i \rightarrow j} \end{cases}$

## Traditional approach

(from “strengths” only)



$$\tilde{w}_{i \rightarrow j} = \frac{s_i^{out} s_j^{in}}{W}$$

margins: **OK**, topology: **BAD**

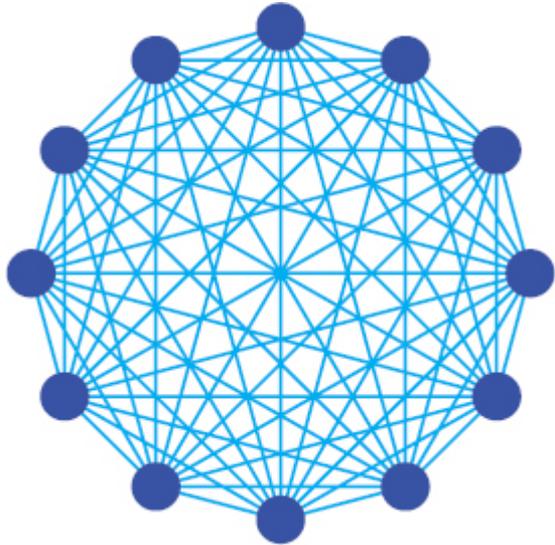
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## Traditional approach

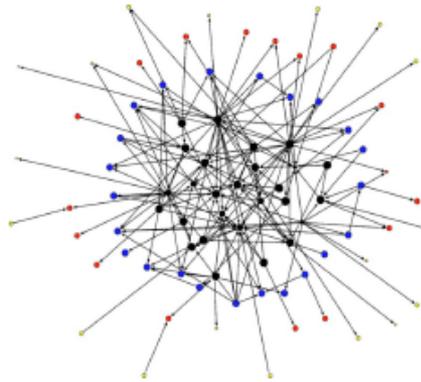
(from “strengths” only)



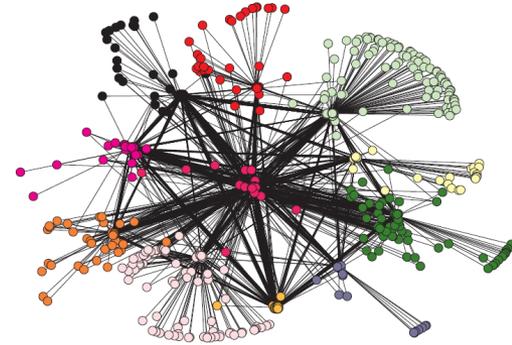
$$\tilde{w}_{i \rightarrow j} = \frac{s_i^{out} s_j^{in}}{W}$$

margins: **OK**, topology: **BAD**

With respect to real networks,  
**links are too many**  
**and thus too weak:**



Italian interbank  
network



Austrian interbank  
network

**⇒ Underestimation  
of systemic risk !**

# Probabilistic method: Maximum-entropy principle

Impose available info as **constraint**

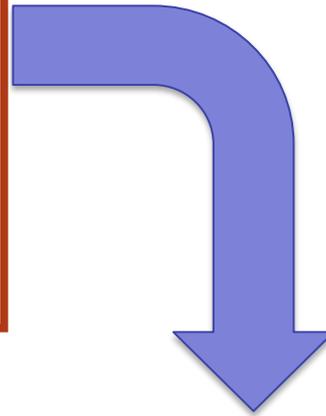
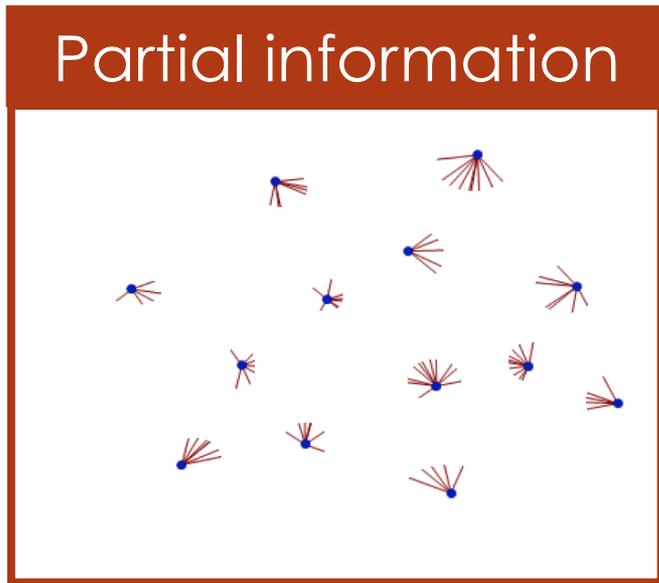
$$\vec{C}(G^*)$$

Maximize Shannon's **entropy**

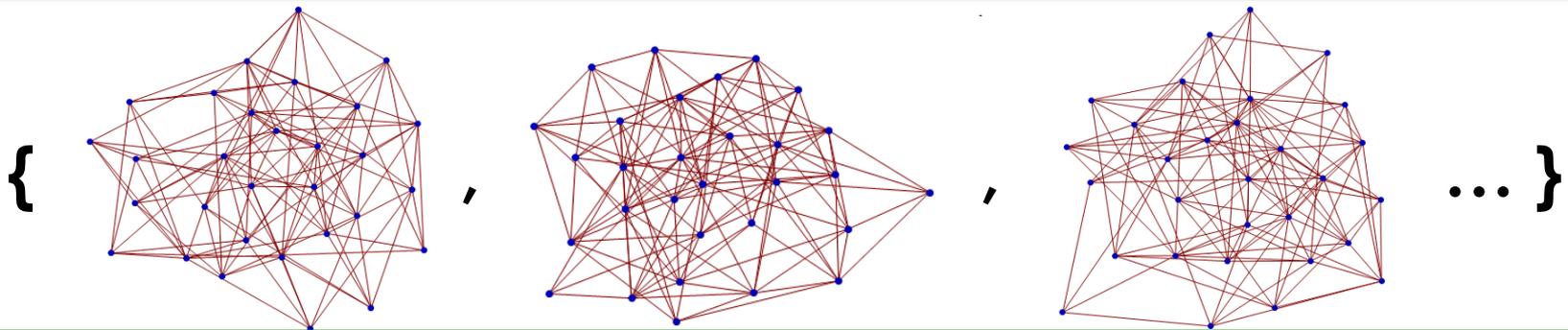
$$S = - \sum_G P(G) \log P(G)$$

Find **unbiased** probability

$$P(G)$$



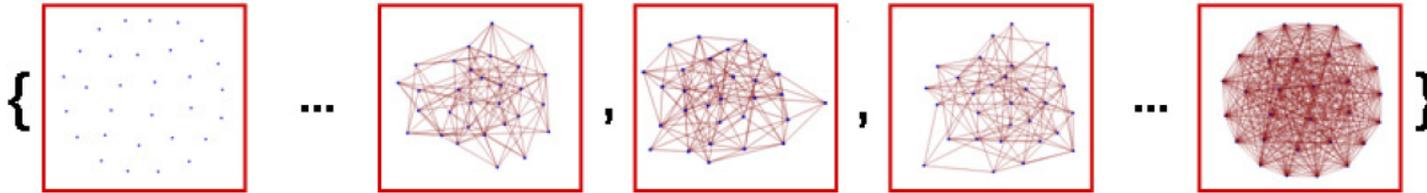
Constrained ensemble of networks



# Microcanonical vs Canonical ensembles

**Microcanonical**  
(hard constraints)

$$P_{\text{mic}}(\mathbf{G}) = \begin{cases} 1/\Omega_{\vec{C}^*}, & \text{if } \vec{C}(\mathbf{G}) = \vec{C}^*, \\ 0, & \text{else,} \end{cases}$$

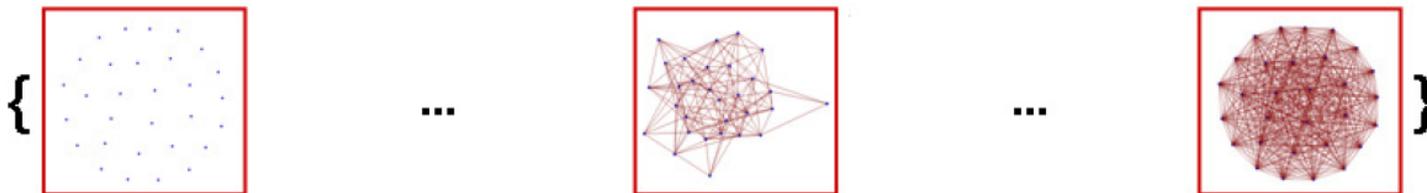


VS

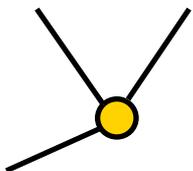
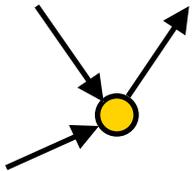
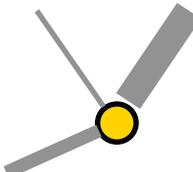
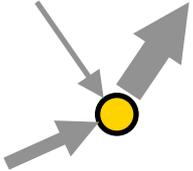
**Canonical**  
(soft constraints)

$$P_{\text{can}}(\mathbf{G}) = \frac{\exp[-H(\mathbf{G}, \vec{\theta}^*)]}{Z(\vec{\theta}^*)}$$

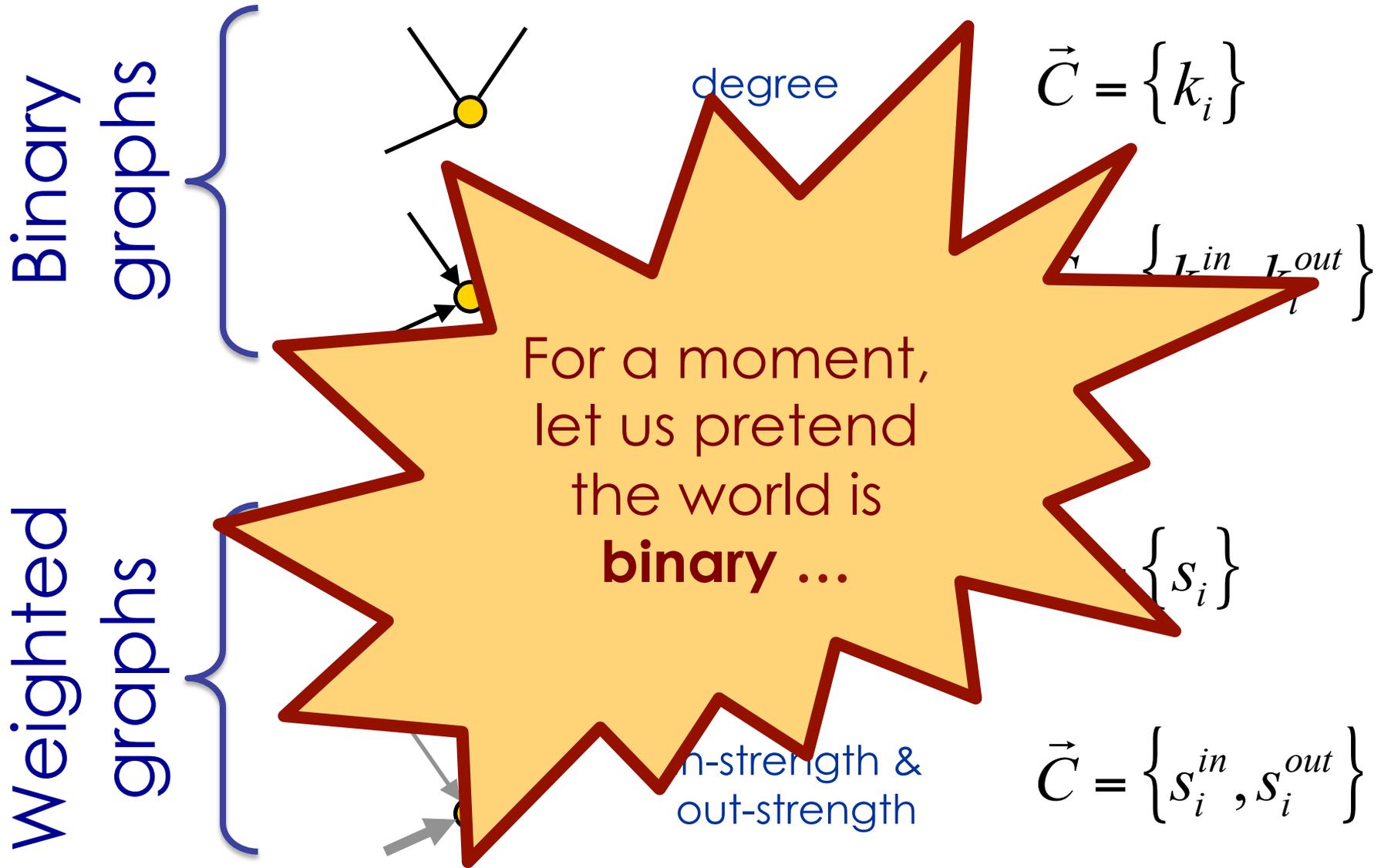
$$H(\mathbf{G}, \vec{\theta}) \equiv \sum_a \theta_a C_a(\mathbf{G})$$



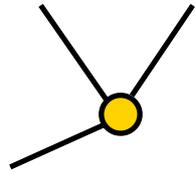
# Possible choices of **local** constraints

