The Recoverability of Network Controllability

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Outline

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▪ 2. Purpose
▪ 3. Work
▪ 4. Conclusion
Introduction
The Recoverability of Controllability

What is Controllability?

What is Controllability of Networks?

- Control Theory + Network Science
- Driver nodes

The Recoverability of Controllability

How?

Controllability of Networks $\rightarrow$ Maximum Matching problem

✓ Number of driver nodes ($N_D$) = Unmatched nodes

Controllability of Networks in Analytical Expression

✓ Fraction of driver nodes ($n_D$): $n_D = G_{out}(1 - \hat{w}_1) + G_{in}(w_2) - 1 + k\hat{w}_1(1 - w_2)$

The Recoverability of Controllability

\[ R = \frac{1 - n_D}{1 - n_{D_0}} \quad R \in [0,1] \]

- \( n_{D_0} \): fraction of driver nodes at the start
- \( n_D \): fraction of driver nodes during the attack/recovery process

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- **Scenario A:** recovery of any alternative link
- **Scenario B:** recovery of attacked links
Purpose
Purpose

1. Analytically express the controllability for the network after removing/adding links

2. Analytically approximate $n_D$ and compute R-value during attack process, and recovery process in Scenario A & B

3. Compare different recovery strategies in Scenario A & B
Work
Networks used in work

Swarm Signalling Networks (SSNs):

- $P(k_{out}) = \delta(k - k_{out})$
- $P(k_{in}) = e^{-k} \frac{k^{k_{in}}}{k_{in}!}$
- $G_{out}(x) = x^k$
- $G_{in}(x) = e^{-k(1-x)}$

SSN after removing links

\[ n_D = (p + (1 - p)(1 - e^{-k(1-p)(1-w_2)}))k - 1 + e^{-k(1-p)(1-w_2)} + k(1 - p)(1 - w_2)e^{-k(1-p)(1-w_2)} \]

\[ p = \frac{m}{L} \]
SSN after adding links

\[ n_D = e^{-\tilde{k}(1-w_2)} + (1 - e^{-\tilde{k}(1-w_2)}) k (1 - f e^{-\tilde{k}(1-w_2)}) \frac{N-1-k}{1 + \tilde{k}(1-w_2)e^{-\tilde{k}(1-w_2)}} \]

\[ \tilde{k} = k + f (N - 1 - k) \]

\[ f = \frac{m}{N(N-1)-L} \]

SSN(N=10,000) add f-percent links

![Graph showing the fraction of driver nodes vs. degree with analytical values and simulations for f=3 \times 10^{-5}.](image)
Recoverability

How to express the degree distributions after removal/recovery of links?

Scenario A: recovery of any alternative link

\[ G(x) \begin{cases} 
\text{attack: } \tilde{G}(x) = G(p + (1 - p)x) \\
\text{recovery: } \bar{G}(x) = (1 - f(1 - x))^{N-1} \bar{G}\left(\frac{x}{1 - f(1 - x)}\right) 
\end{cases} \]

Scenario B: recovery of attacked links

\[ G(x) \begin{cases} 
\text{attack: } \tilde{G}(x) = G(p + (1 - p)x), \quad p = \frac{i}{L}, 0 < i \leq M \\
\text{recovery: } \bar{G}(x) = \tilde{G}(p + (1 - p)x), \quad p = \frac{2M - i}{L}, M < i \leq 2M 
\end{cases} \]
SSN’s recoverability

Scenario A

Original Network

• $G_{out}(x) = x^k$
• $G_{in}(x) = e^{-k(1-x)}$
• $n_d = G_{in}(1 - \omega_1) - 1 + G_{out}(\bar{\omega}_2) + k \cdot \omega_1(1 - \bar{\omega}_2)$

Attack Phase

• $G_{out}(x) = (p + (1 - p)x)^k$
• $G_{in}(x) = e^{-k(1-p)(1-x)}$
• $n_d = G_{in}(1 - \omega_1) - 1 + G_{out}(\bar{\omega}_2) + k(1-p) \cdot \omega_1(1 - \bar{\omega}_2)$

Recovery Phase

• $G_{out}(x) = (1 - f(1-x))^{N-1}(p + (1 - p)\frac{x}{1-f(1-x)})^k$
• $G_{in}(x) = (1 - f(1-x))^{N-1} \cdot e^{-k(1-p)(1-x)}$
• $n_d = G_{in}(1 - \omega_1) - 1 + G_{out}(\bar{\omega}_2) + (k(1-p) + f(N - 1 - k(1-p))) \cdot \omega_1(1 - \bar{\omega}_2)$
SSN’s recoverability

Scenario B

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Original Network

\[ G_{out}(x) = x^k \]
\[ G_{in}(x) = e^{-k(1-x)} \]
\[ n_d = G_{in}(1 - \omega_1) - 1 + G_{out}(\omega_2) + k \cdot \omega_1(1 - \omega_2) \]

Attack Phase

\[ G_{out}(x) = (p + (1 - p)x)^k \]
\[ G_{in}(x) = e^{-k(1-p)(1-x)} \]
\[ n_d = G_{in}(1 - \omega_1) - 1 + G_{out}(\omega_2) + k(1 - p) \cdot \omega_1(1 - \omega_2) \]
\[ p = \frac{i}{L}, 0 < i \leq M \]

Recovery Phase

\[ G_{out}(x) = (p + (1 - p)x)^k \]
\[ G_{in}(x) = e^{-k(1-p)(1-x)} \]
\[ n_d = G_{in}(1 - \omega_1) - 1 + G_{out}(\omega_2) + k(1 - p) \cdot \omega_1(1 - \omega_2) \]
\[ p = \frac{2M-i}{L}, M < i \leq 2M \]
SSN’s Recoverability

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SSN N=500 k=2 R-threshold=0.9
Recover Strategies-scenario A

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Random: 15.2 s/time
Greedy: 8531.9 s/time
Connect: 0.04 s/time
1. Finding independent matching paths
2. Ordering them
3. Linking two independent matching paths in order in each step
Recover strategies-scenario B

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Conclusion
Conclusion

✓ Analytical express $n_D$ after removing/adding links

✓ Analytical express $n_D$ during attack/recovery process in two different scenarios

✓ Connect Strategy in Scenario A & Greedy Strategy in Scenario B
Thanks!
Q&A