Binary graphs		degree	$\vec{C} = \{k_i\}$
		in-degree & out-degree	$\vec{C} = \{k_i^{in}, k_i^{out}\}$
Weighted graphs		strength	$\vec{C} = \{s_i\}$
		in-strength & out-strength	$\vec{C} = \{s_i^{in}, s_i^{out}\}$

# Possible choices of **local** constraints



# Binary configuration model (BCM): fixed degrees



$$\vec{C} = \{k_i\} = \left\{ \sum_{j \neq i} a_{ij} \right\}$$

$$H(G) = \sum_{i=1}^N \alpha_i k_i = \sum_{i < j} a_{ij} (\alpha_i + \alpha_j)$$

$\alpha_{ij} = 0, 1$

$$P(G) = e^{-H(G)} / Z$$

$$= \prod_{i < j} \frac{e^{-(\alpha_i + \alpha_j) a_{ij}}}{1 + e^{-(\alpha_i + \alpha_j)}}$$

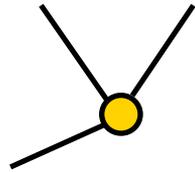
$$= \prod_{i < j} p_{ij}^{a_{ij}} (1 - p_{ij})^{1 - a_{ij}}$$

where

$$p_{ij} = \frac{e^{-(\alpha_i + \alpha_j)}}{1 + e^{-(\alpha_i + \alpha_j)}} = \frac{x_i x_j}{1 + x_i x_j}$$

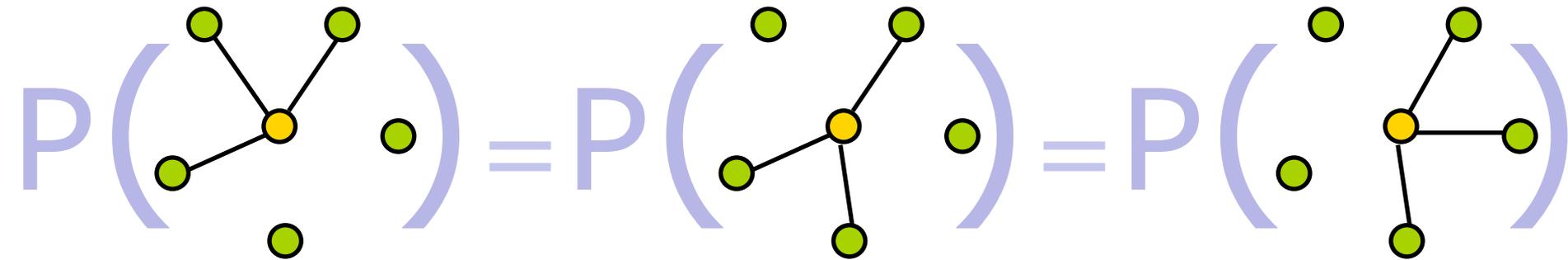
**FERMI-  
DIRAC !**

# Binary configuration model (BCM): fixed degrees



$$\vec{C} = \{k_i\}$$

Equiprobable configurations:

$$P(\text{Diagram 1}) = P(\text{Diagram 2}) = P(\text{Diagram 3})$$


(must hold for all vertices simultaneously)

# Using the BCM for network reconstruction: **Maximum-Likelihood Principle**

Given the real network  $\mathbf{G}^*$   
find  $\theta^*$  that maximizes  $\mathcal{L}(\vec{\theta}) \equiv \ln P(\mathbf{G}^* | \vec{\theta})$

## **For a generic ensemble**

Solution: 
$$\langle \vec{C} \rangle_{\vec{\theta}^*} = \sum_{\mathbf{G}} \vec{C}(\mathbf{G}) P(\mathbf{G} | \vec{\theta}^*) = \vec{C}(\mathbf{G}^*)$$

Garlaschelli & Loffredo, *Phys. Rev. E* **78**, 015101 (2008)

## **For the Binary Configuration Model**

Solution: 
$$\sum_{j \neq i} \frac{x_i^* x_j^*}{1 + x_i^* x_j^*} = k_i(\mathbf{A}^*)$$

Squartini & Garlaschelli, *NJP* **13**, 083001 (2011)

**codes available:**

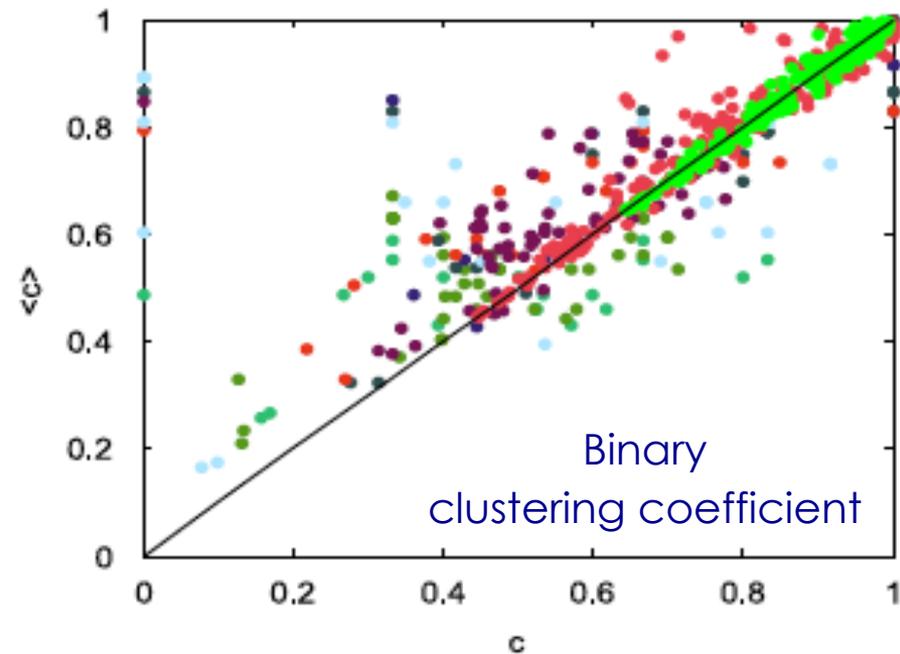
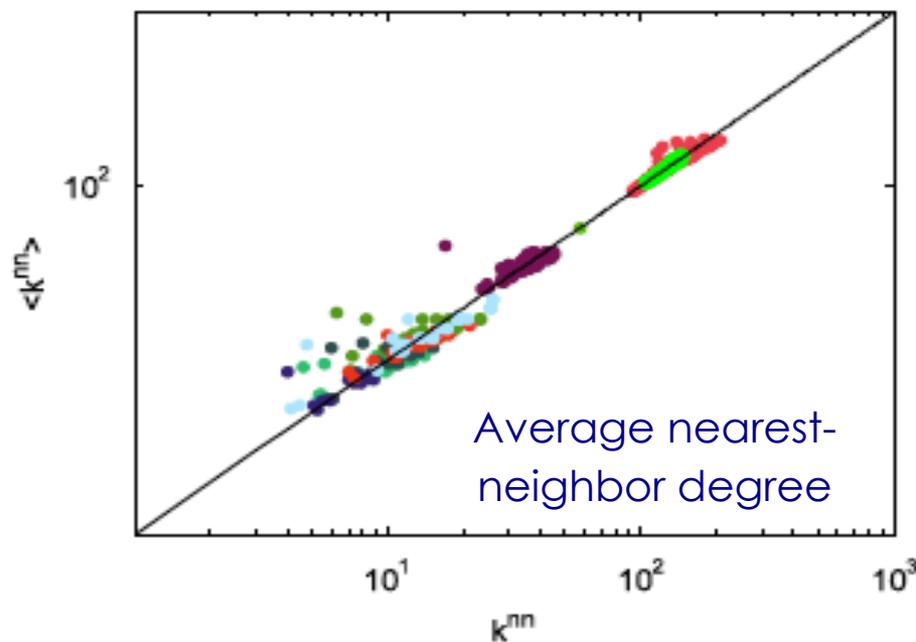
Squartini, Mastrandrea, Garlaschelli, *NJP* **17**, 023052 (2015)

# Using the BCM for network reconstruction

**Result:** good prediction of higher-order properties (from degrees only) in binary graphs

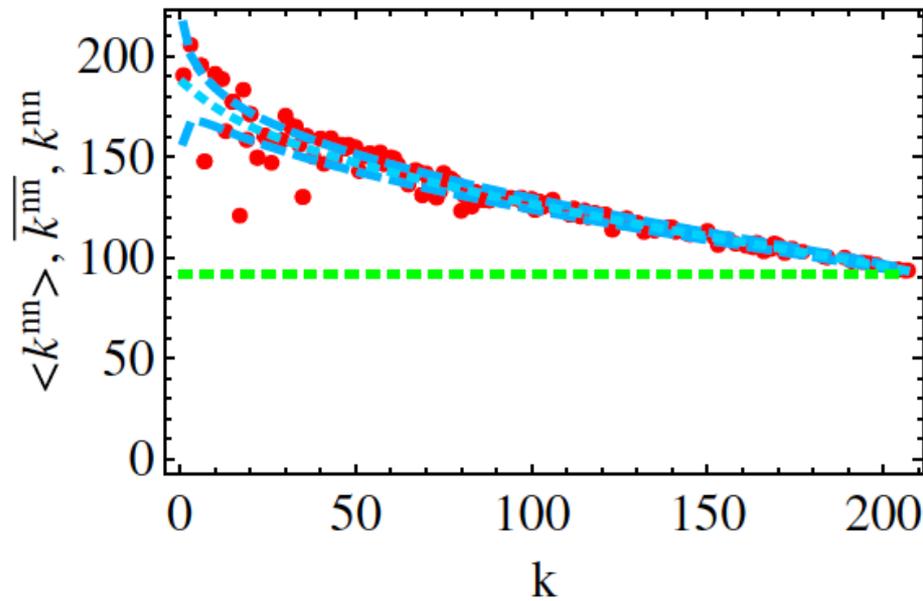
## Network

- Office social network [27]
- Research group social network [27]
- Fraternity social network [27]
- Maspalomas Lagoon food web [28]
- Chesapeake Bay food web [28]
- Crystal River (control) food web [28]
- Crystal River food web [28]
- Michigan Lake food web [28]
- Mondego Estuary food web [28]
- Everglades Marshes food web [28]
- Italian interbank network (1999) [26]
- World Trade Web (2000) [20]

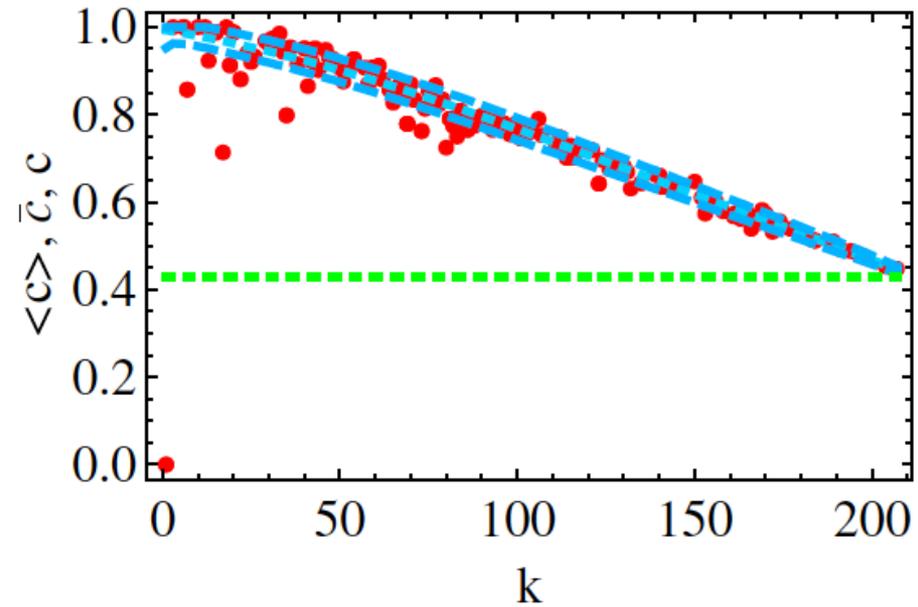


# Zooming in on the eMID interbank network

Average nearest-neighbor degree



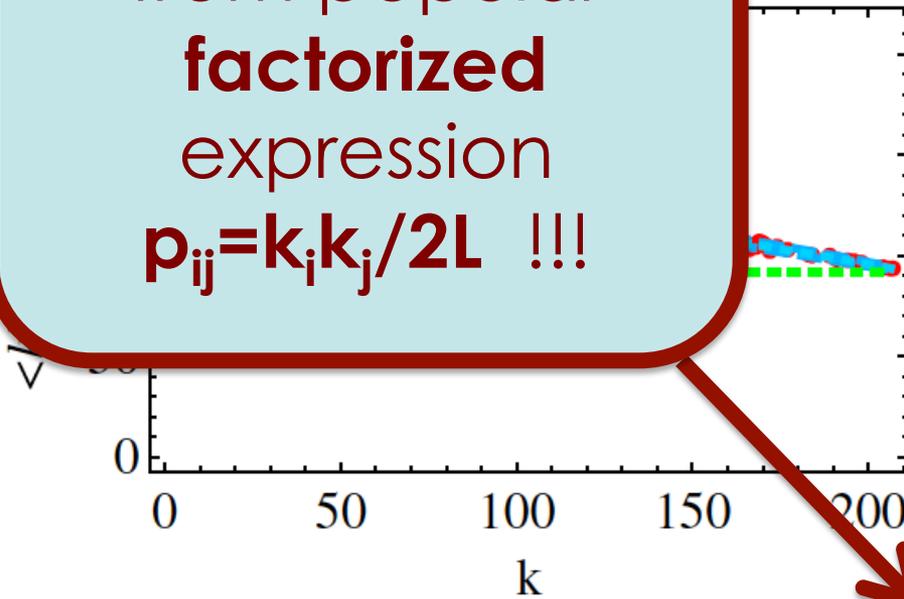
Binary clustering coefficient



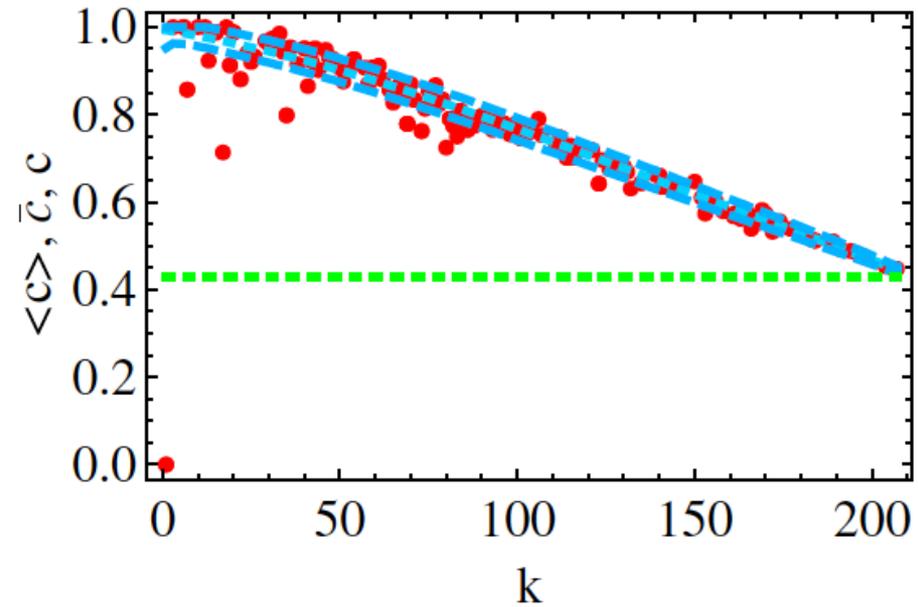
$$P_{ij} = \frac{x_i x_j}{1 + x_i x_j}$$

# Zooming in on the eMID interbank network

**NOTE:** different from popular factorized expression  $p_{ij} = k_i k_j / 2L$  !!!



Binary clustering coefficient

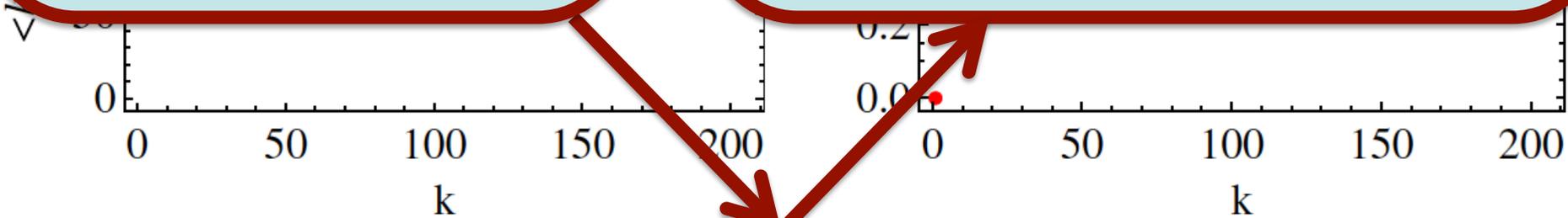


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# Zooming in on the eMID interbank network

**NOTE:** different from popular **factorized** expression  $p_{ij} = k_i k_j / 2L$  !!!

So, even if constraints are only **local** (configuration model), here degree “correlations”, clustering, cycles, and other higher-order properties are **automatically accounted for**.



$$P_{ij} = \frac{x_i x_j}{1 + x_i x_j}$$

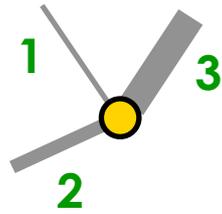
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So, even if constraints are only **local** (configuration model), here degree “correlations”, clustering, cycles, and other higher-order properties are **automatically accounted for**.

**GOOD,**  
but what about  
**link weights?**

# Weighted configuration model (WCM): fixed strengths

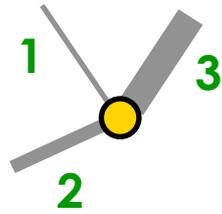


$$\vec{C} = \{s_i\} = \left\{ \sum_{j \neq i} w_{ij} \right\}$$

$$H(G) = \sum_i \beta_i s_i = \sum_{i < j} (\beta_i + \beta_j) w_{ij}$$

└──────────┘  $w_{ij} = 0, 1, \dots, w_*$

# Weighted configuration model (WCM): fixed strengths



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$$H(G) = \sum_i \beta_i s_i = \sum_{i < j} (\beta_i + \beta_j) w_{ij}$$

$\hookrightarrow w_{ij} = 0, 1, \dots, w_*$

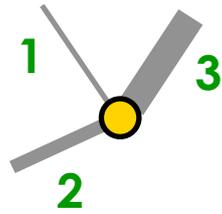
$$P(G) = e^{-H(G)} / Z$$
$$= \prod_{i < j} q_{ij}(w_{ij})$$

$$q_{ij}(w) = \frac{(y_i y_j)^w (1 - y_i y_j)}{1 - (y_i y_j)^{w_* + 1}}$$

(probability of  $w$  'occupations')

[Garlaschelli & Loffredo, *Phys. Rev. Lett.* **102**, 038701 (2009)]

# Weighted configuration model (WCM): fixed strengths



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(probability of  $w$  'occupations')

[Garlaschelli & Loffredo, *Phys. Rev. Lett.* **102**, 038701 (2009)]

If  $w_* = +\infty$ , the expected occupation number is

$$\langle w_{ij} \rangle = \frac{y_i y_j}{1 - y_i y_j}$$

**BOSE-EINSTEIN !**

[Park & Newman, *Phys. Rev. E* **70**, 066117 (2004)]

# Weighted configuration model (WCM): fixed strengths

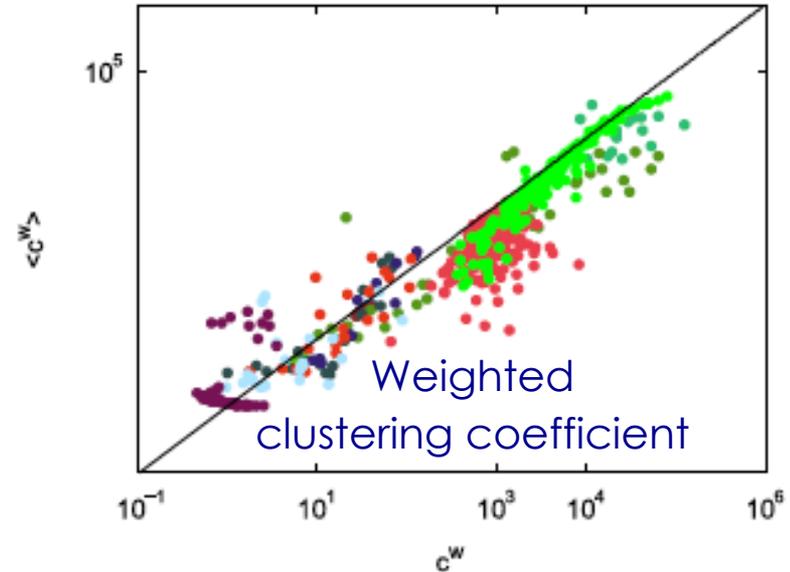
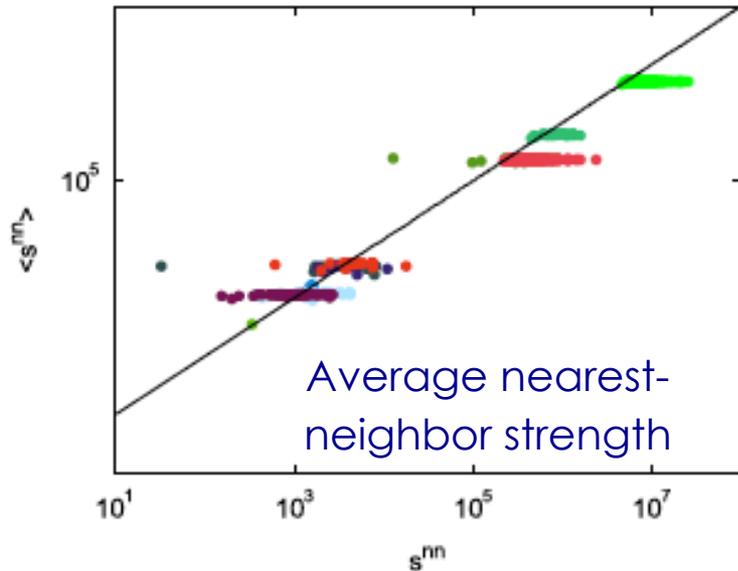
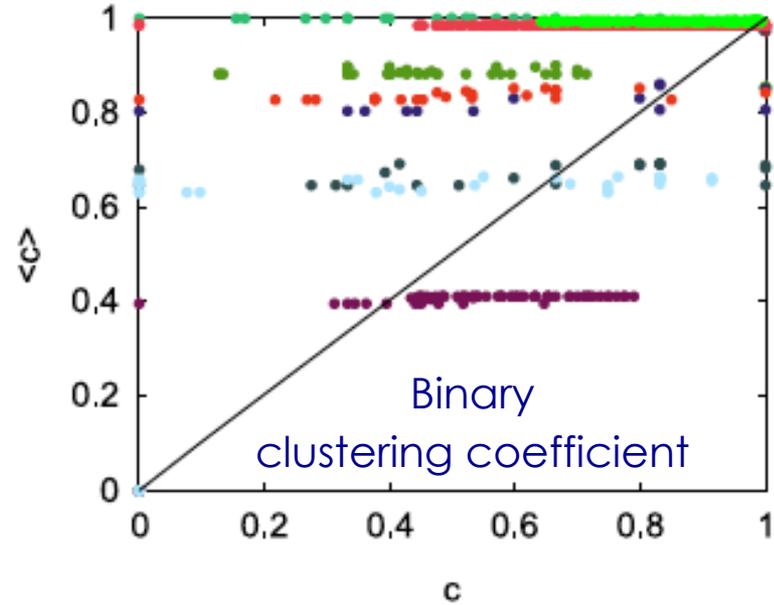
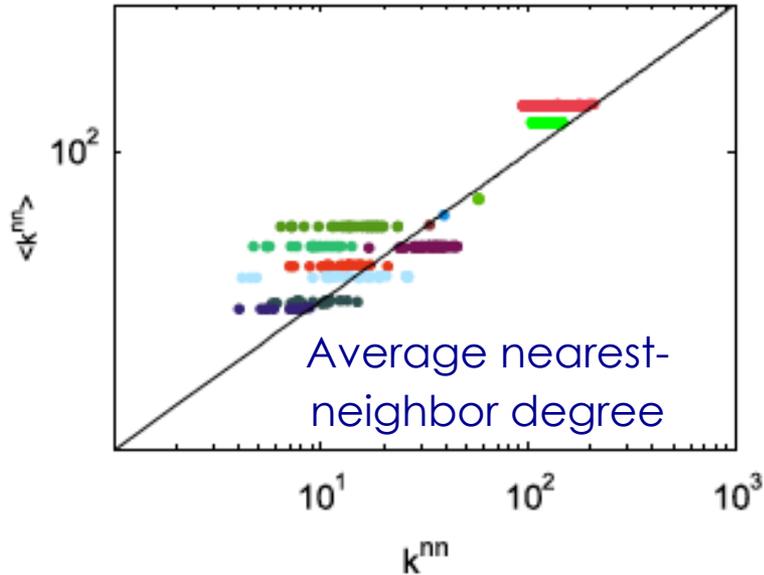


Equiprobable configurations:

$$\begin{aligned} P\left(\begin{array}{c} 1 \\ \diagup \\ \bullet \\ \diagdown \\ 2 \end{array} \begin{array}{c} 3 \\ \diagup \\ \bullet \\ \diagdown \\ 2 \end{array}\right) &= P\left(\begin{array}{c} 1 \\ \diagup \\ \bullet \\ \diagdown \\ 1 \end{array} \begin{array}{c} 4 \\ \diagup \\ \bullet \\ \diagdown \\ 1 \end{array}\right) = P\left(\begin{array}{c} 3 \\ \diagup \\ \bullet \\ \diagdown \\ 3 \end{array}\right) \\ &= P\left(\begin{array}{c} 2 \\ \diagup \\ \bullet \\ \diagdown \\ 2 \end{array} \begin{array}{c} 2 \\ \diagup \\ \bullet \\ \diagdown \\ 2 \end{array}\right) = P\left(\begin{array}{c} 6 \\ \diagup \\ \bullet \\ \diagdown \\ 6 \end{array}\right) \end{aligned}$$

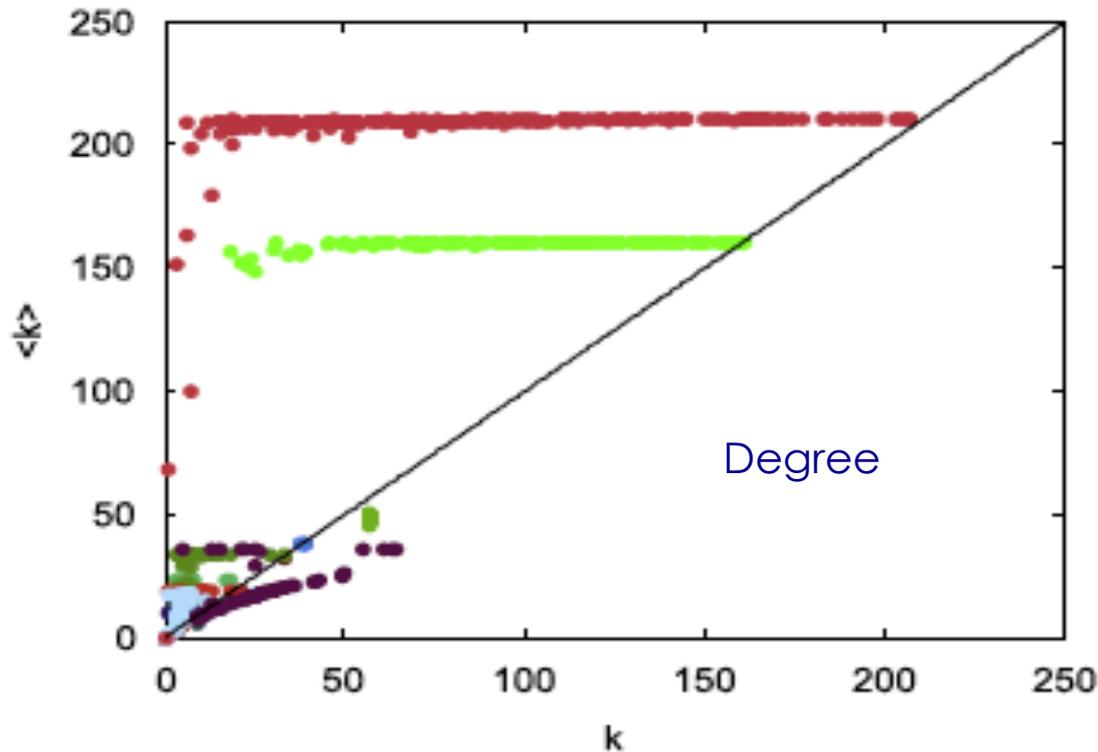
(must hold for all vertices simultaneously)

# Result: *bad* reconstruction



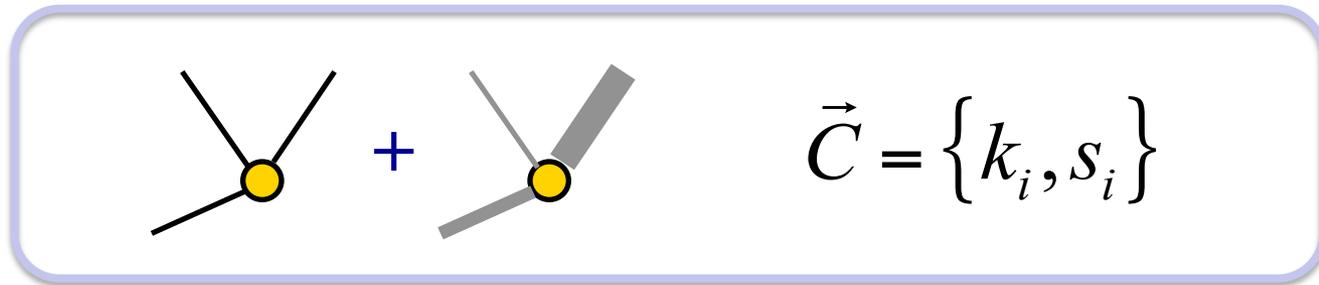
## Reason:

poor **binary** reconstruction from strengths only



**Naive expectation** that aggregate **weighted** properties are more informative than **binary** ones **is incorrect!**

# Doubling the constraints (degrees + strengths): **Enhanced Configuration Model (ECM)**



$$H(W) = \sum_{i < j} [\alpha_{ij} \Theta(w_{ij}) + \beta_{ij} w_{ij}]$$

→  $w_{ij} = 0, 1, \dots, w_*$

→  $\Theta(w_{ij}) = a_{ij} = 0, 1$

- **Combination** of Fermionic and Bosonic constraints;
- Extra energy (+ or -) for **first occupation** (i.e. edge weight);
- (+): Initial barrier/threshold; (-): saturation/aging.

# The generalized **Bose-Fermi** distribution

$$H(W) = \sum_{i < j} [\alpha_{ij} \Theta(w_{ij}) + \beta_{ij} w_{ij}]$$

Full probability:

$$\begin{aligned} P(W) &= \frac{\prod_{i < j} e^{-\alpha_{ij} \Theta(w_{ij}) - \beta_{ij} w_{ij}}}{\sum_{W'} \prod_{i < j} e^{-\alpha_{ij} \Theta(w'_{ij}) - \beta_{ij} w'_{ij}}} \\ &= \prod_{i < j} \frac{e^{-\alpha_{ij} \Theta(w_{ij}) - \beta_{ij} w_{ij}}}{\sum_{w'_{ij}=0}^{w_*} e^{-\alpha_{ij} \Theta(w'_{ij}) - \beta_{ij} w'_{ij}}} \\ &= \prod_{i < j} \frac{e^{-\alpha_{ij} \Theta(w_{ij}) - \beta_{ij} w_{ij}}}{1 + e^{-\alpha_{ij}} \sum_{w'_{ij}=1}^{w_*} e^{-\beta_{ij} w'_{ij}}} = \prod_{i < j} q_{ij}(w_{ij}) \end{aligned}$$

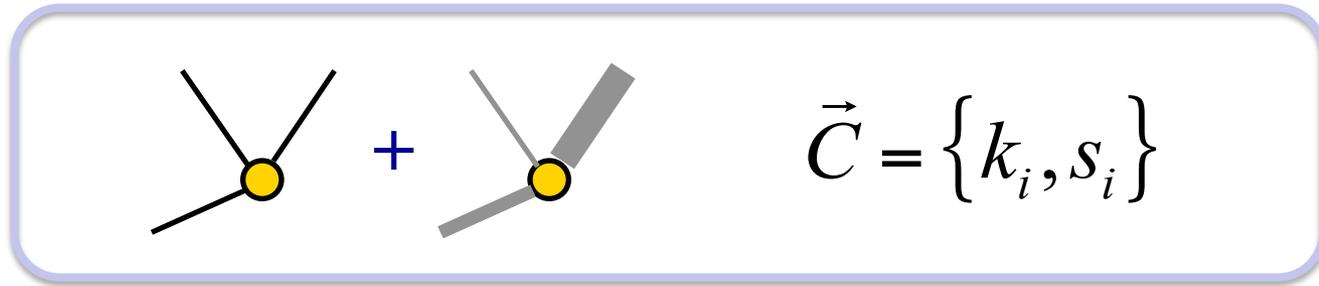
Probability of  $w$  occupations:

$$q_{ij}(w) = \frac{x_{ij}^{\Theta(w)} y_{ij}^w}{1 + x_{ij} \sum_{w'=1}^{w_*} y_{ij}^{w'}}$$

Fugacities:

$$x_{ij} \equiv e^{-\alpha_{ij}} \quad \text{and} \quad y_{ij} \equiv e^{-\beta_{ij}}$$

# Doubling the constraints (degrees + strengths): Enhanced Configuration Model (ECM)



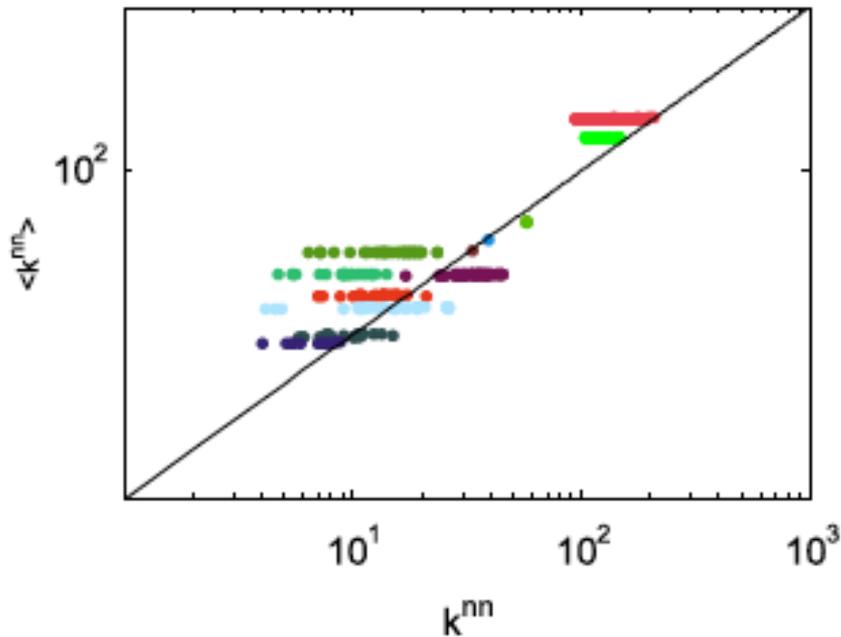
$$P\left(\begin{array}{c} 1 \\ \diagup \\ \bullet \\ \diagdown \\ 2 \end{array} \begin{array}{c} 3 \\ \diagup \\ \bullet \\ \diagdown \\ 3 \end{array}\right) \neq P\left(\begin{array}{c} \diagup \\ \bullet \\ \diagdown \\ 3 \end{array} \begin{array}{c} 3 \\ \diagup \\ \bullet \\ \diagdown \\ 3 \end{array}\right) \neq P\left(\begin{array}{c} \bullet \\ \diagdown \\ 6 \end{array}\right)$$

$$P\left(\begin{array}{c} 1 \\ \diagup \\ \bullet \\ \diagdown \\ 2 \end{array} \begin{array}{c} 3 \\ \diagup \\ \bullet \\ \diagdown \\ 3 \end{array}\right) = P\left(\begin{array}{c} 2 \\ \diagup \\ \bullet \\ \diagdown \\ 2 \end{array} \begin{array}{c} 2 \\ \diagup \\ \bullet \\ \diagdown \\ 2 \end{array}\right) = P\left(\begin{array}{c} 1 \\ \diagup \\ \bullet \\ \diagdown \\ 1 \end{array} \begin{array}{c} 4 \\ \diagup \\ \bullet \\ \diagdown \\ 1 \end{array}\right)$$

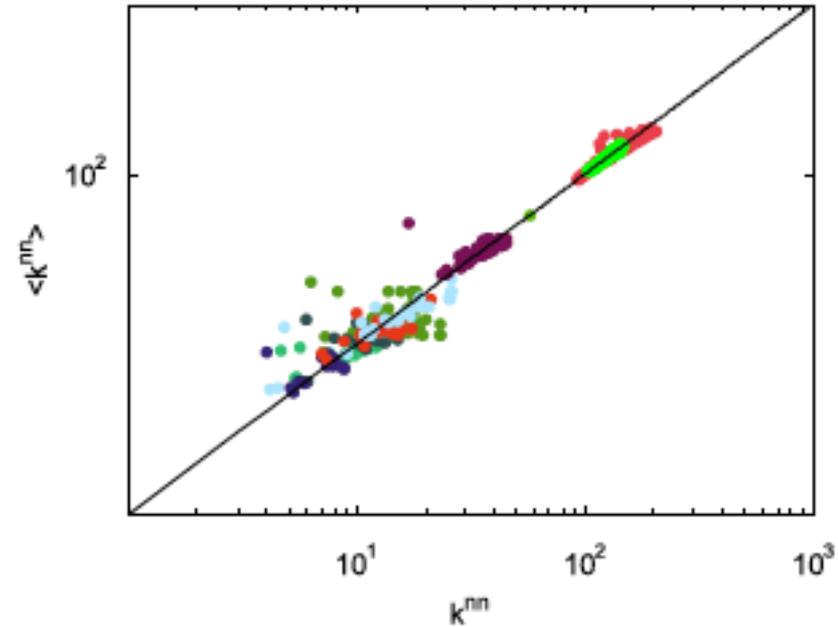
Note: equivalent to minimizing Kullback-Leibler distance from **“topological prior”** (BCM) rather than uniform prior

# Reconstruction greatly improved by the ECM:

Standard reconstruction  
from strengths only:  
Bose distribution (WCM)



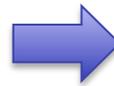
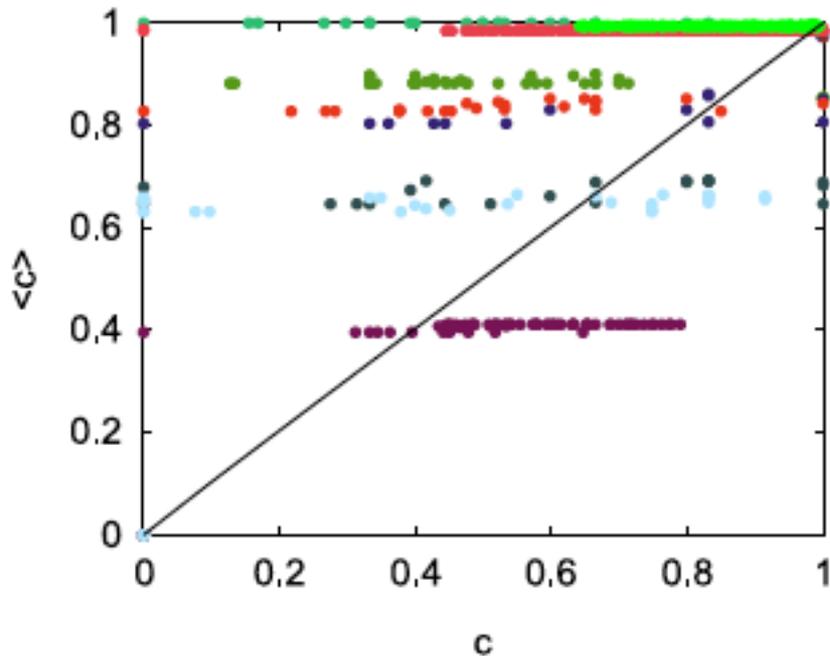
Enhanced reconstruction  
from strengths and degrees:  
Bose-Fermi distribution (ECM)



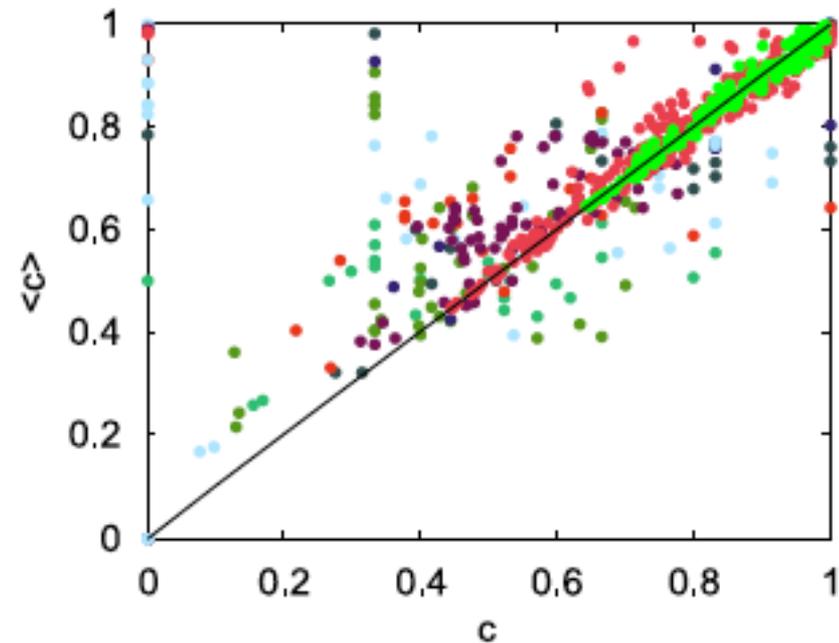
Average nearest-neighbor degree

# Reconstruction greatly improved by the ECM:

Standard reconstruction  
from strengths only:  
Bose distribution (WCM)



Enhanced reconstruction  
from strengths and degrees:  
Bose-Fermi distribution (ECM)

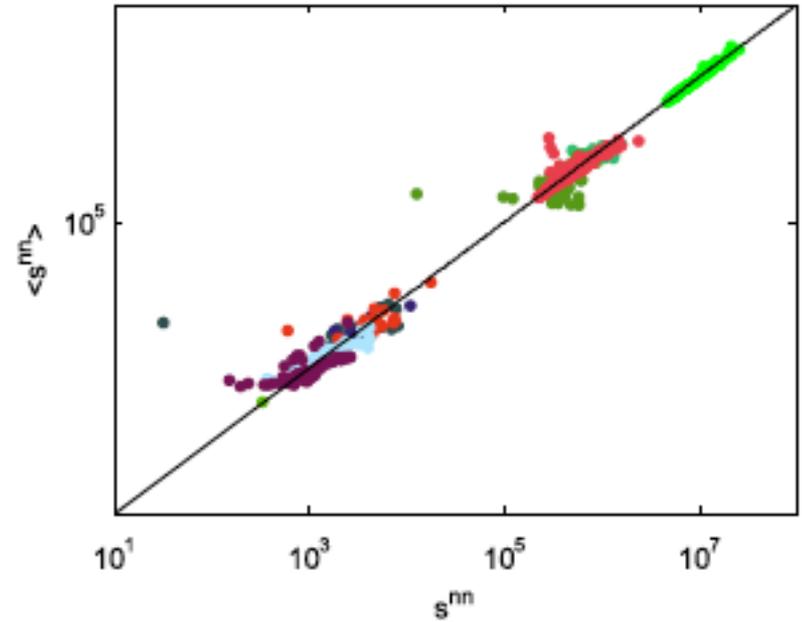
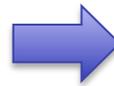
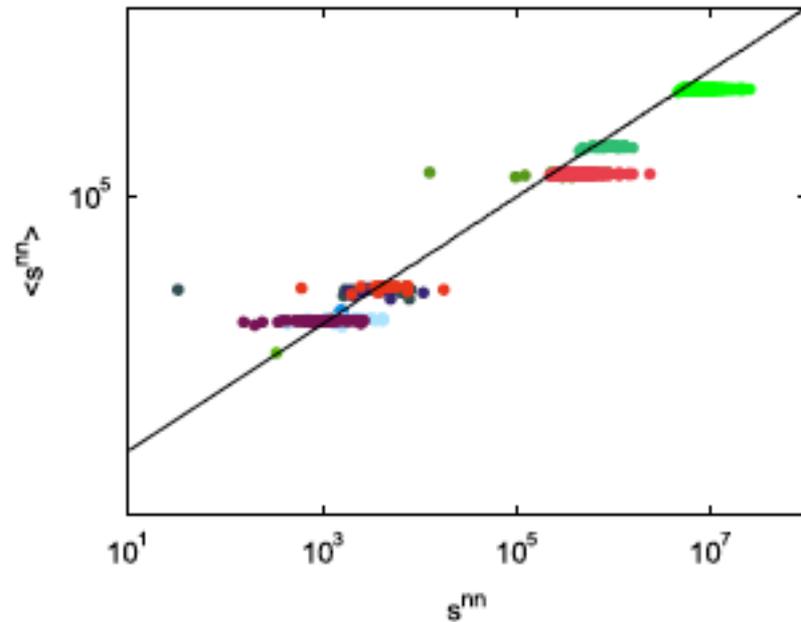


Binary clustering coefficient

# Reconstruction greatly improved by the ECM:

Standard reconstruction  
from strengths only:  
Bose distribution (WCM)

Enhanced reconstruction  
from strengths and degrees:  
Bose-Fermi distribution (ECM)

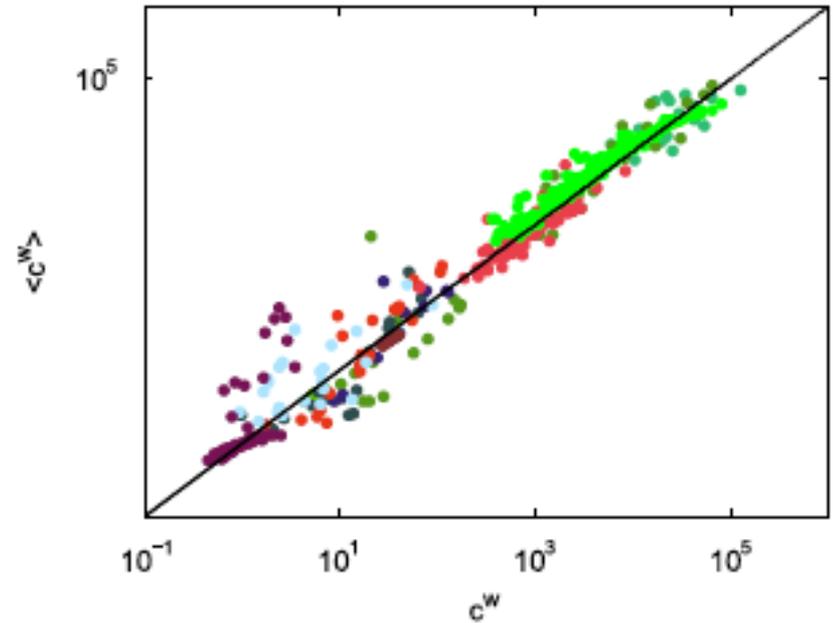
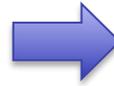
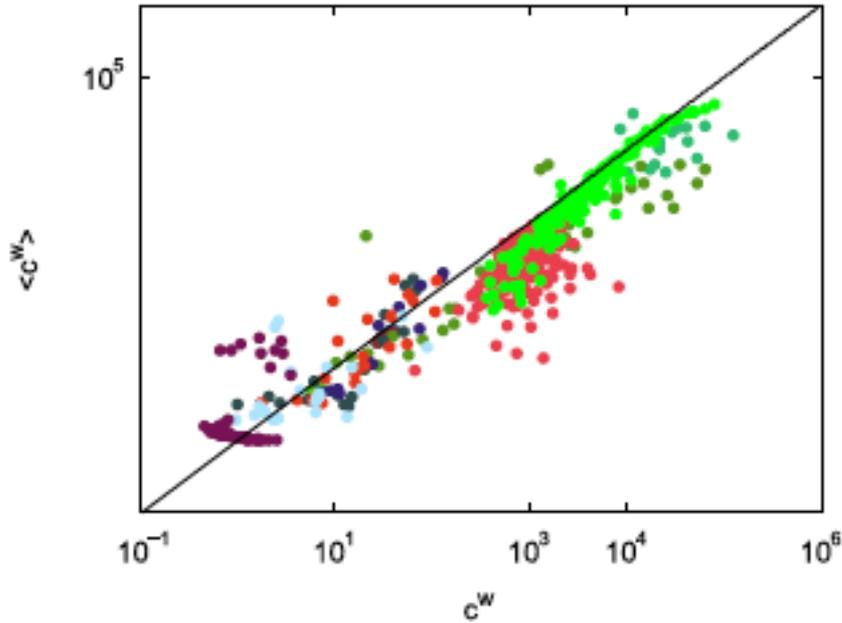


Average nearest-neighbor strength

# Reconstruction greatly improved by the ECM:

Standard reconstruction  
from strengths only:  
Bose distribution (WCM)

Enhanced reconstruction  
from strengths and degrees:  
Bose-Fermi distribution (ECM)



Weighted clustering coefficient

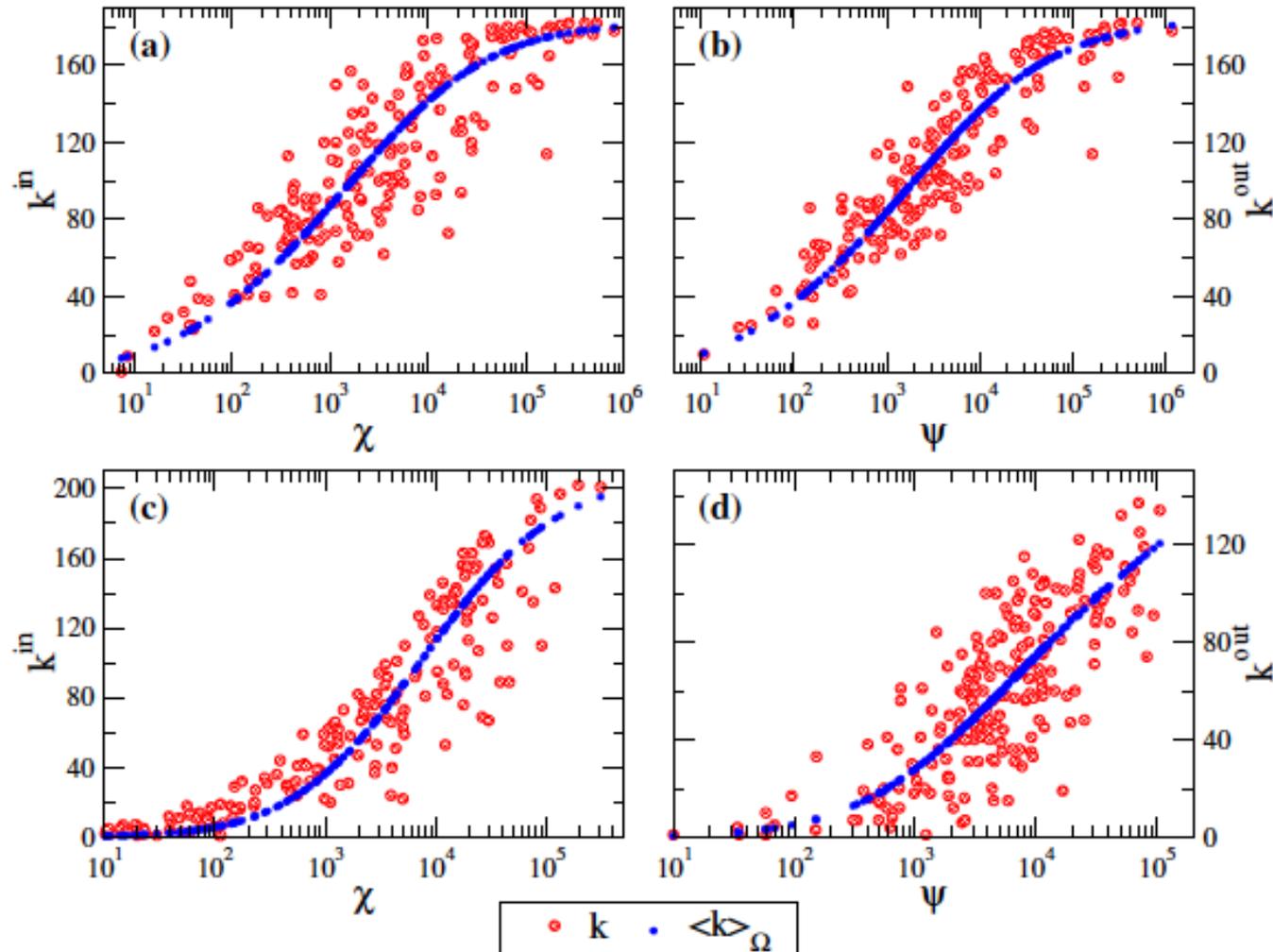
# Reducing the required input info

inference from strengths and only some proxy of link density

$$P_{ij} = \frac{x_i x_j}{1 + x_i x_j}$$



$$P_{ij} = (z s_i^{\text{out}} s_j^{\text{in}}) / (1 + z s_i^{\text{out}} s_j^{\text{in}})$$



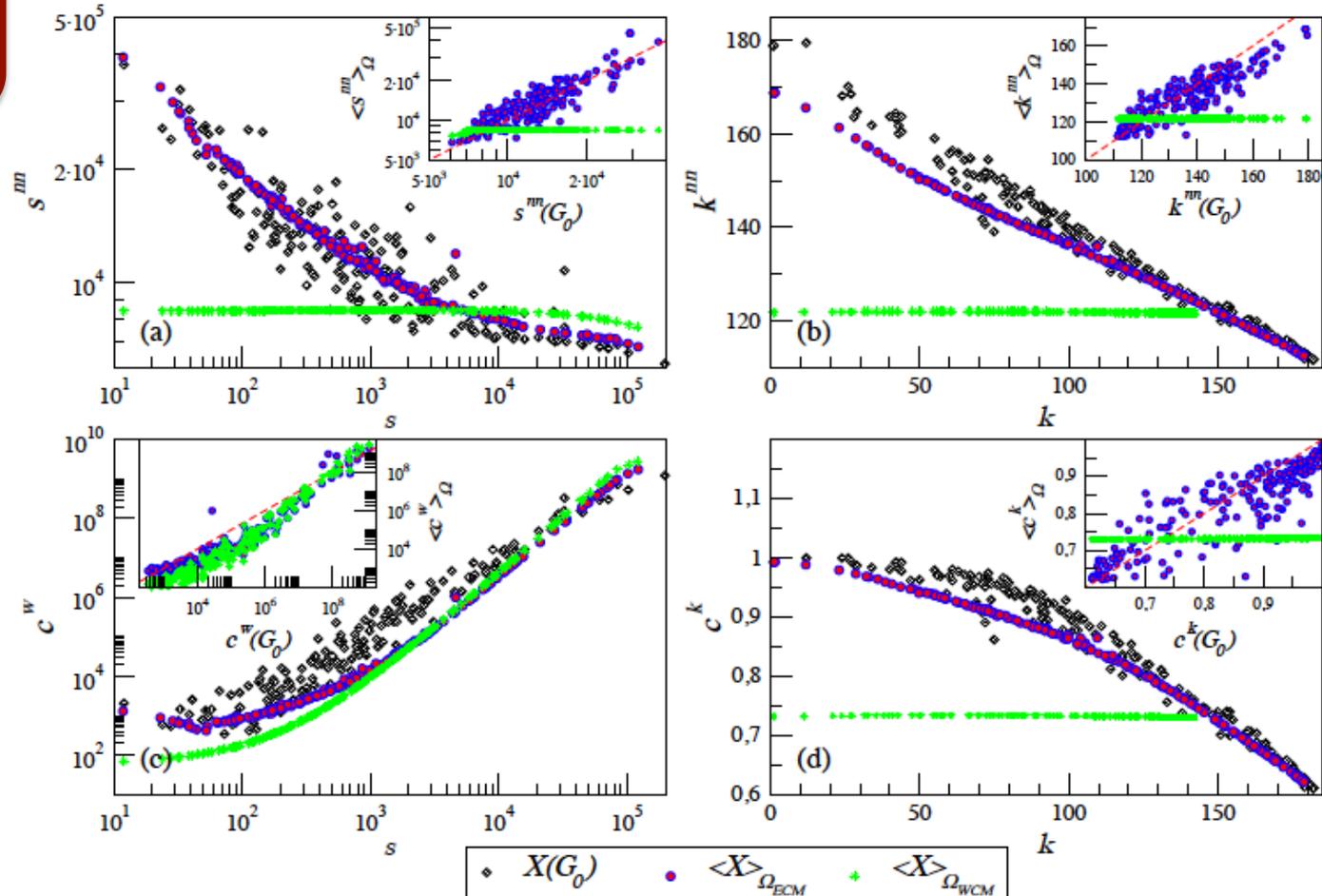
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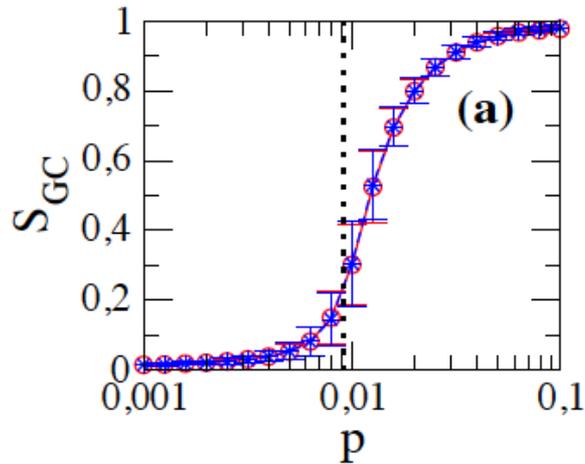
$$P_{ij} = (z s_i^{\text{out}} s_j^{\text{in}}) / (1 + z s_i^{\text{out}} s_j^{\text{in}})$$



# Predicting systemic risk estimators

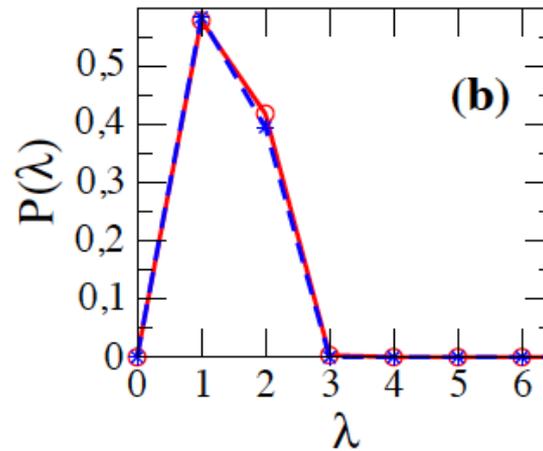
## Percolation

(relative size of giant component vs occupation probability  $p$ )



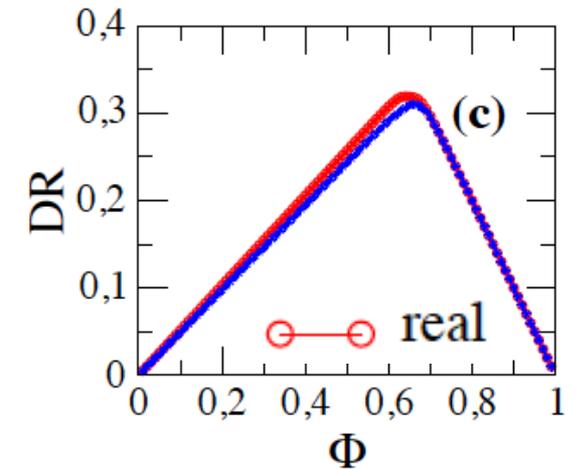
## Path length

(distribution of shortest distances  $\lambda$  among pairs of nodes)

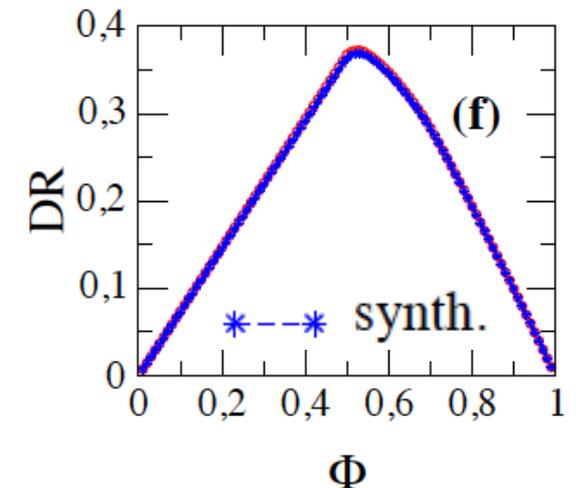
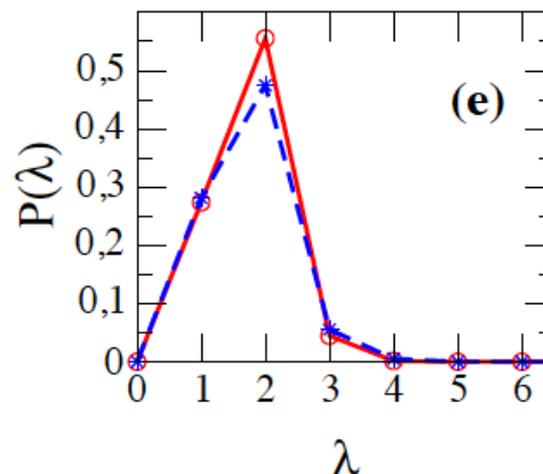
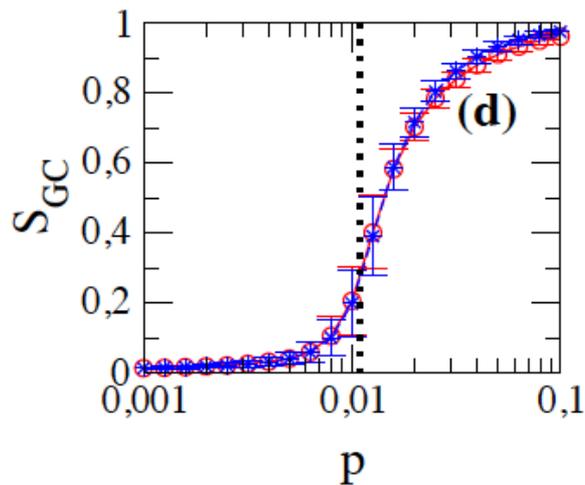


## Group DebtRank

(total devaluation induced by an initial devaluation  $\Phi$ )  
[Battiston et al. 2012]

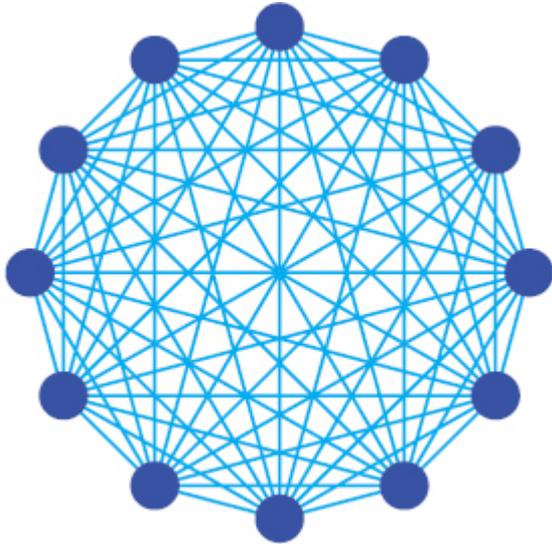


E-mid



## Traditional approach

(dense: many links, but weak)

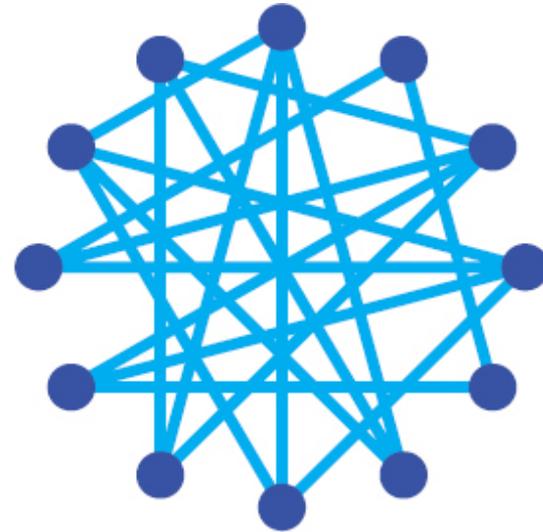


$$\tilde{W}_{i \rightarrow j} = \frac{s_i^{\text{out}} s_j^{\text{in}}}{W}$$

margins: **OK**, topology: **BAD**

## Enhanced method

(sparse: few links, but strong)



$$\tilde{W}_{i \rightarrow j} = \begin{cases} 0 & \text{with probability } 1 - p_{ij}, \\ \frac{z^{-1} + s_i^{\text{out}} s_j^{\text{in}}}{W} & \text{with probability } p_{ij} \end{cases}$$

where  $p_{ij} = (z s_i^{\text{out}} s_j^{\text{in}}) / (1 + z s_i^{\text{out}} s_j^{\text{in}})$

margins: **OK**, topology: **OK**

# Independent tests of our method

The method

*“outperforms the other methods  
when the same input information is used”*

[Mazzarisi et al 2017]

and *“is the clear winner  
among probabilistic methods”*

[Anand et al. 2017]

Anand et al, “The missing links: A global study on uncovering financial network structure from partial data”, (2017) [Journal of Financial Stability, in press].

Mazzarisi P., F. Lillo “Methods for Reconstructing Interbank Networks from Limited Information: A Comparison”, in *Econophysics and Sociophysics: Recent Progress and Future Directions*, New Economic Windows, Springer International Publishing (2017).



# Leiden econophysics model tested best by central banks

June 2, 2017

26

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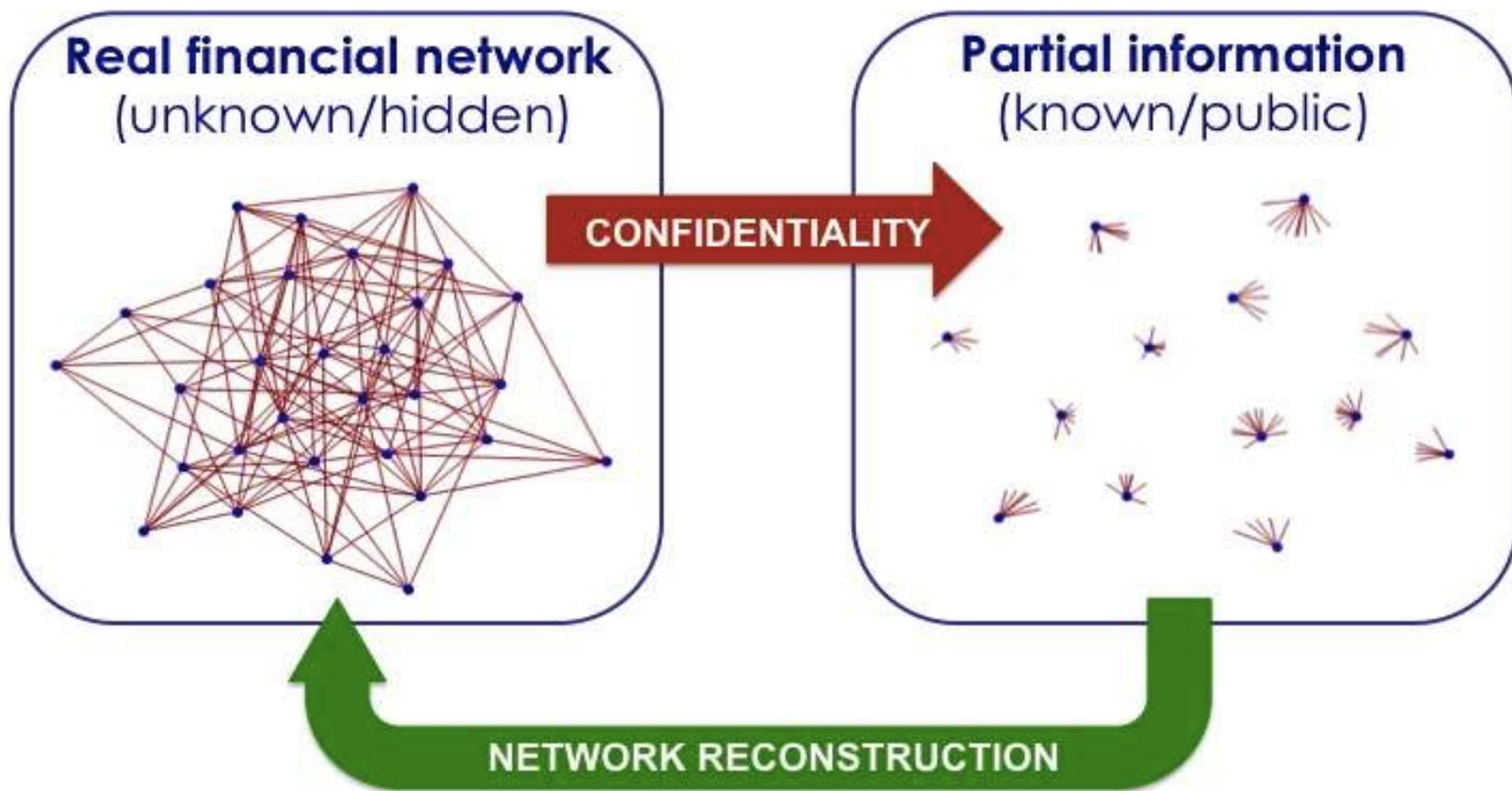
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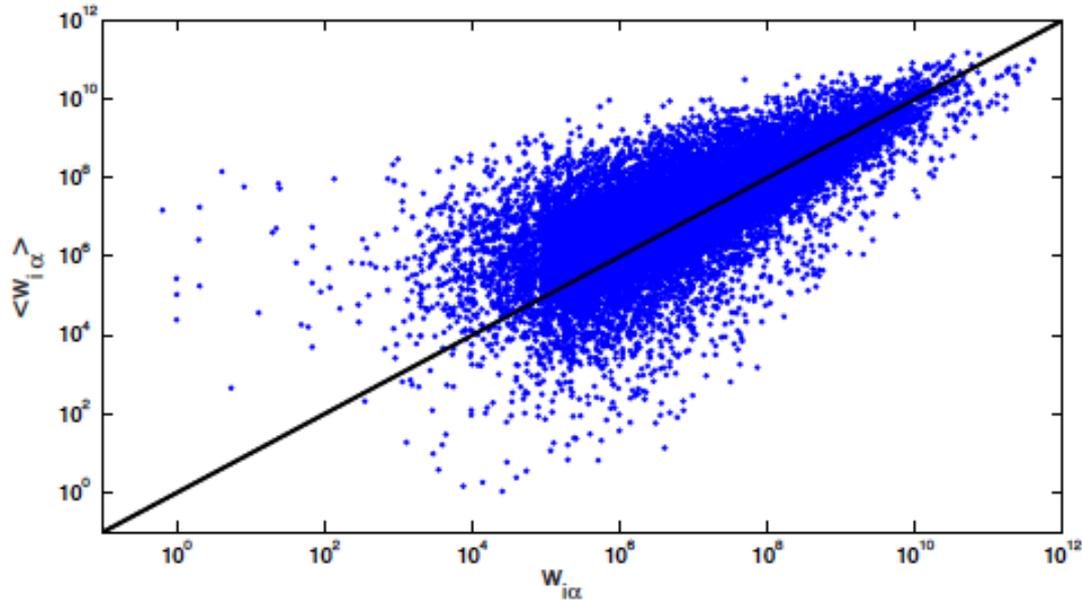
Print



Credit: Leiden Institute of Physics

Snortify

# Extension to the bipartite European network



**Data (ECB):**

Security Holding Statistics (SHS)

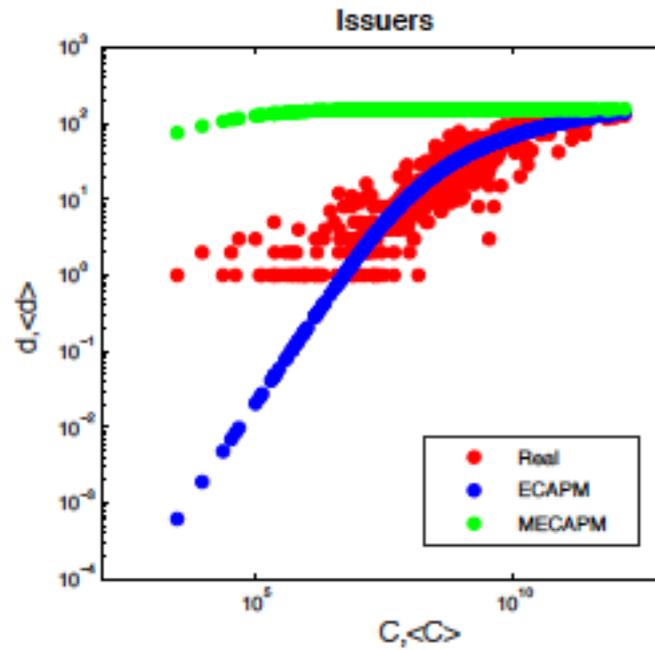
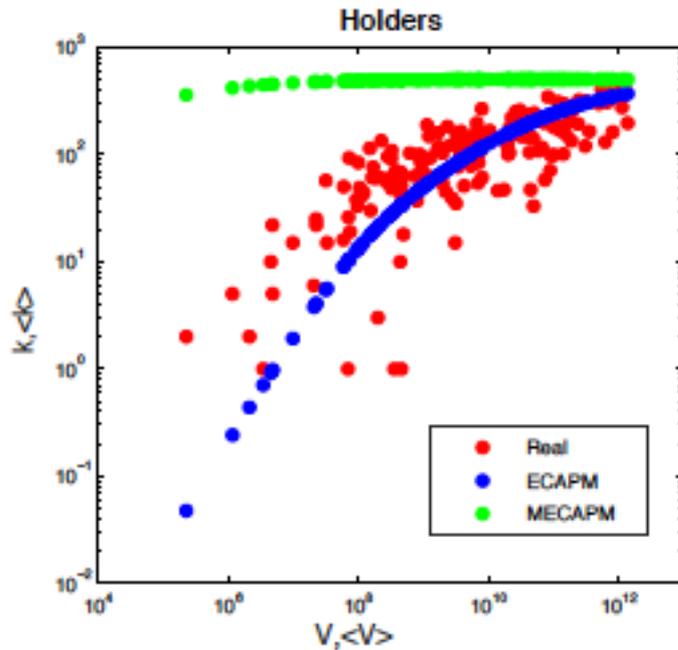
**Links:**

Long-term security bonds

**Nodes:**

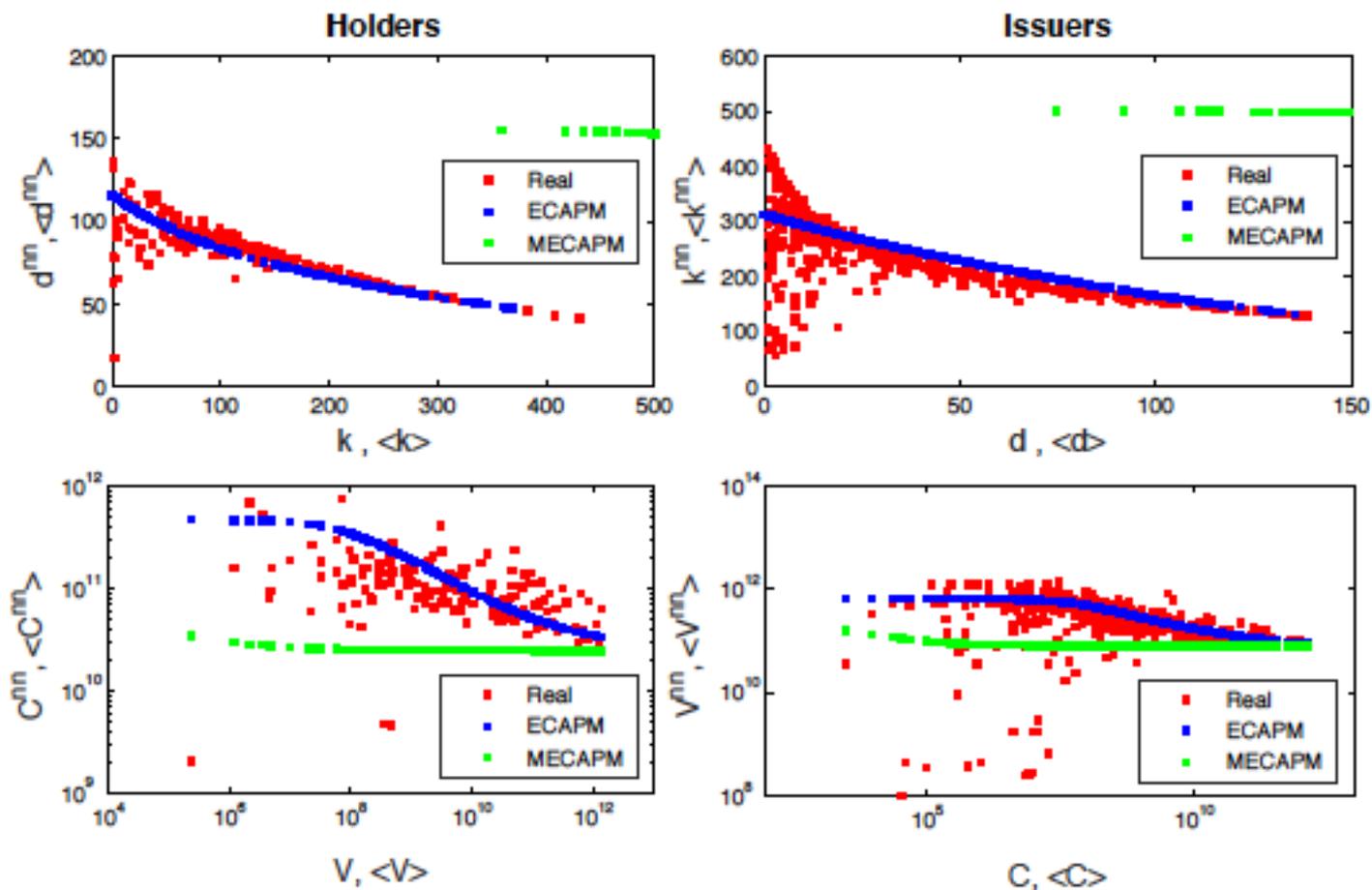
N=266 holders (country-sector)

M=3136 issuers (country-sector)



# Extension to the bipartite European network

## Enhanced Capital Asset Pricing Model (ECAPM)



arXiv:1606.07684v2

## COMPLEX SYSTEMS

# Complexity theory and financial regulation

Economic policy needs interdisciplinary network analysis and behavioral modeling

By Stefano Battiston,<sup>1\*</sup> J. Dooyne Farmer,<sup>2,3</sup> Andreas Flache,<sup>4</sup> Diego Garlaschelli,<sup>5</sup> Andrew G. Haldane,<sup>6</sup> Hans Heesterbeek,<sup>7</sup> Cars Hommes,<sup>8,9\*</sup> Carlo Jaeger,<sup>10,11,12</sup> Robert May,<sup>13</sup> Marten Scheffer<sup>14</sup>

Traditional economic theory could not explain, much less predict, the near collapse of the financial system and its long-lasting effects on the global economy. Since the 2008 crisis, there has been increasing interest in using ideas from complexity theory to make sense of economic and financial markets. Concepts, such as tipping points, networks, contagion, feedback, and resilience have entered the financial and regulatory lexicon, but

**POLICY** actual use of complexity models and results remains at an early stage. Recent insights and techniques offer potential for better monitoring and management of highly interconnected economic and financial systems and, thus, may help anticipate and manage future crises.

**TIPPING POINTS, WARNING SIGNALS.** Financial markets have historically exhibited sudden and largely unforeseen collapses, at a systemic scale. Such “phase transitions” may in some cases have been triggered by unpredictable stochastic events. More often, however, there have been endogenous underlying processes at work. Analyses of complex systems ranging from the climate to ecosystems reveal that, before a major transition, there is often a gradual and unnoticed loss of resilience. This makes the system brittle: A small disruption can trigger a domino effect that propagates through the system and propels it into a crisis state.

Recent research has revealed generic empirical quantitative indicators of resilience that may be used across complex systems to detect tipping points. Markers include rising correlation between nodes in a network and rising temporal correlation, variance, and skewness of fluctuation patterns. These indicators were first predicted mathematically and subsequently demonstrated experimentally in real complex systems, including living systems (1). A recent study of the Dutch interbank network (2) showed that standard analysis using a homogeneous network model could only lead to late detection of the 2008 crisis, although a more realistic and heterogeneous network model could identify an early warning signal 3 years before the crisis (see the chart).

Ecologists have developed tools to quantify the stability, robustness, and resilience of food webs and have shown how these depend on the topology of the network and the strengths of interactions (3). Epidemiologists have tools to gauge the potential for events to propagate in systems of interacting entities, to identify superspreaders and core groups relevant to infection persistence, and to design strategies to prevent or limit the spread of contagion (4).

Extrapolating results from the natural sciences to economics and finance presents challenges. For instance, publication of an early warning signal will change behavior and affect future dynamics [the Lucas critique (5)]. But this does not affect the case where indicators are known only to regulators or when the goal is to build better network barriers to slow contagion.

**TOO CENTRAL TO FAIL.** Network effects matter to financial-economic stability because shock amplification may occur via strong cascading effects. For example, the Bank of International Settlements recently developed a framework drawing on data on the interconnectedness between banks to gauge the systemic risk posed to the financial network by Global Systemically Important Banks. Recent research on contagion in financial networks has shown that network topology and positions of banks matter; the global financial network may collapse even when individual banks appear safe (6). Capturing these effects is essential for quantifying stress on individual banks and for looking at systemic risk for the network as

a whole. Despite on-going efforts, these effects are unlikely to be routinely considered anytime soon.

Information asymmetry within a network—e.g. where a bank does not know about troubled assets of other banks—can be problematic. The banking network typically displays a core-periphery structure,

**“...policies and financial regulation [that] weaken positive feedback... stabilize experimental macroeconomic systems...”**

with a core consisting of a relatively small number of large, densely interconnected banks that are not very diverse in terms of business and risk models. This implies that core banks' defaults tend to be highly correlated. That, in turn, can generate a collective moral hazard problem (i.e., players take on more risk, because others will bear the costs in case of default), as banks recognize that they are likely to be supported by the authorities in situations of distress, the likelihood amplifies their incentives to herd in the first place.

Estimating systemic risk relies on granular data on the financial network. Unfortunately, business interactions between banks are often hidden because of confidentiality issues. Tools being developed to reconstruct networks from partial information and to estimate systemic risk (7) suggest that publicly available bank information does not allow reliable estimation of systemic risk. The estimate would improve greatly if banks publicly reported the number of connections with other banks, even without disclosing their identity.

In addition to data, understanding the effects of interconnections also relies on integrative quantitative metrics and concepts that reveal important network aspects, such as systemic repercussions of the failure of individual nodes. For example, DebtRank, which measures the systemic importance of individual institutions in a financial network (8), shows that the issue of too-central-to-fail may be even more important than too-big-to-fail.



**AGENTS AND BEHAVIOR.** Agent-based models (ABMs) are computer models in which the behavior of agents and their interactions are explicitly represented as decision rules mapping agents' observations onto actions. Although ABMs are less well established in analyzing financial-economic systems than in, e.g., traffic control, epidemiology, or battlefield conflict analyses, they have produced promising results. Axtell (9) developed a simple ABM that explains more than three dozen empirical properties of firm formation without recourse to external shocks. ABMs provide a good explanation for why the volatility of prices is clustered and time-varying (10) and have been used

Laboratory experiments with human subjects can provide empirical validation of individual decision rules of agents, their interactions, and emergent macro behavior. Recent experiments studying behavior of a group of individuals in the laboratory show that economic systems may deviate significantly from rational efficient equilibrium at both individual and aggregate levels (14). This generic feature of positive feedback systems leads to persistent deviations of prices from equilibrium and emergence of speculation-driven bubbles and crashes, strongly amplified by coordination on trend-following and herding behavior (15). There is strong empirical evidence of

monetary and fiscal policies and financial regulation designed to weaken positive feedback are successful in stabilizing experimental macroeconomic systems when properly calibrated (16). Complexity theory provides mathematical understanding of these effects.

**POLICY DASHBOARD.** It is an opportune time for academic economists, complexity scientists, social scientists, ecologists, epidemiologists, and researchers at financial institutions to join forces to develop tools from complexity theory, as a complement to existing economic modeling approaches (17). One ambitious option would be an online, financial-economic dashboard that integrates data, methods, and indicators. This might monitor and stress-test the global socioeconomic and financial system in something close to real time, in a way similar to what is done with other complex systems, such as weather systems or social networks. The funding required for essential policy-relevant and fundamental interdisciplinary progress in these areas would be trivial compared with the costs of systemic financial failures or the collapse of the global financial-economic system. ■

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# Complexity theory and financial regulation

Economic policy needs interdisciplinary network analysis and behavioral modeling

By Stefano Battiston,<sup>1\*</sup> J. Doyne Farmer,<sup>2,3</sup> Andreas Flache,<sup>4</sup> Diego Garlaschelli,<sup>5</sup> Andrew G. Haldane,<sup>6</sup> Hans Heesterbeek,<sup>7</sup> Cans H. Hommes,<sup>8,9†</sup> Carlo Jaeger,<sup>10,11</sup> Robert May,<sup>12</sup> Marten Scheffer<sup>13</sup>

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**TIPPING POINTS, WARNING SIGNALS.** Financial markets have historically experienced sudden and largely unforeseen collapses on a systemic scale. Such “phase transitions” may in some cases have been triggered by unpredictable stochastic events. Often, however, there have been clear underlying processes at work. As complex systems ranging from traffic to ecosystems reveal that, before a transition, there is often a gradual, unnoticed loss of resilience. This makes them brittle: A small disruption can trigger a domino effect that propagates through the system and propels it into a crisis.

<sup>1</sup>Department of Banking and Finance, University of Zurich, Switzerland. <sup>2</sup>Institute for New Economic Thinking, Oxford Martin School, and Mathematical Institute of Oxford, Oxford OX1 2JD, UK. <sup>3</sup>Santa Fe Institute, Santa Fe, NM 87501, USA. <sup>4</sup>Department of Sociology, University of Groningen, 9712 TG Groningen, Netherlands. <sup>5</sup>Lorentz Institute for Theoretical Physics, University of Leiden, 2333 CA Leiden, Netherlands. <sup>6</sup>Bank of England, London, EC2R 8AH, UK. <sup>7</sup>Faculty of Veterinary Medicine, University of Utrecht, 3512 JE Utrecht, Netherlands. <sup>8</sup>Amsterdam School of Economics, University of Amsterdam, 1018 WB Amsterdam, Netherlands. <sup>9</sup>Tinbergen Institute, 1082 MS Amsterdam, Netherlands. <sup>10</sup>Beijing Normal University, 100875 Beijing, China. <sup>11</sup>Potsdam University, 14469 Potsdam, Germany. <sup>12</sup>Global Climate Forum 10178 Berlin, Germany. <sup>13</sup>Department of Zoology, University of Oxford, Oxford OX1 2JD, UK. <sup>†</sup>Environmental Sciences, Wageningen University 6708 PB Wageningen, Netherlands. \*Authors are in alphabetical order. †Corresponding author. E-mail: C.H.Hommes@uva.nl

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**ACKNOWLEDGMENTS**  
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## Challenge 1:

Far from critical events, the (maximum-entropy) reconstruction of interbank networks is **reliable**;

As crises approach, reconstruction becomes unreliable and actually **prevents** the detection of early-warning signals;

In any case, maximum-entropy methods appear **crucial** to **construct** the early-warning signal itself!

ILLUSTRATION: C. SMITH/SCIENCE

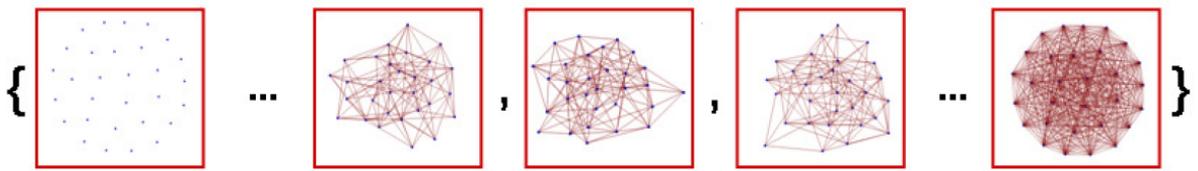
dynamics can be linked with ABM work on opinion dynamics in the social sciences (13) to understand how propagation of opinions through social networks affects emergent macro behavior, which is crucial to managing the stability and resilience of socioeconomic systems.

A simple behavioral model, with agents gradually switching to better performing heuristics, explains individual, as well as emergent, macro behavior in these laboratory economies. The experiments also provide a general mechanism for managing social contagion in such systems. For example,

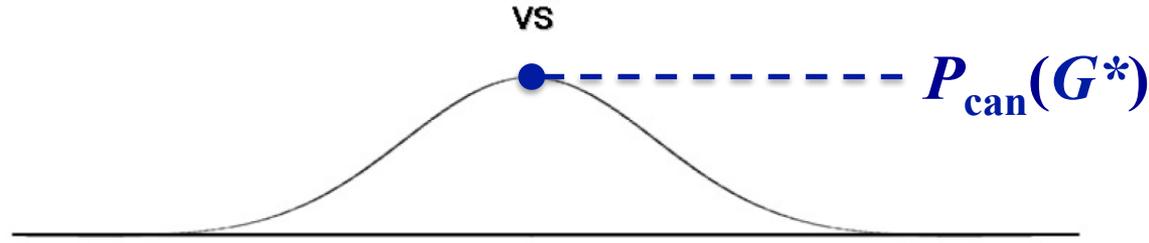
# Challenge 2: ensemble nonequivalence

$$s \equiv \lim_{N \rightarrow \infty} \frac{S_N(P_{\text{mic}} || P_{\text{can}})}{N} = 0 \iff \lim_{N \rightarrow \infty} \frac{1}{N} \ln P_{\text{mic}}(\mathbf{G}^*) = \lim_{N \rightarrow \infty} \frac{1}{N} \ln P_{\text{can}}(\mathbf{G}^*)$$

Microcanonical ensemble,  $P_{\text{mic}}(\mathbf{G})$



Canonical ensemble,  $P_{\text{can}}(\mathbf{G})$



vs

# Summary/conclusions

- 1) **binary** graphs are often **well** reconstructed from degrees
- 2) **weighted** graphs are **badly** reconstructed from strengths
- 3) weighted graphs require **topological prior info** (degrees)
- 4) strengths+degrees = **BOSE-FERMI** = Enhanced CM
- 5) degrees can be **inferred** from strengths (and n. of links)
- 6) reconstruction may **deteriorate** as **crises** approach
- 7) statistical ensembles are **not equivalent**